New Keynesian models of open-economy monetary policy

- Example: Clarida-Gali-Gertler 02 (Benigno-Benigno 02)
- Model
  - Two symmetric countries, H and F; different size, $1 - \gamma$ and $\gamma$.
  - Preferences, technologies same, shocks different, specialization of production, consumption of home and foreign goods
  - Complete markets (nominal bonds enough)
  - Production in two stages: final (perfect competition) and intermediate (monopolistic competition)
  - Sticky nominal prices (Calvo), cost-push shocks (exogenous time-varying wage markup), PCP (complete pass-through)
  - Optimization under discretion (Commitment in Benigno-Benigno 02)
- Results
  - Max welfare $\implies$ flexible domestic (rather than CPI) inflation targeting
    * Approximately strict domestic IT? ($\lambda \approx 0$)
  - International Nash equilibrium (noncooperative): Monetary policy isomorphic to closed economy
  - Gains to international cooperation in some cases
  - Benigno-Benigno: Domestic flexible IT under commitment can implement the optimal cooperative equilibrium, even if there are gains to cooperation

Households (home)

- $C_t$ index of consumption of home ($H$) and foreign ($F$) goods:
  \[ C_t \equiv C_{H,t}^{1-\gamma} C_{F,t}^\gamma \]  
- $P_t$ consumer price index (CPI):
  \[ P_t = k^{-1} P_{H,t}^{1-\gamma} P_{F,t}^\gamma \]
  \[ k \equiv (1 - \gamma)^{1-\gamma} \gamma^\gamma \]
- $S_t \equiv \frac{P^H_{F,t}}{P^H_{H,t}}$ terms of trade (TOT improvement $\iff S_t \downarrow$)
- $N_t(h)$ household’s $h$ labor hours, $W_t(h)$ nominal wage
- Preferences household $h$
  \[ E_t \sum_{j=t}^{\infty} \beta^{j-t} [U(C_j) - V(N_j)] \] 
- Sequence of budget constraints
  \[ P_t C_t + E_t \{ Q_{t,t+1} D_{t+1} \} = W_t(h) N_t(h) + D_t - T_t + \Gamma_t \]  
- $D_{t+1}$ random nominal payoff of the portfolio purchased at $t$, $Q_{t,t+1}$ stochastic nominal discount factor (complete risk-sharing), $T_t$ nominal lump sum taxes, $\Gamma_t$ nominal profits from intermediate-goods firms (monopolistic competition)
Demand for $h$ labor (from intermediate-goods firms) (to be derived below)

$$N_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\eta_t} N_t$$ (6)

$N_t$ per capita employment, $W_t \equiv \left( \frac{1}{1-\gamma} \int_0^{1-\gamma} W_t(h)^{1-\eta_t} dh \right)^{-\eta_t}$ wage index, $\eta_t > 1$ exogenous stochastic demand elasticity, same for all $h$

Specific period utility function

$$U(C_t) - V(N_t(h)) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\phi}}{1+\phi}$$ (7)

$\frac{1}{\sigma}$ CIES of consumption ($\sigma$ relative risk aversion), $\frac{1}{\phi}$ CIES of labor

FOCs

$$P_{H,t}C_{H,t} = (1-\gamma)P_tC_t$$ (8)

$$P_{F,t}C_{F,t} = \gamma P_tC_t$$ (9)

$$\beta(C_{t+1}/C_t)^{-\sigma} (P_t/P_{t+1}) = Q_{t+1}$$ (10)

$R_t$ gross nominal yield on a one-period discount bond,

$$R_t^{-1} = E_t Q_{t+1}$$

Expectations of (10): Euler equation

$$1 = \beta R_t E_t [(C_{t+1}/C_t)^{-\sigma} (P_t/P_{t+1})]$$ (11)

FOC for labor supply (monopolistic competition)

$$\frac{W_t(h)}{P_t} = (1 + \mu_t^{h})N_t(h)^\phi C_t^\sigma$$ (12)

$\mu_t^h \equiv \frac{1}{h_t-1}$ optimal wage markup, exogenous stochastic

MRS of $C$ for $N$:

$$\frac{dC}{dN}\frac{d(U-V)}{d(U-V)} = \frac{V_N}{U_C}$$

Wages flexible (different from Erceg-Henderson-Levin 00)

Symmetry: $\forall h \in [0, 1-\gamma], \forall t$

$$W_t(h) = W_t$$

$$N_t(h) = N_t$$ (hours/capita)
Foreign country, similar FOCs

\[
\beta(C_{t+1}^* / C_t^*)^{-\sigma}(P_{t+1}^* / P_t^*) = Q_{t,t+1} \mathcal{E}_{t+1} / \mathcal{E}_t = Q_{t,t+1}^* \tag{13}
\]

$\mathcal{E}_t$ nominal exchange rate

LOOP

\[
P_t = \mathcal{E}_t P_t^* \tag{14}
\]

Suitable initial conditions $\Rightarrow$

\[
C_t = C_t^* \quad \forall t \tag{15}
\]

Perfect risk-sharing

\begin{itemize}
  \item Firms
  \begin{itemize}
  \item Final goods
  \[
  Y_t = \left( \int_0^1 Y_t(f)(\xi + df)^{-\xi} \right)^{\xi / (1 - \xi)} \tag{16}
  \]
  \end{itemize}
  \end{itemize}

$Y_t$ aggregate output of home goods ($H$), $Y_t(f)$ input of intermediate good $f \in [0, 1]$

Quantities measured per capita, $1 - \gamma$

Perfect competition, profit maximizing (cost minimizing), demand for intermediate input $f$

\[
Y_t(f) = \left( \frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\xi} Y_t \tag{17}
\]

Domestic price index

\[
P_{H,t} = \left( \int_0^1 P_{H,t}(f)^{1-\xi} df \right)^{\xi / (1 - \xi)} \tag{18}
\]
– Intermediate goods
  Linear technology
  \[ Y_t(f) = A_t N_t(f) \]  
  \[ A_t \] is an exogenous productivity, \( N_t(f) \) labor aggregate
  \[ N_t(f) = \left( \frac{1}{1-\gamma} \int_0^{1-\gamma} N_t(f, h) \frac{\eta - 1}{\eta} dh \right)^{\frac{\eta}{\eta-1}} \]

\[ N_t(f, h) \] labor \( h \) used in firm \( f \), \( \eta_t > 1 \) exogenous, stochastic
Cost minimizing
  \[ N_t(f, h) = \left( \frac{W_t(h)}{W_t} \right)^{-\eta_t} N_t(f) \]

\( N_t \equiv \int_0^1 N_t(f) df \), \( N_t(h) \equiv \int_0^1 N_t(f, h) df \), results in (6)
Symmetry ⇒ intermediate-goods firms described in terms of \( N_t(f) \) and \( W_t \)

Monopolistic competition
Subsidy \( \tau \) on wage bill (to achieve optimal employment)

Price-setting (Calvo 83), \( 0 < \theta < 1 \) probability not change price in given period

\[ MC_t = \frac{(1-\tau)(W_t/P_{H,t})}{A_t} \]
\[ = \frac{(1-\tau)(W_t/P_t)S_t^\gamma}{kA_t} \]

\( MC_t \) real marginal cost (= average cost), same across all firms

\[ P_{H,t}^0 \] optimal price set in period \( t \)

\[ \max_{P_{H,t}^0} E_t \sum_{j=0}^\infty \theta^j Q_{t,t+j} Y_{t+j}(f) \left( P_{H,t}^0 - P_{H,t+j} MC_{t+j} \right) \]
subject to (17).

\[ Y_{t+j}(f) = \left( \frac{P_{H,t}^0}{P_{H,t+j}} \right)^{-\xi} Y_{t+j} \]

FOC

\[ E_t \sum_{j=0}^\infty \theta^j Q_{t,t+j} Y_{t+j}(f) \left( P_{H,t}^0 - P_{H,t+j} MC_{t+j} \right) = 0 \]

\[ \mu^p \equiv \frac{1}{1-\xi} \] markup
[If flexible prices

\[ \frac{P_{H,t}^0}{P_{H,t}} = (1+\mu^p)MC_t \]

Domestic price index (law of large numbers)

\[ P_{H,t} = \left[ \theta P_{H,t-1}^{1-\xi} + (1-\theta)P_{H,t}^{1-\xi} \right]^{\frac{1}{1-\xi}} \]
• Equilibrium conditional on $Y_t, Y^*_t$  
  First equilibrium conditional on $Y_t$ and $Y^*_t$, then flex-price equilibrium, finally sticky-price equilibrium  
  Goods market clearing, home and foreign countries  
  $$(1 - \gamma) Y_t = (1 - \gamma) C_{H,t} + \gamma C^*_{H,t} \quad (26)$$  
  $$\gamma Y^*_t = (1 - \gamma) C_{F,t} + \gamma C^*_{F,t} \quad (27)$$  
  Demand curves for home and foreign goods by home citizens, (8) and (9), foreign analogues, LOOP (PCP, complete pass-through) results in zero trade balance  
  $$P_{H,t} Y_t = P_t C_t \quad (28)$$  
  $$P^*_{H,t} Y^*_t = P^*_t C^*_t \quad (29)$$  
  Combine (2) and (28), and (14), (15), (28), (29), respectively  
  $$Y_t = k^{-1} C_t S_t^\gamma \quad (30)$$  
  $$S_t = \frac{Y_t}{Y^*_t} \quad (31)$$  
  Combining (30), (31) and (11) $\Rightarrow Y_t$ for given $Y^*_t$ and (in principle) real interest rates $\{r_t\}$  
  $$1 = \beta (1 + r_t) E_t (C_{t+1}/C_t)^{-\sigma}$$  
  Combine (30) and (31)  
  $$C_t = k Y_t^{1-\gamma} Y^*_t^{\gamma} \quad (32)$$

Labor: Note $N_t \equiv \int_0^1 N_t(f) df = \int_0^1 \frac{V_t(f)}{Y_t} df \equiv \frac{Y_t}{V_t} \int_0^1 \left( \frac{V_t(f)}{Y_t} \right) df$, use (17)  
  $$Y_t = A_t N_t \quad \frac{Y_t}{V_t} \quad (33)$$  
  $$V_t \equiv \int_0^1 \left( \frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\xi} df \geq 1$$  
  MC (AC): Use (12), (30) and (33) to eliminate $C_t$ and $N_t$ from (21)  
  $$MC_t = (1 - \tau)(1 + \mu_t) \frac{k^{-1} N_t^\sigma C_t^\sigma S_t^\gamma}{A_t} \quad (34)$$  
  $$= (1 - \tau) k^{\sigma - 1} (1 + \mu_t) A_t^{-(1+\phi)} Y_t^{(Y_t^*)&\phi} V_t^\phi \quad (35)$$  
  $$\nu \equiv \frac{MC_t}{\sigma Y_t}, \nu_0 = \frac{MC_t}{\sigma Y^*_t} (\nu \text{ is } \kappa \text{ in paper})$$  
  $$\nu \equiv \sigma (1 - \gamma) + \gamma + \phi \quad (36)$$  
  $$= \sigma + \phi - \nu_0 \quad (37)$$  
  Combining (36), (37) and (32) international considerations depend on $\nu$ and $\nu_0$  

- $\nu = \frac{\sigma MC_t}{\sigma Y_t} \geq 0$ depending on $\sigma \geq 1$: $Y^*_t \uparrow \Rightarrow$  
  * TOT appreciation reduces MC ($-\gamma$)  
  * Consumption increase increases MC ($\sigma \gamma$) (from (32))  
  * $\sigma = 1 \Rightarrow \nu_0 = 0$ (as in closed economy)  

- $\nu \equiv \frac{MC_t}{\sigma Y^*_t}, Y^*_t \uparrow \Rightarrow$  
  * TOT deterioration increases MC ($-\gamma$)  
  * Consumption increase increases MC ($\sigma (1 - \gamma)$) (from (32))  
  * Production increase increases MDUL ($\phi$)
Home: Determined $C_t$, $S_t$, $MC_t$ and $N_t$ conditional on $Y_t$, $V_t$ (output/price dispersion) and $Y^*_t$  
Foreign: Determined $C^*_t$, $S^*_t(\equiv 1/S_t)$, $MC^*_t$ and $N^*_t$ conditional on $Y^*_t$, $V^*_t$ (output/price dispersion) and $Y_t$  
$Y_t$, $V_t$, $Y^*_t$, $V^*_t$ depends on prices and pricing

- **Flex-price equilibrium**  
  Constant average wage markup $\mu^w$ (measure of potential output independent of degree of efficiency; interpret $\mu^w$ as proxy for wage rigidity)  
  Domestic flex-price equilibrium (home country, conditional on $Y^*_t$)  
  World flex-price equilibrium (both countries) (Relevant for noncooperation/cooperation)

  - Domestic flex-price equilibrium ($\overline{X}_t$)
    \[ \frac{P^0_{H,t}}{P_{H,t}} = (1 + \mu)MC_t \]
    \[ \frac{P^0_{H,t}}{P_{H,t}} = 1 \]
    \[ \overline{MC} = \frac{1}{1 + \mu^p} \]  
    (38)
    Symmetry: $Y_t(f) = \bar{Y}_t$, $V_t = 1$, from (33):
    \[ \overline{Y}_t = A_t\overline{N}_t \]  
    (39)
    From (35):
    \[ \frac{\overline{Y}_t}{y_t} = \left( \frac{k^{1-\sigma} A_t^{1+\phi} }{(1-\tau)(1 + \mu^p)(1 + \mu^p)} \right)^{\frac{1}{\mu}} \]  
    (40)
    Note $\frac{\overline{Y}_t}{y_t} = -\frac{\nu_0}{\nu}$

    - World flex-price equilibrium
      \[ \overline{Y}_t = \left( \frac{k^{1-\sigma} A_t^{1+\phi} }{(1-\tau)(1 + \mu^p)(1 + \mu^p)} \right)^{\frac{1}{\mu}} \]
      (41)
      \[ \overline{Y}_t = \overline{y}_t \left( \frac{\overline{Y}_t}{y_t} \right)^{-\frac{\nu_0}{\nu}} \]
      \[ \overline{Y}_t = \bar{y}_t + \frac{\nu_0}{\nu} (y^*_t - \overline{y}_t) \equiv \bar{y}_t + \frac{\nu_0 y^*_t}{\nu} y_t \]
Sticky-price equilibrium

Log-linearize \((x = \ln X)\)

From (30):
\[ y_t = c_t + \gamma s_t \quad (42) \]

From (11):
\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} \left[ i_t - (E_t p_{t+1} - p_t) \right] \]
\[ = E_t c_{t+1} - \frac{1}{\sigma} \left\{ i_t - E_t \left[ (p_{H,t+1} - p_{H,t}) + \gamma (s_{t+1} s_t) \right] \right\} \]
\[ = E_t c_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} + \gamma E_t \Delta s_{t+1}) \quad (43) \]

\(i_t \equiv \ln R_t \) nominal rate of interest \((r_t \text{ in paper})\), \(\pi_t \equiv p_{t+1} - p_t \) is the rate of domestic inflation from \(t\) to \(t+1\), from (31)
\[ s_t = y_t - y_t^* \quad (44) \]

From (33):
\[ y_t = a_t + n_t \quad (45) \]

Combine log-linearized optimal price setting (23) with price index (25)
\[ \pi_t = \delta m c_t + \beta E_t \pi_{t+1} \]
\[ \delta \equiv \frac{(1-\theta)(1-\beta \theta)}{\theta} \]
\[ \bar{y}_t = y_t - \bar{y}_t \] the domestic output gap. From (35) and (45)
\[ m c_t = \mu_t^w + \nu y_t + \nu_0 y_t^* - (1 + \phi) a_t \]
\[ = \nu \bar{y}_t + \mu_t^w \quad (47) \]

where \(\mu_t^w \equiv \ln(1 + \mu_t^w)\) and from (40):
\[ \bar{y}_t \equiv \nu^{-1} [(1 + \phi) a_t - \nu_0 y_t^*] \quad (48) \]

Write in terms of IS and AS curve
\[ \bar{y}_t = E_t \bar{y}_{t+1} - \sigma_0^{-1} (i_t - E_t \pi_{t+1} - \bar{r}_t) \quad (49) \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \bar{y}_t + \nu_0 \]
\[ \sigma_0 \equiv \sigma - \nu_0, \kappa \equiv \delta \nu \] (\(\kappa\) is \(\lambda\) in paper), \(u_t \equiv \delta \mu_t^w\), \(\bar{r}_t\) is the domestic natural real interest rate (conditional on \(y_t^*\) \((rrt \text{ in paper})\)
\[ \bar{r}_t = \sigma_0 E_t \Delta \bar{y}_{t+1} + \nu_0 E_t \Delta y_t^* \quad (51) \]

Similarly for foreign country:
\[ \sigma_0^* \equiv \sigma - \nu_0^*, \nu_0^* \equiv (1 - \gamma)(\sigma - 1), \]
\[ \nu^* \equiv \sigma + \phi - \nu_0^*, \text{ and } \kappa^* \equiv \delta \nu^* \]

Assume
\[ u_t = \rho u_{t-1} + \varepsilon_t, \quad 0 < \rho < 1, \quad \varepsilon_t \text{ iid} \quad (52) \]

- Open economy effects:
  - Small open economy: Isomorphic to closed economy (CGG 01, Galf-Monacelli 02)
  - Open economy: \(\sigma_0 = \sigma - \nu_0, \nu = \sigma + \phi - \nu_0, \nu_0 = \gamma(\sigma - 1)\)
    * Interest elasticity of domestic demand \((\sigma_0^{-1})\)
    * Slope of short-run Phillips curve, \(\kappa \equiv \delta \nu \)
    * Impact of \(y_t^*\) on \(\bar{y}_t\) and \(\bar{r}_t\)
    * Log utility \((\sigma_0 = \sigma = 1, \nu_0 = 0)\), identical to closed economy

Terms of trade and the output gap:
\[ s_t = (\bar{y}_t - \bar{y}_t^*) + (\bar{y}_t - \bar{y}_t^*) \quad (53) \]
\[ s_t - \bar{a}_t = \bar{y}_t - \bar{y}_t^* \]
\[ \bar{a}_t \equiv (\bar{y}_t - \bar{y}_t^*) \]
\(\bar{a}_t\) is the natural level of the terms of trade
• Welfare and optimal policy in non-cooperative case

Domestic central bank maximizes utility of home household under discretion, taking foreign variables \(Y_t^*\) as given

Assume optimal subsidy (appendix)

\[
(1 - \tau)(1 + \mu^w)(1 + \mu^p)(1 - \gamma) = 1 \tag{54}
\]

\[
(1 - \tau^*)(1 + \mu^w)(1 + \mu^p)\gamma = 1 \tag{55}
\]

(Offsets both incentive to increase output and to improve terms of trade, cf. previous lecture)

Central bank’s objective function, 2nd order approximation of the utility of representative household around the domestic flex-price equilibrium (appendix). Normalization, welfare loss as fraction of steady state consumption:

\[
\mathcal{W}_t^H \equiv - (1 - \gamma)\frac{\Lambda}{2} E_t \sum_{j=t}^{\infty} \beta^j \left( \pi_j^2 + \lambda \delta_j^2 \right) \tag{56}
\]

\(\Lambda \equiv \frac{\xi}{\beta}, \lambda \equiv \frac{\nu \delta}{\xi} \equiv \frac{\xi}{\bar{\epsilon}} (\lambda \text{ is } \alpha \text{ in paper})\)

Similar decision problem as for closed economy \((Y_t^* \text{ exogenous})\)

Flexible domestic inflation targeting \((\lambda > 0) \text{ (} \lambda \text{ depends on } \nu_0 = \sigma - \sigma_0)\)

Isomorphic to closed economy

Optimization under discretion

Targeting rule (Euler condition)

\[
0 = \pi_t + \frac{\Lambda}{\kappa} \bar{y}_t \tag{57}
\]

\[
\equiv \pi_t + \frac{1}{\xi} \bar{y}_t \tag{58}
\]

\(\xi \text{ demand elasticity of intermediate good, independent of open economy}\)

Combine with (50), solve forward

\[
\pi_t = \psi u_t \tag{59}
\]

\[
\bar{y}_t = - \xi \psi u_t \tag{60}
\]

\(\psi \equiv \frac{1}{(1 - \rho^2) + \kappa \xi} > 0 (\rho \text{ serial correlation in } u_t)\)

Similar expressions hold for the foreign economy.

- Targeting rule identical to closed economy
- Tradeoff between inflation gap and output gap
- Higher demand elasticity \(\xi\), more inflation stabilization
  - Intuition: Source of distortion is domestic intermediate-good price dispersion due to inflation. Welfare loss for given price deviation increases in demand elasticity.
- Openness affects \(\lambda\) and \(\kappa\) similarly \((\lambda \equiv \frac{\nu}{\xi})\)
- Equilibrium response to \(u_t\) \((\psi)\) depends on open economy only via \(\kappa\)
- \(\pi_t\) and \(\bar{y}_t\) independent on foreign shocks, but not \(y_t = \bar{y}_t + \bar{y}_t\)
Corresponding (implicit) instrument rule (one of many) (from 49)

\[ i_t = \bar{r}_t + E_t \pi_{t+1} + \sigma_0 (E_t \bar{y}_{t+1} - \bar{y}_t) \]
\[ = \bar{r}_t + E_t \pi_{t+1} + \sigma_0 (-\xi E_t \pi_{t+1} + \xi \pi_t) \]
\[ = \bar{r}_t + \psi E_t \pi_{t+1} \]  \hspace{1cm} (61)

\[ \psi \equiv 1 + \frac{\sigma_0(1-\rho)}{\rho} > 1 \]
- Implicit instrument rule: Equilibrium condition
- Depends on precise disturbance, not robust
- Foreign shocks enter via \( \bar{r}_t \)
- "Wicksell rule" rather than Taylor rule...

Proposition in paper:

**Proposition 1** In the Nash equilibrium, the policy problem a country faces is isomorphic to the one it would face if it were a closed economy. As in the case of a closed economy, the optimal policy rule under discretion may be expressed as a Taylor rule that is linear in the domestic natural real interest rate and expected domestic inflation. Open economy considerations affect the slope coefficient on domestic inflation in the rule, as well as the behavior of the domestic natural real interest rate.

- Terms of trade and the nominal exchange rate
- Symmetric case: \( \gamma = \frac{1}{2} (\sigma_0 = \sigma_0^*) \)
- Equilibrium terms of trade

\[ s_t = (\bar{y}_t - \bar{y}_t^*) + (\bar{y}_t - \bar{y}_t^*) \]  \hspace{1cm} (62)
\[ = -\xi \psi (u_t - u_t^*) + (a_t - a_t^*) \]
- Equilibrium terms of trade depends on both productivity differentials and relative cost push shocks. (Recall: Forward-looking variables depend on (all) predetermined variables)
- A positive cost push shock in the home country induces an appreciation in the terms of trade.

- Nominal exchange rate should respond to relative differences in cost push shocks, in addition to the productivity shocks
- \( \xi > 1 \): Relatively high cost-push shock should engineer short run appreciation, followed by eventual permanent depreciation. Initial appreciation is a consequence of TOT appreciation resulting from the contraction in the output gap that is needed to dampen inflationary pressures. The eventual depreciation to a permanently lower level results from the permanent effect of the shock on the domestic price level, which has a unit root under the optimal time consistent policy.
  - Flexible exchange rate
  - Exchange rate nonstationary: Inflation targeting, unit root in price level (IT under commitment sometimes results in stationary price level and exchange rate)
International cooperation

Potential gain from cooperation: Foreign effect on domestic MC

Common optimization

\[ U(C_t) - (1 - \gamma)V(N_t) - \gamma V(N_t^*) \]

(64)

No surprise TOT improvement

Common subsidy \( \tau \) (output incentive only)

\[ (1 - \tau)(1 + \mu^w)(1 + \mu^p) = 1 \]

(65)

Results

– There is a gain from cooperation unless \( \sigma_0 = \sigma = 1, \nu_0 = 0 \).
– Flexible exchange rates
– Quantitative importance?

Difference from C-P 02

– \( \sigma \neq 1 \)
– PCP

Output gap relative to world flex-price potential output

\[ \tilde{y}_t = \tilde{y}_t - \frac{\nu_0}{\nu}y_t \]

(66)

Objective function

\[ \mathcal{W}_C \equiv -\frac{1}{2} \lambda E_0 \sum_{t=0}^{\infty} \beta^t \left[ (1 - \gamma) \left( \pi_t^2 + \lambda (\tilde{y}_t)^2 \right) + \gamma (\pi_t^2 + \lambda^* (\tilde{y}_t^*)^2) - 2 \Phi \tilde{y}_t \tilde{y}_t^* \right] \]

(67)

\[ \Lambda \equiv \frac{\delta}{\xi}, \lambda \equiv \frac{\xi}{\xi}, \lambda^* \equiv \frac{\xi^*}{\xi}, \Phi \equiv \frac{\delta(1-\sigma)(1-\gamma)}{\xi} \]

Aggregate supply

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t + \kappa_0 \tilde{y}_t^* + u_t \]

(68)

\[ \pi_t^* = \beta E_t \pi_{t+1}^* + \kappa^* \tilde{y}_t + \kappa_0^* \tilde{y}_t^* + u_t^* \]

(69)

\[ \kappa_0 \equiv \delta \nu_0, \kappa_0^* \equiv \delta^* \nu_0^* \]

FOCs/targeting rules

\[ \tilde{y}_t = -\xi \pi_t - \frac{\nu_0}{\nu} \left( \tilde{y}_t + \xi \pi_t^* \right) \]

(70)

\[ \tilde{y}_t^* = -\xi \pi_t^* - \frac{\nu_0^*}{\nu^*} \left( \tilde{y}_t + \xi \pi_t \right) \]

(71)

Rewrite

\[ \pi_t + \frac{1}{\xi} \tilde{y}_t = 0 \]

(72)

\[ \pi_t^* + \frac{1}{\xi} \tilde{y}_t^* = 0 \]

(73)

More details in paper
Benigno and Benigno 02 derive the welfare-optimizing cooperative policy under commitment

- Targeting rules

\[ \pi_t + \frac{1}{\xi} (\bar{y}_t - \bar{y}_{t-1}) = 0 \]
\[ \pi^*_t + \frac{1}{\xi} (\bar{y}^*_t - \bar{y}^*_{t-1}) = 0 \]

\( \bar{y}_t \) and \( \bar{y}^*_t \) are domestic and foreign output gaps relative to efficient output (resulting in flex-price equilibrium with zero markup, actual markup close to zero) (\( \xi \) is \( \sigma \) in B-B)

Corresponds to flexible domestic-inflation targeting under commitment

- Comments (cf. Nelson 02)
  - Elegant analysis, eventually simple
  - Some simple and striking results
  - Calvo: Persistence in inflation and output gap (if persistent cost-push shocks)
  - Some differences from C-P 02 (discretion, \( \sigma \neq 1 \))
  - Discretion only, but arguably more realistic
  - Limited distortions (price dispersion for intermediate goods, positive markups); other costs of inflation
  - Parameterization: (Woodford-Rotemberg, Gali-Monacelli), \( \lambda \) quite small, \( \approx 0.05 \), close to strict IT
  - Real world: CPI flexible inflation targeting
  - Explicit welfare maximization not operational
  - No lags, inflation control exaggerated
  - Complete risk-sharing, current account balance
  - Source of cost-push shocks
  - Import final goods only (Nelson-McCallum, imported intermediate goods, CPI IT)
• Monetary policy in small open economies
  – Svensson 00b (flexible CPI), Weerapana 00 (2-country version), Flamini 01 (openness)
  – Closing of SOE: Ghironi 00, Schmitt-Grohe and Uribe 01
    * Exogenous world real interest rate, current account dynamics \((\beta, \frac{1}{1+r})\)
      - Incomplete asset markets: Steady state depends on initial conditions, random walk in dynamics
    * Endogenous rate of time-preference, increasing in wealth (Uzawa)
    * Debt-elastic interest-rate premium
    * Convex portfolio-adjustment costs
    * Complete asset markets
    * OLG: Yaari, Blanchard (aggregation)
  – Gali and Monacelli 02
    * Predecessor of CGG 02
    * Foreign country large, approximately closed, \(1 - \gamma^* \neq 1 - \gamma\), \(\gamma^* \to 1\), \(c_t^*, y_t^*\) exogenous for home country
    * Complete risksharing, \(c_t = c_t^* + \frac{1-\gamma^*}{\sigma} s_t\) (\(\gamma\) is \(\alpha\), degree of openness, in paper)
    * Trade balance, \(nx_t = y_t - c_t - \gamma s_t \neq 0\)
    * No cost-push shocks
    * Examines strict CPI IT, strict domestic IT, fixed exchange rate, welfare optimizing policy

– Smets and Wouters 01
  * Implications for monetary policy of sticky import prices and imperfect pass-through
  * Import sector: Import homogenous good, produce differentiated import goods
  * Calvo-type pricing of imported goods
  * Imports both intermediate and final goods
  * From Ghironi 00: Blanchard-Yaari, Blanchard, Frenkel-Razin OLG
    - Linearized around PF solution, added shocks. Not linearized around stochastic solution
  * Calibration and estimation on euro-area data
  * Analysis of optimal monetary policy in calibrated model
  * Not complete welfare optimization. Minimize average resource cost from distortions due to price variability in domestic and imported goods, average of variability of inflation and inflation changes for domestic and imported goods