Forecast targeting

An example with forward-looking variables

(Svensson-Woodford 99, “Implementing optimal policy through inflation-forecast targeting”)

- Inflation-targeting monetary-policy rules that achieve (socially) optimal equilibrium
  - Transparency (relation to ultimate goals)
  - Robustness (to model perturbations)
  - Determinacy

Preview of results

- General targeting rules
  - Minimize social loss function under discretion: Inefficient
  - Minimize modified loss function: Consistent with optimal equilibrium
    * Determinacy, if additional information (announced forecasts)

- Specific targeting rules
  - First-order condition: Consistent with optimal equilibrium
    * Determinacy, if additional information (announced forecasts)

- Instrument rules
  - Exogenous interest rate: Indeterminacy
  - Respond to first-order condition: Determinacy

- Transparent relation to goals
  - Higher-level formulations

- Robustness
  - Higher-level formulations

- Determinacy
  - Lower-level formulations

- “Hybrid” rule attractive?
Outline
2 The model: Social optimum
3 Commitment to modified loss function
4 Commitment to specific targeting rule
5 Commitment to explicit instrument rule
6 Concluding remarks: Optimality, determinacy, transparency, robustness

2 The model
Forward-looking aggregate-supply equation (AS), inflation predetermined
\[ \pi_{t+1} = \beta \pi_{t+2|t} + \kappa x_{t+1|t} + u_{t+1} \] (2.1)
\[ z_{t+2|t} = \text{E}_t z_{t+2|t} \]
Exogenous cost-push shock, special case AR(1)
\[ u_{t+1} = \rho u_t + \zeta_{t+1} \] (2.2)
Forward-looking aggregate-demand equation (IS), output gap predetermined
\[ x_{t+1} = x_{t+2|t} - \sigma (i_{t+1|t} - \pi_{t+2|t} - r^n_{t+1}) \] (2.3)
Exogenous natural interest rate, special case AR(1)
\[ r^n_{t+1} = \omega r^n_t + \eta_{t+1} \] (2.4)
Realized values and private-sector plans
\[ \pi_{t+1} = \pi_{t+1|t} + u_{t+1} - u_{t+1|t} \] (2.5)
\[ x_{t+1} = x_{t+1|t} + \sigma (r^n_{t+1} - r^n_{t+1|t}) \] (2.6)
- Private sector plans (active decisions): \( \pi_{t+1|t}, x_{t+1|t} \)
- Private sector expectations (passive): \( i_{t+1|t}, \pi_{t+1|t}, x_{t+1|t}, \tau \geq 2 \).
- Central bank instrument (plan): \( i_{t+1|t} \)
Intertemporal social loss function

\[
E \sum_{t=0}^{\infty} \beta^{t-t_0} L_t
\]  

(2.7)

Period loss function (flexible inflation targeting)

\[
L_t = \frac{1}{2}[\pi_t^2 + \lambda(x_t - x^*)^2]
\]

(2.8)

Information in period \( t \)

- Central bank: \( h_{t-1}, u_t, r^n_t \). Decides \( i_{t+1 \mid t} \)
- Private sector: \( h_{t-1}, u_t, r^n_t, i_{t+1 \mid t} \). Decides \( \pi_{t+1 \mid t}, x_{t+1 \mid t} \)
- No circularity

Model extremely forward-looking

- No (essential) endogenous predetermined variable
- Exaggerates commitment/discretion difference, indeterminacy problem

State-space form

- Predetermined variables: \( u_t, r^n_t, \pi_t, x_t \)
- Forward-looking variables: \( \pi_{t+1 \mid t}, x_{t+1 \mid t} \)
- Instrument: \( i_{t+1 \mid t} \)
- Target variables: \( \pi_t, x_t \)

\[
\begin{align*}
\pi_{t+2 \mid t} &= \frac{1}{\beta} \pi_{t+1 \mid t} - \frac{\kappa}{\beta} x_{t+1 \mid t} - \frac{1}{\beta} u_{t+1 \mid t} \\
&= \frac{1}{\beta} \pi_{t+1 \mid t} - \frac{\kappa}{\beta} x_{t+1 \mid t} - \rho u_t \\
x_{t+2 \mid t} &= x_{t+1 \mid t} + \sigma i_{t+1 \mid t} - \sigma \pi_{t+2 \mid t} - \sigma r^n_{t+1 \mid t} \\
&= x_{t+1 \mid t} + \sigma i_{t+1 \mid t} - \sigma \left( \frac{1}{\beta} \pi_{t+1 \mid t} - \frac{\kappa}{\beta} x_{t+1 \mid t} - \rho u_t \right) - \sigma \omega r^n_t
\end{align*}
\]

\[
\begin{bmatrix}
u_{t+1} \\
r^n_{t+1} \\
\pi_{t+1} \\
x_{t+1} \\
\pi_{t+2 \mid t} \\
x_{t+2 \mid t}
\end{bmatrix} =
\begin{bmatrix}
\rho & 0 & 0 & 0 & 0 & 0 \\
0 & \omega & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-\frac{\rho}{\beta} & 0 & 0 & 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} \\
\frac{\sigma \rho}{\beta} & -\sigma \omega & 0 & 0 & -\frac{\sigma}{\beta} & 1 + \frac{\sigma \kappa}{\beta}
\end{bmatrix}
\begin{bmatrix}
u_t \\
r^n_t \\
\pi_t \\
x_t \\
\pi_{t+1 \mid t} \\
x_{t+1 \mid t}
\end{bmatrix}
+ \begin{bmatrix}0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}i_{t+1 \mid t} \\
\varepsilon_{t+1}
\end{bmatrix}
+ \begin{bmatrix}0 \\
\eta_{t+1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\pi_t \\
x_t
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
u_t \\
r^n_t \\
\pi_t \\
x_t \\
\pi_{t+1 \mid t} \\
x_{t+1 \mid t}
\end{bmatrix}
+ \begin{bmatrix}0 \\
0 \lambda
\end{bmatrix}
\begin{bmatrix}\pi_t \\
x_t
\end{bmatrix}
\]

\[
L_t = \frac{1}{2} \begin{bmatrix}\pi_t & x_t \end{bmatrix} \begin{bmatrix}1 & 0 \\
0 & \lambda
\end{bmatrix} \begin{bmatrix}\pi_t \\
x_t
\end{bmatrix}
\]
Simplify model: Consider \(x_{t+1|t}\) the instrument

- Predetermined variables: \(u_t, r_t^n, \pi_t, x_t\)
- Forward-looking variable: \(\pi_{t+1|t}\)
- Instrument: \(x_{t+1|t}\)
- Target variables: \(\pi_t, x_t\)

\[
\begin{bmatrix}
    u_{t+1} \\
    r_t^n \\
    \pi_{t+1} \\
    x_{t+1} \\
    \pi_{t+2|t}
\end{bmatrix} =
\begin{bmatrix}
    \rho & 0 & 0 & 0 & 0 \\
    0 & \omega & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 \\
    0 & 0 & 0 & 0 & 0 \\
    -\beta & 0 & 0 & 0 & \frac{1}{\beta}
\end{bmatrix}
\begin{bmatrix}
    u_t \\
    r_t^n \\
    \pi_t \\
    x_t \\
    \pi_{t+1|t}
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    0 \\
    1 \\
    0 \\
    0
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_{t+1} \\
    \eta_{t+1} \\
    \varepsilon_{t+1} \\
    \sigma \eta_{t+1} \\
    0
\end{bmatrix}
+ \begin{bmatrix}
    x_{t+1|t}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \pi_t \\
    x_t
\end{bmatrix} =
\begin{bmatrix}
    0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
    u_t \\
    r_t^n \\
    \pi_t \\
    x_t \\
    \pi_{t+1|t}
\end{bmatrix}
\]

\[
L_t = \frac{1}{2} \begin{bmatrix}
    \pi_t & x_t
\end{bmatrix}
\begin{bmatrix}
    1 & 0 \\
    0 & \lambda
\end{bmatrix}
\begin{bmatrix}
    \pi_t \\
    x_t
\end{bmatrix}
\]

2.1 The (socially) optimal equilibrium

(Private-sector) equilibrium: Stochastic processes \(\{\pi_t, x_t, i_t\}\) fulfilling (2.1) and (2.3) [and (2.5) and (2.6)]

\[
E_t \left[ L_{t+1} \right] = \frac{1}{2} \left[ \pi_{t+1|t+1} + \lambda (x_{t+1|t+1} - \bar{x})^2 \right]
\]

\[
= \frac{1}{2} \left[ \left( \pi_{t+1|t} + \varepsilon_{t+1} \right)^2 + \lambda (x_{t+1|t} + \sigma \eta_{t+1} - \bar{x})^2 \right]
\]

\[
= \frac{1}{2} \left[ \pi_{t+1|t} + \lambda (x_{t+1|t} - \bar{x})^2 \right] + \frac{1}{2} \left( \sigma^2 + \lambda \sigma^2 \sigma^2 \right)
\]

\[
t_0\text{-optimality}
\]

Commitment in period \(t_0\) for \(t \geq t_0\)

\[
\min E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t+1-t_0} \frac{1}{2} \left[ \pi_{t+1|t} + \lambda (x_{t+1|t} - \bar{x})^2 \right]
\]

Restriction (AS equation)

\[
\pi_{t+1|t} = \beta \pi_{t+2|t} + \kappa x_{t+1|t} + u_{t+1|t}
\]

(2.9)

Lagrangian

\[
\mathcal{L}_{t_0} \equiv E \sum_{t=t_0}^{\infty} \beta^{t+1-t_0} \left\{ \frac{1}{2} \pi_{t+1|t}^2 + \lambda (x_{t+1|t} - \bar{x})^2 \right\} + \varphi_{t+1} \left[ \beta \pi_{t+2|t} + \kappa x_{t+1|t} + u_{t+1|t} - \pi_{t+1|t} \right]
\]

(2.10)

Relate the Lagrange multiplier \(\varphi_t\) to the previous multiplier \(\Xi_t\),

\[
\varphi_{t+1} \equiv \Xi_t.
\]

Note that \(\varphi_{t+1}\) depends on information available in period \(t\).
First-order conditions \((t \geq t_0)\)

\[
\pi_{t+1|t} - \varphi_{t+1} + \varphi_t = 0, \tag{2.11}
\]

\[
\lambda(x_{t+1|t} - x^*) + \kappa \varphi_{t+1} = 0, \tag{2.12}
\]

Initial condition

\[
\varphi_{t_0} = 0. \tag{2.13}
\]

Difference equation for \(\varphi_{t+1}^{'}\)

\[
\varphi_{t+2|t} - \frac{1}{\alpha\beta}\varphi_{t+1} + \frac{1}{\beta}\varphi_t = -\frac{1}{\beta}u_{t+1|t} - \frac{\kappa}{\beta}x^*, \tag{2.14}
\]

\[
0 < \alpha \equiv \frac{\lambda}{\kappa^2 + \lambda(1 + \beta)} < 1.
\]

Characteristic equation,

\[
\mu^2 - \frac{1}{\alpha\beta}\mu + \frac{1}{\beta} = 0, \tag{2.15}
\]

Two roots (eigenvalues), \(\mu \equiv (1 - \sqrt{1 - 4\alpha^2\beta^2})/(2\alpha\beta)\) and \(1/(\beta\mu)\), such that

\[
0 < \mu < 1 < 1/\beta < 1/(\beta\mu),
\]

Solution

\[
\varphi_{t+1} - \varphi^* = \mu \sum_{j=0}^{\infty} (\beta\mu)^j u_{t+1+j|t} + \mu(\varphi_t - \varphi^*) \tag{2.16}
\]

\[
\varphi^* \equiv \frac{\lambda}{\kappa}x^*
\]

Special case, \(u_t\) AR(1)

\[
\varphi_{t+1} - \varphi^* = \frac{\rho\mu}{1 - \beta\rho\mu}u_t + \mu(\varphi_t - \varphi^*) \tag{2.17}
\]

\[
= \frac{\rho\mu}{1 - \beta\rho\mu} \sum_{j=0}^{t-0} \mu^j u_{t-j} - \mu^{t+1-t_0}\varphi^* \tag{2.18}
\]

\[
\pi_{t+1|t} = \frac{\rho\mu}{1 - \beta\rho\mu}u_t - (1 - \mu)(\varphi_t - \varphi^*) \tag{2.19}
\]

\[
= \frac{\rho\mu}{1 - \beta\rho\mu} \left[ u_t - (1 - \mu) \sum_{j=1}^{t-0} \mu^{j-1} u_{t-j} \right] + (1 - \mu)\mu^{t-t_0}\varphi^* \tag{2.20}
\]

\[
x_{t+1|t} = -\frac{\kappa}{\lambda} \frac{\rho\mu}{1 - \beta\rho\mu}u_t - \frac{\kappa}{\lambda}\mu(\varphi_t - \varphi^*) \tag{2.21}
\]

\[
- \frac{\kappa}{\lambda} \frac{\rho\mu}{1 - \beta\rho\mu} \sum_{j=0}^{t-0} \mu^j u_{t-j} + \frac{\kappa}{\lambda}\mu^{t+1-t_0}\varphi^* \tag{2.22}
\]

t_0\)-optimality: Dependence on \(t_0\), one-time surprise
2.2 Optimality from a “timeless perspective”

Time-less perspective (Woodford 99d): Select policy rule for $t \geq t_0$ to which it would have been optimal to commit oneself in a period far in the past.

Select $\varphi_{t_0}$ accordingly

Let $t_0 \rightarrow -\infty$ in (2.18), $\varphi_{t+1} - \varphi^* = \frac{\rho\mu}{1 - \beta \rho \mu} \sum_{j=0}^{\infty} \mu^j u_{t-j}$

Select $\varphi_{t_0}$: $\varphi_{t_0} - \varphi^* = \frac{\rho\mu}{1 - \beta \rho \mu} \sum_{j=0}^{\infty} \mu^j u_{t_0-1-j}$

Resulting $\pi_{t+1|t}$, $x_{t+1|t}$

\[
\pi_{t+1|t} = \frac{\rho \mu}{1 - \beta \rho \mu} \left[ u_{t} - (1 - \mu) \sum_{j=1}^{\infty} \mu^{j-1} u_{t-j} \right], \quad (2.23)
\]

\[
x_{t+1|t} = \frac{\kappa \rho \mu}{\lambda (1 - \beta \rho \mu)} \sum_{j=0}^{\infty} \mu^j u_{t-j}. \quad (2.24)
\]

Optimal equilibrium

- No average inflation bias
- History-dependence

2.3 Interest rates in an optimal equilibrium

Rewrite IS equation

\[
i_{t+1|t} = r_{t+1|t}^n + \pi_{t+2|t} + \frac{1}{\sigma} (x_{t+2|t} - x_{t+1|t}) \quad (2.27)
\]

Substitute equilibrium values, special case $r_t^n$ AR(1)

\[
i_{t+1|t} = i^*_{t+1} \equiv \omega r_{t}^n + \frac{\lambda \sigma - \kappa}{\lambda \sigma} \frac{\rho \mu}{1 - \beta \rho \mu} \left[ p u_t - (1 - \mu) \sum_{j=0}^{\infty} \mu^j u_{t-j} \right] \quad (2.29)
\]

Assume

\[
i_{t+1} = i_{t+1|t}
\]
2.4 The problem of indeterminacy

Keep track of determinacy properties for different policy rules
Consider commitment to reaction function $i_{t+1} = i^*_{t+1}$, exogenous
Combine with expectation of (2.1) and (2.3). Dynamic system

$$z_{t+1|t} = Mz_t + Ns_t,$$

$$z_t = \begin{bmatrix} \pi_t+1|t \\ x_t+1|t \end{bmatrix}, \quad s_t = \begin{bmatrix} u_t+1|t \\ r_t+1|t \\ i_t+1|t \end{bmatrix},$$

$$M = \begin{bmatrix} 1/\beta & -\kappa/\beta \\ -\sigma/\beta & 1 + \kappa\sigma/\beta \end{bmatrix}.$$

Characteristic equation [Note correction!]

$$\nu^2 - \frac{1 + \beta + \kappa\sigma}{\beta} \nu + \frac{1}{\beta} = 0$$

Eigenvalues, $|\nu_1|, |\nu_2| > 1$ if and only if

$$g_\pi + \frac{1 - \beta}{\kappa}g_x > 1$$

“Taylor principle” (Woodford): $di/d\pi > 1$.

Next: Consequences of alternative decision procedures for monetary policy: “Forecast-targeting” rules

Example: Taylor rule (not optimal, since no history-dependence)

$$i_{t+1} = g_\pi \pi_{t+1|t} + g_x x_{t+1|t}, \quad g_x, g_\pi \geq 0$$

$$M = \begin{bmatrix} 1/\beta & -\kappa/\beta \\ -\sigma/\beta & 1 + \kappa\sigma/\beta + \sigma g_x \end{bmatrix}.$$
3 Commitment to a modified loss function

3.1 Forecast targeting

Construct conditional inflation and output-gap forecasts in period \( t \) for alternative interest-rate paths

Interest rate path: \( i_t^\tau \equiv \{i_{t+\tau,t}\}_{\tau=1}^{\infty} \)

Inflation and output-gap (mean) forecasts \( \pi_t^\tau \equiv \{\pi_{t+\tau,t}\}_{\tau=1}^{\infty}, x_t^\tau \equiv \{x_{t+\tau,t}\}_{\tau=1}^{\infty} \)

Distinguish forecasts and private-sector expectations

Forecast model (for \( \tau \geq 1 \))

\[
\pi_{t+\tau,t} = \beta \pi_{t+\tau+1,t} + \kappa x_{t+\tau,t} + u_{t+\tau,t} \\
x_{t+\tau,t} = x_{t+\tau+1,t} - \sigma (i_{t+\tau,t} - \pi_{t+\tau+1,t} - \rho_{t+\tau,t})
\]  

(3.1)

\[
u_t^\tau \equiv \{u_{t+\tau,t}\}_{\tau=1}^{\infty}, \rho_t^\tau \equiv \{\rho_{t+\tau,t}\}_{\tau=1}^{\infty} \]

\[u_{t+\tau,t} \equiv E_t u_{t+\tau,t}, \rho_{t+\tau,t} \equiv E_t \rho_{t+\tau,t} \]

- Feasible set of conditional forecasts, \( \mathcal{Y}_t \)

\[
\pi_{t+\tau,t} = \beta \pi_{t+\tau+1,t} + \kappa x_{t+\tau,t} + u_{t+\tau,t} \\
= \beta^T \pi_{t+\tau+T,t} + \sum_{j=0}^{T-1} \beta^j \kappa x_{t+\tau+j,t} + \sum_{j=0}^{T-1} \beta^j u_{t+\tau+j,t}
\]

Restrict \( \pi_{t+\tau+T,t} \to 0 \) when \( T \to \infty \)

\[
\pi_{t+\tau,t} = \sum_{j=0}^{\infty} \beta^j \kappa x_{t+\tau+j,t} + \sum_{j=0}^{\infty} \beta^j u_{t+\tau+j,t} \\
= \sum_{j=0}^{\infty} \beta^j \kappa x_{t+\tau+j,t} + \frac{1}{1-\beta \rho} u_{t+\tau,t} \\
= \sum_{j=0}^{\infty} \beta^j \kappa x_{t+\tau+j,t} + \frac{\rho^j}{1-\beta \rho} u_t
\]

(3.1)

\[ (\ast) \]

\[ \mathcal{Y}_t = \{\pi^\tau, x^\tau | \pi^\tau \text{ and } x^\tau \text{ fulfill } (\ast) \text{ and are bounded} \} \]

The corresponding \( i^\tau \) follows from (3.2)

3.2 Minimizing the social loss function under discretion

Evaluate forecasts with social loss function

\[
L_{t+\tau,t} = \frac{1}{2} [\pi_{t+\tau,t}^2 + \lambda (x_{t+\tau,t} - x^*)^2] \\
\min_{\tau=1}^{\infty} \sum_{\tau=1}^{\infty} \beta^\tau L_{t+\tau,t}
\]  

(3.3)

Find \( \pi^\tau \) and \( x^\tau \), given (3.1). Then find \( i^\tau \) from (3.2)

\[
i_{t+\tau,t} = r_{t+\tau,t}^{\tau} + \pi_{t+\tau+1,t} + \frac{1}{\sigma} (x_{t+\tau+1,t} - x_{t+\tau,t})
\]

(3.5)

Lagrangian

\[
L_t \equiv \sum_{\tau=1}^{\infty} \beta^\tau \left\{ \frac{1}{2} [\pi_{t+\tau,t}^2 + \lambda (x_{t+\tau,t} - x^*)] + \varphi_{t+\tau,t} (\beta \pi_{t+\tau+1,t} + \kappa x_{t+\tau,t} + u_{t+\tau,t} - \pi_{t+\tau,t}) \right\}
\]

(3.6)
First-order conditions \((\tau \geq 1)\)

\[
\begin{align*}
\pi_{t+\tau,t} - \varphi_{t+\tau,t} + \varphi_{t+\tau-1,t} & = 0, \quad (3.7) \\
\lambda(x_{t+\tau,t} - x^*) + \kappa \varphi_{t+\tau,t} & = 0 \quad (3.8)
\end{align*}
\]

Initial condition

\[
\varphi_{t,t} = 0 \quad (3.9)
\]

For \(\tau = 1\)

\[
\pi_{t+1,t} - \varphi_{t+1,t} = 0
\]

“Surprise” in period \(t\). Reoptimize in period \(t+1\), “surprise” also in period \(t+1\).

First-order condition from reoptimization in period \(t + \tau\)

\[
\pi_{t+1+\tau,t+\tau} - \varphi_{t+1+\tau,t+\tau} = 0
\]

Internalize in period \(t\)

\[
\pi_{t+\tau,t} - \varphi_{t+\tau,t} = 0 \quad (3.13)
\]

Use (3.8) and (3.13) in (3.1)

\[
\varphi_{t+\tau,t} = \frac{\beta \lambda}{\kappa^2 + \lambda} \varphi_{t+\tau-1,t} + \frac{\lambda}{\kappa^2 + \lambda} u_{t+\tau,t} + \frac{\lambda \kappa}{\kappa^2 + \lambda} x^*
\]

Solve forward, special case, \(u\) AR(1)

\[
\varphi_{t+\tau,t} = \frac{\lambda \kappa}{\kappa^2 + \lambda (1 - \beta)} x^* + \frac{\lambda}{\kappa^2 + \lambda (1 - \beta \rho)} \rho^\tau u_t
\]

Resulting \(\pi^t, x^t\)

\[
\begin{align*}
\pi_{t+\tau,t} & = \frac{\lambda \kappa}{\kappa^2 + \lambda (1 - \beta)} x^* + \frac{\lambda}{\kappa^2 + \lambda (1 - \beta \rho)} \rho^\tau u_t, \\
x_{t+\tau,t} & = -\frac{\lambda \kappa}{\kappa^2 + \lambda (1 - \beta \rho)} \rho^\tau u_t.
\end{align*}
\]

Instrument plan \(i^t\)

\[
i_{t+\tau,t} = \omega^\tau r_t + \frac{\rho \lambda + (1 - \rho) \kappa / \sigma}{\kappa^2 + \lambda (1 - \beta \rho)} \rho^\tau u_t. \quad (3.16)
\]

Corresponds to inefficient discretion equilibrium

- Average inflation bias, if \(x^* > 0\)
- Stabilization bias
- No history-dependence
3.3 Minimizing a modified loss function

Modify period loss function for $\tau = 1$

$$\hat{L}_{t+1,t} = \frac{1}{2}[\pi_{t+1,t}^2 + \lambda(x_{t+1,t} - x^*)^2] + \varphi_{t,t-1}(\pi_{t+1,t} - \pi_{t+1,t-1})$$  (3.19)

$$\min \beta \bar{L}_{t+1,t} + \sum_{\tau=2}^{\infty} \beta^\tau L_{t+\tau,t}$$

Initial condition (3.9) for $\tau = 1$ replaced by

$$\varphi_{t,t} = \varphi_{t,t-1}$$  (3.20)

Results in first-order conditions corresponding to social optimum

- No average inflation bias
- No stabilization bias
- History-dependence
- Internalize cost of deviation from previous forecast
- $\varphi_{t,t-1}$ marginal loss in period $t - 1$ of increasing forecast $\pi_{t+1,t-1}$
- Transparency, predictability, continuity (King)
- Not deviate w/o good reason
3.3.1 An explicit decision procedure

In period $t$

- Recall $i_{t,t-1}$, $\varphi_{t,t-1}$. Set $i_t = i_{t,t-1}$. Observe $u_t, r_t^n$. Form $u^t, r^{nt}$
- Calculate $\pi^t, x^t, i^t$, record $i_{t+1,t}, \varphi_{t+1,t}$

3.3.2 The implied reaction function and determinacy

Implied reaction function

$$i_{t+1} = i^*_t$$

- $i^*_{t+1}$ exogenous, indeterminacy, optimal equilibrium one of many equilibria

Additional announcements, transparency, focal point

- CB announces $\pi_{t+2,t}, x_{t+2,t}, i_{t+1,t}$
- If announcements credible, private-sector expectations fulfill
  $$\pi_{t+2|t} = \pi_{t+2,t}, x_{t+2|t} = x_{t+2,t}, \ i_{t+1|t} = i_{t+1,t}$$
- Private sector determines $\pi_{t+1|t}, x_{t+1|t}$

4 Commitment to a specific targeting rule

Condition for target variables (forecasts), no optimizing

Consolidated first-order condition for optimal equilibrium

$$\pi_{t+1|t} + \frac{\lambda}{\kappa}(x_{t+1|t} - x_{t|t-1}) = 0 \quad (2.28)$$

Forecasts

$$[\pi_{t+1|t} + \frac{\lambda}{\kappa}(x_{t+1|t} - x_{t|t-1})]_t = 0 \quad (4.1)$$

Targeting rule for CB forecasts

$$\pi_{t+1,t} + \frac{\lambda}{\kappa}(x_{t+1,t} - x_{t|t-1}) = 0 \quad (4.2)$$

Find $\pi^t$ and $x^t$ that fulfill (3.1) and (4.2). Difference equation for $x_{t+\tau,t}$ $(\tau \geq 1)$

$$x_{t+\tau+1,t} - \frac{1}{\alpha \beta} x_{t+\tau,t} + \frac{1}{\beta} x_{t+\tau-1,t} = \frac{\kappa}{\lambda \beta} u_{t+\tau,t},$$

Roots $\mu$ and $1/(\beta \mu)$
Solution

\[ x_{t+\tau,t} = -\frac{\kappa}{\lambda} \frac{\rho \mu}{1 - \beta \rho \mu} \frac{\mu^\tau - \rho^\tau}{\mu - \rho} u_t + \mu^\tau x_{t,t-1} \]

\[ \pi_{t+\tau,t} = \frac{\rho \mu}{1 - \beta \rho \mu} \frac{(1 - \rho)\rho^{\tau-1} - (1 - \mu)\mu^{\tau-1}}{\mu - \rho} u_t + \frac{\lambda}{\kappa} (1 - \mu)\mu^{\tau-1} x_{t,t-1} \]

Instrument path

\[ i_{t+\tau,t} = \omega^\tau r_t^n + \frac{\lambda \sigma - \kappa}{\lambda \sigma} \frac{\rho \mu}{1 - \beta \rho \mu} \frac{(1 - \rho)\rho^{\tau} - (1 - \mu)\mu^{\tau}}{\mu - \rho} u_t \]

\[ + \frac{\lambda \sigma - \kappa}{\kappa \sigma} (1 - \mu)\mu^{\tau} x_{t,t-1}. \]

Implied reaction function

\[ i_{t+1} = \bar{i}_{t+1} \equiv \omega^\tau r_t^n + \frac{\lambda \sigma - \kappa}{\lambda \sigma} \frac{\rho \mu}{1 - \beta \rho \mu} (\rho + \mu - 1) u_t + f x_{t,t-1} \quad (2.30, 4.3) \]

\[ f \equiv \frac{\lambda \sigma - \kappa}{\kappa \sigma} (1 - \mu) \mu \quad (2.31) \]

4.1 Determinacy under the specific targeting rule

Implied reaction function results in determinacy if and only if

\[ f \equiv \frac{\lambda \sigma - \kappa}{\kappa \sigma} (1 - \mu) \mu > \frac{\kappa}{1 - \beta} \quad (4.5) \]

(Taylor principle)

Announce more, credible announcements, focal point

- Announce targeting rule
- Announce forecasts

4.2 A hybrid targeting-instrument rule

(Perceived) CB reaction to out-of-equilibrium behavior

\[ i_{t+1} = \bar{i}_{t+1} + g \left[ \pi_{t+1}\mid_t + \frac{\lambda}{\kappa} (x_{t+1}\mid_t - x_{t,t-1}) \right] \quad (4.7) \]

Determinacy if \( g > \bar{g} > 0 \)
5 Commitment to an explicit instrument rule

No optimizing, no forecasts

“Interest-rate targeting,” \( \hat{i}_{t+1} \) interest rate target

\[
L_{t+1} = \frac{1}{2} (\hat{i}_{t+1} - \hat{i}_{t+1})^2
\]

First-order condition

\[
i_{t+1} = \hat{i}_{t+1}
\]

1. Commit to implied reaction function \( i_{t+1}^* \)

\[
i_{t+1} = \hat{i}_{t+1}^*\]

Indeterminacy (\( i_{t+1}^* \) exogenous)

2. Respond to consolidated first-order condition

\[
i_{t+1} = g \left[ \pi_{t+1|t} + \frac{\lambda}{\kappa} (x_{t+1|t} - x_{t|t-1}) \right]
\] (5.1)

Determinacy if and only if \( g > 1 \) (Taylor principle)

Not optimal unless \( g \to \infty \), mistakes!

3. Alternative

\[
i_{t+1} = \hat{i}_{t+1}^* + g \left[ \pi_{t+1|t} + \frac{\lambda}{\kappa} (x_{t+1|t} - x_{t|t-1}) \right]
\] (5.2)

Determinacy if and only if \( g > 1 \) (Taylor principle)

4. Hybrid rule

\[
i_{t+1} = \tilde{i}_{t+1} + g \left[ \pi_{t+1|t} + \frac{\lambda}{\kappa} (x_{t+1|t} - x_{t|t-1}) \right]
\] (5.3)

Determinacy if \( g > \bar{g} \).
6 Concluding remarks

Optimality

• Several alternatives

Determinacy

• Problem for higher-level formulations
  – Mitigate by transparency, focal point
• Instrument rules: Respond to deviations from targeting rule
  – Hybrid rule
• Model may exaggerate indeterminacy

Transparent connection to ultimate objectives

• Higher-level formulations more transparent
• Minimizing social loss function (inefficient)
• Minimizing modified loss function
  – Internalizing cost of deviating from previous forecast
  – CB thinking and rhetoric
  – More complex model, more forward-looking variables/multipliers
• Specific targeting rule

\[
\pi_{t+1|t} = -\frac{\lambda}{\kappa}(x_{t+1|t} - x_{t|t-1})
\]  

(2.28)

Less transparent

– Price-level targeting (CGG, Svensson, Vestin, Woodford)

\[
p_{t+1|t} = -\frac{\lambda}{\kappa}x_{t+1|t}
\]

• Explicit instrument rules least transparent
Robustness (to model perturbation)

- Higher-level formulations more robust
- Minimizing social loss function robust (degree of inefficiency not robust)
- Minimizing modified loss function robust
  - Number of multipliers not robust
- Specific targeting rule
  - Depends on loss function and AS equation, not on IS equation
  - Robust to number/nature of additive exogenous disturbances
- Instrument rules involving $i_{t+1}^*, \bar{i}_{t+1}$ not robust

Summary

- Suitably designed inflation-forecast targeting rules can achieve social optimum
- Targeting rules more transparent and robust than competing instrument rules
- Hybrid rule potential compromise
- Announcing forecasts, improve determinacy, focal point?