Model uncertainty

- Multiplicative uncertainty: No certainty equivalence
- Random coefficients (Brainard, Chow, Blinder, Söderström)
- Robust control (Hansen-Sargent, Onatski-Williams)
- Markov Jump Linear-Quadratic Systems (Svensson-Williams)

1 A simple example of model uncertainty

(Brainard 67, Chow 75, Blinder 95, Svensson 99, "Extensions...")
Model (period: year)

$$\pi_{t+1} = \pi_t + \alpha_{yt} y_t + \varepsilon_{t+1} \tag{1}$$

$$y_{t+1} = \tilde{\beta}_{yt} y_t - \beta_{rt}\left(i_t - \pi_{t+1|t}\right) + \eta_{t+1} \equiv y_{t+1,t} + \eta_{t+1}, \tag{2}$$

α_y, $\tilde{\beta}_y$ and β_r dated according to the year they refer to. ε_t and η_t iid and zero mean.
For simplicity, consider only strict inflation targeting ($\lambda = 0$).

$$\min_{i_t} \delta^2 \mathrm{E}_t \left[\frac{1}{2}\left(\pi_{t+2} - \pi^*\right)^2\right] \tag{3}$$

subject to

$$\pi_{t+2} = \pi_{t+2,t}\left(i_t\right) + \varepsilon_{t+1} + \alpha_{y,t+1}\eta_{t+1} + \varepsilon_{t+2},$$

$$\begin{aligned}\pi_{t+2,t}\left(i_t\right) &= \pi_{t+1|t} + \alpha_{y,t+1} y_{t+1,t} \\ &= \pi_{t+1|t} + \tilde{a}_{y,t+1} y_t - a_{r,t+1}\left(i_t - \pi_{t+1|t}\right) \\ \pi_{t+1|t} &\equiv \mathrm{E}_t \pi_{t+1} = \pi_t + \alpha_{yt} y_t, \end{aligned} \tag{4}$$

$$\tilde{a}_{y,t+1} = \alpha_{y,t+1}\tilde{\beta}_{yt} \text{ and } a_{r,t+1} = \alpha_{y,t+1}\beta_{rt}$$

Note that $\pi_{t+1|t}$ is predetermined.

No uncertainty: α_{yt}, $\tilde{\beta}_{yt}$ and β_{rt} constant. FOC (targeting rule)

$$\pi_{t+2|t}(i_t) = \mathrm{E}_t \pi_{t+2,t}(i_t) = \pi^*. \tag{6}$$

Uncertainty in year t about $\tilde{a}_{y,t+1}$ and $a_{r,t+1}$, resulting from uncertainty about the coefficients α_{yt}, $\tilde{\beta}_{yt}$ and β_{rt}.
Let the α_{yt} be known at t, and let

$$\begin{aligned} \alpha_{y,t+1} &= \alpha_y + \nu_{\alpha y,t+1} \\ \tilde{\beta}_{yt} &= \tilde{\beta}_y + \nu_{\beta yt} \\ \beta_{rt} &= \beta_r + \nu_{\beta rt} \end{aligned}$$

$\nu_{\alpha y,t+1}$, $\nu_{\beta yt}$ and $\nu_{\beta rt}$ are iid stochastic disturbances with zero means and given variances/covariar
The realizations of these disturbances become known in year $t+1$.
Assume that $\nu_{\alpha y,t+1}$ is uncorrelated with $\nu_{\beta yt}$ and $\nu_{\beta rt}$.

$$\begin{aligned} \tilde{a}_{y,t+1} &= \tilde{a}_y + \nu_{y,t+1} \\ a_{r,t+1} &= a_r + \nu_{r,t+1}, \end{aligned}$$

$\nu_{y,t+1}$ and $\nu_{r,t+1}$ are zero mean iid disturbances, and

$$\tilde{a}_y = \alpha_y \tilde{\beta}_y, \ a_r = \alpha_y \beta_r. \tag{7}$$

In year t, the parameters in the current Phillips curve are known, but not those of next year's Phillips curve, and not those of the current aggregate demand equation. These are instead known in year $t+1$. That is, we assume that all uncertainty relevant for the policy decision in year t is resolved in year $t+1$.

There is a new realization of the stochastic disturbance terms each year, with unchanged variances and covariances. Therefore, there is nothing that can be learned to reduce the uncertainty, and there is no point in experimenting in order to learn more about the stochastic properties of the model. The fact that there is no role for experimentation and learning simplifies the analysis considerably.
Constraint in year t can be written

$$\pi_{t+2} = \pi_{t+1|t} + (\tilde{a}_y + \nu_{y,t+1}) y_t - (a_r + \nu_{r,t+1}) \left(i_t - \pi_{t+1|t}\right) + \varepsilon_{t+1} + \alpha_{y,t+1}\eta_{t+1} + \varepsilon_{t+2}, \tag{8}$$

$\pi_{t+1|t}$ given by (4), predetermined.
$\nu_{y,t+1}$ and $\nu_{r,t+1}$ variances and covariance σ_y^2, σ_r^2 and σ_{yr}
Covariance of $\nu_{r,t+1}$ with $\varphi_{t+1} \equiv \varepsilon_{t+1} + \alpha_y \eta_{t+1}$ is $\sigma_{\varphi r}$.
2-year conditional inflation forecast

$$\pi_{t+2|t}(i_t) \equiv \mathrm{E}_t \pi_{t+2,t}(i_t) = \pi_{t+1|t} + \tilde{a}_y y_t - a_r \left(i_t - \pi_{t+1|t}\right). \tag{9}$$

Minimize (3) subject to (8). FOC:

$$\begin{aligned}
0 \;&=\; \delta^2 \mathrm{E}_t \left[(\pi_{t+2} - \pi^*) \frac{\partial \pi_{t+2}}{\partial i_t} \right] \\
&=\; -\,\delta^2 \mathrm{E}_t \left\{ \left[\pi_{t+1|t} + (\tilde{a}_y + \nu_{y,t+1})\, y_t - (a_r + \nu_{r,t+1})\,(i_t - \pi_{t+1|t}) \right.\right. \\
&\qquad\qquad \left.\left. +\, \varphi_{t+1} + \varepsilon_{t+2} - \pi^* \right] \right\} (a_r + \nu_{r,t+1}) \\
&=\; -\,\delta^2 [\pi_{t+2|t}(i_t) - \pi^*] a_r - \delta^2 \sigma_{yr} y_t + \delta^2 \sigma_r^2 \left(i_t - \pi_{t+1|t} \right) - \delta^2 \sigma_{\varphi r}.
\end{aligned}$$

Rewrite:

$$\pi_{t+2|t}(i_t) - \pi^* = -\,\frac{\sigma_{yr}}{a_r} y_t + \frac{\sigma_r^2}{a_r} \left(i_t - \pi_{t+1|t} \right) - \frac{\sigma_{\varphi r}}{a_r}. \qquad (10)$$

Different from (6). No certainty equivalence.

Optimal policy function (reaction function): Use (9) in (10),

$$i_t = \pi_{t+1|t} + \frac{1}{(1+v_r)\, a_r} \left(\pi_{t+1|t} - \pi^* \right) + \frac{\tilde{a}_y + \sigma_{yr}/a_r}{(1+v_r)\, a_r} y_t + \frac{\sigma_{\varphi r}/a_r}{(1+v_r)\, a_r}, \qquad (11)$$

Coefficient of variation of the policy multiplier a_r

$$v_r = \frac{\sigma_r^2}{a_r^2}.$$

Consider special case of "independent multiplier uncertainty": $\sigma_r^2 > 0$; ν_r uncorrelated with ν_y or φ: $\sigma_{yr} = \sigma_{\varphi r} = 0$.

Uncertainty in β_{rt} alone, β_{rt} is uncorrelated with φ_{t+1}. (11) simplifies to

$$i_t = \pi_{t+1|t} + \frac{1}{(1+v_r)\, a_r} \left(\pi_{t+1|t} - \pi^* \right) + \frac{\tilde{a}_y}{(1+v_r)\, a_r} y_t. \qquad (12)$$

More uncertainty (higher v_r) leads to a more conservative/attenuated (less activist/aggressive) policy, in the sense of lower magnitude of the response coefficients.

- Parameters are stochastic with a known stationary distribution: No scope for learning and experimentation
- For particular covariance patterns for the disturbances to the different parameters, the policy attenuation can be overturned and a more aggressive policy can be optimal.
- Söderström 02 (more general treatment of random coefficients): Uncertainty about $\alpha_{\pi t}$ ($\pi_{t+1} = \alpha_{\pi t}\pi_t + \alpha_{yt} y_t + \varepsilon_{t+1}$), flexible inflation targeting, more aggressive response to π_t

2 Robust control made simple

(Svensson 99b)
M set of feasible models of the transmission mechanism of monetary policy,
$m \in M$ particular feasible model
F feasible set of policies, $f \in F$ particular monetary policy
Let $V(f, m)$ denotes the (expected) loss of policy f in model m.

2.1 Bayesian approach to optimal policy under model uncertainty

(as in Brainard 67)
Prior probability measure, Φ, on the feasible set of models M.
Expected loss for given policy f

$$\mathrm{E}_M V(f, m) \equiv \int_{m \in M} V(f; m) d\Phi(m).$$

Optimal policy f^*

$$f^* = \arg\min_{f \in F} \mathrm{E}_M V(f; m).$$

The optimal policy will be a function of M, Φ and V, $f^*(M, \Phi, V)$.

2.2 Robust control

(Hansen-Sargent, Onatski-Stock)
No prior probability measure on the feasible set of models. Instead, it focus on the maximum loss for any given policy f (assume max exists, else sup)

$$\max_{m \in M} V(f; m).$$

Optimal policy $\hat{f}$ minimizes maximum loss,

$$\hat{f} = \arg\min_{f \in F} \max_{m \in M} V(f; m).$$

Worst possible model for a given policy, $\hat{m}(f)$,

$$\hat{m}(f) = \arg\max_{m \in M} V(f; m),$$

Optimal policy satisfies

$$\hat{f} = \arg\min_{f \in F} V(f; \hat{m}(f)).$$

The optimal policy will be a function of M and V, $\hat{f}(M, V)$.
Use a simple model to illustrate

2.2.1 Optimal policy with known model

Simple backward-looking Phillips curve,

$$\pi_{t+1} = \gamma\pi_t + \alpha x_t + \varepsilon_{t+1}, \tag{13}$$

π_t and x_t inflation and output gap, $\alpha > 0$, $\gamma > 0$, ε_t is an iid shock, $\mathrm{E}[\varepsilon_t] = 0$, $\mathrm{Var}[\varepsilon_t] = \sigma_\varepsilon^2$.
Period loss function ($\lambda \geq 0$)

$$L_t = \pi_t^2 + \lambda x_t^2, \tag{14}$$

Social loss function

$$\mathrm{E}[L_t] = \mathrm{E}[\pi_t^2] + \lambda \mathrm{E}[x_t^2] = \mathrm{Var}[\pi_t] + \lambda \mathrm{Var}[x_t], \tag{15}$$

(last equality follows if policy restricted to result in $\mathrm{E}[\pi_t] = \mathrm{E}[x_t] = 0$)
x_t control variable, π_t endogenous predetermined variable.
Optimal policy when α and γ are known. Linear in π_t,

$$x_t = -f\pi_t, \tag{16}$$

response coefficient f to be determined

$$\pi_{t+1} = (\gamma - \alpha f)\pi_t + \varepsilon_t.$$

Optimal policy will satisfy $0 \leq f \leq \frac{\gamma}{\alpha}$.

Variance inflation and the output gap

$$\mathrm{Var}[x_t] = f^2 \mathrm{Var}[\pi_t], \tag{17}$$

$$\mathrm{Var}[\pi_t] = \frac{1}{1 - (\gamma - \alpha f)^2}\sigma_\varepsilon^2, \tag{18}$$

Social loss will equal

$$\mathrm{E}[L_t] = \frac{1 + \lambda f^2}{1 - (\gamma - \alpha f)^2}\sigma_\varepsilon^2 \equiv V(f; \alpha, \gamma; \lambda). \tag{19}$$

Optimal policy $f = f^*(\alpha, \gamma; \lambda)$ given by

$$f^*(\alpha, \gamma; \lambda) \equiv \arg\min_f V(f; \alpha, \gamma; \lambda).$$

First-order condition

$$V_f(f^*; \alpha, \gamma; \lambda) \equiv 2\frac{\lambda f^*[1 - (\gamma - \alpha f^*)^2] - \alpha(1 + \lambda f^{*2})(\gamma - \alpha f^*)}{[1 - (\gamma - \alpha f^*)^2]^2} = 0 \tag{20}$$

Second-order condition

$$V_{ff}(f^*; \alpha, \gamma) > 0.$$

Easy to show properties of optimal policy

$$f^*_\alpha(\alpha, \gamma; \lambda) \; < \; 0,$$
$$f^*_\gamma(\alpha, \gamma; \lambda) \; > \; 0,$$
$$f^*_\lambda(\alpha, \gamma; \lambda) \; < \; 0.$$

$$f^*(\alpha, \gamma; 0) \; = \; \frac{\gamma}{\alpha},$$
$$f^*(\alpha, \gamma; \infty) \; \equiv \; \lim_{\lambda \to \infty} f^*(\alpha, \gamma; \lambda) \; = \; 0.$$

2.2.2 Optimal policy under robust control

Assume known supports of α and γ, but not their (constant) true values. (True model assumed constant over time; time-varying model in appendix.)
Supports $\alpha \in [\alpha_1, \alpha_2]$ and $\gamma \in [\gamma_1, \gamma_2]$, where $0 < \alpha_1 < \alpha_2$ and $0 < \gamma_1 < \gamma_2$.
The bounds of the supports restricted to satisfy condition

$$\alpha_2\gamma_2 - \alpha_1\gamma_1 < \alpha_1 + \alpha_2. \tag{21}$$

(As shown below, sufficient for the existence of a policy with bounded inflation and output-gap variance for all possible models.)
Rewrite (21)

$$\gamma_2 < 1 + (1 + \gamma_1)\frac{\alpha_1}{\alpha_2}$$

(γ_2 must not be too large, satisfied for any $\gamma_2 \leq 1$)
Models can be indexed by $m = (\alpha, \gamma)$.
Feasible set of models M

$$M \equiv \{m = (\alpha, \gamma) \,|\, \alpha_1 \leq \alpha \leq \alpha_2, \;\gamma_1 \leq \gamma \leq \gamma_2\}.$$

Feasible set of policies F: (16) with $f \geq 0$.

$$F = \{f \,|\, f \geq 0\}.$$

Optimal policy under robust control

$$\hat{f} = \arg\min_{f \in F} \max_{(\alpha,\gamma)\in M} V(f;\alpha,\gamma;\lambda).$$

(as shown below, $f \geq 0$ will not be binding)

2.2.3 The worst possible model

From (19) follows that, for any given f, the worst possible model is the one that maximizes $|\gamma - \alpha f|$. Let $\hat{m}(f)$ denote the worst possible model for any given $f \in F$, that is,

$$\hat{m}(f) \equiv \arg\max_{(\alpha,\gamma)\in F} |\gamma - \alpha f|.$$

We have

$$|\gamma - \alpha f| = \begin{cases} \gamma - \alpha f & \text{if } \gamma - \alpha f \geq 0, \\ \alpha f - \gamma & \text{if } \gamma - \alpha f < 0. \end{cases}$$

Because $f \geq 0$, maximum of $|\gamma - \alpha f|$ for a given f is the maximum of $\gamma_2 - \alpha_1 f$ and $\alpha_2 f - \gamma_1$,

$$\max_{(\alpha,\gamma)} |\gamma - \alpha f| = \max(\gamma_2 - \alpha_1 f, \alpha_2 f - \gamma_1).$$

$\hat{m}(f)$ is either (α_1, γ_2) or (α_2, γ_1), depending on whether $\gamma_2 - \alpha_1 f \gtreqless \alpha_2 f - \gamma_1$, that is, whether $f \lesseqgtr \bar{f}$, where

$$\bar{f} \equiv \frac{\gamma_1 + \gamma_2}{\alpha_1 + \alpha_2} > 0. \tag{22}$$

$\hat{m}(f)$ is given by

$$\hat{m}(f) = \begin{cases} (\alpha_1, \gamma_2) & \text{if } f \leq \bar{f}, \\ (\alpha_2, \gamma_1) & \text{if } f \geq \bar{f}. \end{cases} \tag{23}$$

The worst model is on the boundary of feasible set of models; hence depends crucially on the assumed feasible set of models.
Worst possible model is unique if $f \neq \bar{f}$ but can take either of the two values in (23) if $f = \bar{f}$.
Condition (21) insures that

$$\max_{(\alpha,\gamma)\in M} |\gamma - \alpha \bar{f}| < 1,$$

which is sufficient for the existence of a policy that results in finite inflation and output-gap variance for all feasible models.

2.2.4 Strict inflation targeting

Simplest case, $\lambda = 0$,

$$V(f; \alpha, \gamma, 0) \equiv \frac{1}{1 - (\gamma - \alpha f)^2} \sigma_\varepsilon^2.$$

$$\hat{f} = \arg\min_{f\in F} \max_{(\alpha,\gamma)\in M} |\gamma - \alpha f| = \arg\min_{f\in F} \max(\gamma_2 - \alpha_1 f, \alpha_2 f - \gamma_1).$$

To minimize $\max(\gamma_2 - \alpha_1 f, \alpha_2 f - \gamma_1)$, it is best to select $f = \hat{f}$ where $\hat{f}$ fulfills

$$\gamma_2 - \alpha_1 \hat{f} = \alpha_2 \hat{f} - \gamma_1,$$

that is,

$$\hat{f} = \bar{f},$$

where $\bar{f}$ is given by (22).

2.2.5 Flexible inflation targeting

$\lambda > 0$, $V(f;\alpha,\gamma;\lambda)$ is given by (19).

Since the term λf^2 in the numerator adds to the social loss, we realize that, for sufficiently large λ, it is optimal to lower f below $\bar{f}$, and to trade off some increase in the denominator for a reduction in the numerator. For $f < \bar{f}$, we then have $\hat{m}(f) = (\alpha_1, \gamma_2)$, hence we have

$$\hat{f} = \arg\min_{f\in F} \max_{(\alpha,\gamma)\in M} V(f;\alpha,\gamma;\lambda) = \arg\min_{f\in F} V(f;\alpha_1,\gamma_2;\lambda) = f^*(\alpha_1,\gamma_2;\lambda),$$

where $f^*(\alpha_1,\gamma_2;\lambda)$ denotes the optimal policy if the true model is known to be (α_1,γ_2). Thus, the optimal policy for an arbitrary λ, $\hat{f}(\lambda)$, is given by

$$\hat{f}(\lambda) = \min[f^*(\alpha_1,\gamma_2;\lambda), \bar{f}].$$

Note that, for small λ, if (α_1,γ_2) were the true model, the optimal policy would imply $f^*(\alpha_1,\gamma_2;\lambda) > \bar{f}$. Indeed, we know that $f^*(\alpha_1,\gamma_2;0) = \frac{\gamma_2}{\alpha_1} > \bar{f} = \frac{\gamma_1+\gamma_2}{\alpha_1+\alpha_2}$. This is because, if (α_1,γ_2) were the true model, one would not need to consider that the worst possible model shifts to (α_2,γ_1) when f rises above $\bar{f}$.

Put differently, there is a level $\bar{\lambda} > 0$ for the weight on output-gap stabilization for which

$$f^*(\alpha_1,\gamma_2;\bar{\lambda}) = \bar{f}. \tag{24}$$

Then we have

$$\hat{f}(\lambda) = \begin{cases} \bar{f} \le f^*(\alpha_1,\gamma_2;\lambda) & \text{if } \lambda \le \bar{\lambda}, \\ f^*(\alpha_1,\gamma_2;\lambda) < \bar{f} & \text{if } \lambda > \bar{\lambda}. \end{cases} \tag{25}$$

Since $f^*(\alpha_1,\gamma_2;\lambda) \ge 0$ and $\bar{f} > 0$, this also confirms that the restriction $\hat{f} \ge 0$ imposed above is not binding.

If time-varying model (appendix of "Robust control made simple"), under strict inflation targeting, worst model is not unique but includes both the values (α_1,γ_2) and (α_2,γ_1). Under flexible inflation targeting, if $\lambda > \bar{\lambda}$, the worst model is (α_1,γ_2) and constant over time. Thus, surprisingly little of the analysis changes if the model is allowed to vary over time.

2.3 Discussion

- In this example, the worst possible model ends up being on the boundary of the assumed feasible set of models.
- Prior assumptions about the feasible set of models are crucial for the outcome.
- Robust control has been proposed as a non-Bayesian alternative that utilizes less prior assumptions about the model. OK if the worst possible model somehow ended up in the interior of the feasible set of models. Not if worst model on boundary.
- If a Bayesian prior probability measure were to be assigned to the feasible set of models, probability assigned to the models on the boundary exceedingly small.
- Then, highly unlikely models can come to dominate the outcome of robust control.
- For $\lambda \leq \bar{\lambda}$, optimal policy is given by $\bar{f}$ in (22), "middle" γ, $\frac{\gamma_1+\gamma_2}{2}$, divided by "middle" α, $\frac{\alpha_1+\alpha_2}{2}$. Policy is "intermediate" in this sense, neither particularly "aggressive" nor particularly "cautious" (although still depending on the bounds of the supports).
- For $\lambda > \bar{\lambda}$, optimal policy given by the optimal policy when the economy has a low α and a high γ. Worst model has largest serial correlation and smallest effect of output gap on inflation. Since response coefficient is decreasing in α and increasing in γ, the response coefficient is the largest for the different models (when the model is known). In this sense, the policy under robust control ends up being aggressive.

- Under this kind of robust control, commitment to the reaction function is made initially and forever, given only an assumption about the feasible set of models. Better to observe the economy for some time and learn more about the model before making a commitment to a given policy, especially if the model is constant over time. This leads to optimal policy with learning.

3 More general approach

(Hansen-Sargent, Onatski-Williams)
Reference model

$$\begin{aligned} x_{t+1} &= A(L)x_t + B_1(L)u_t + B_2(L)\varepsilon_t \\ y_t &= C(L)x_t + D(L)\varepsilon_t \end{aligned}$$

x_t predetermined variables
y_t observable variables (indicators)
Loss function

$$L_t = \mathrm{E}_t \sum_{\tau=0}^{\infty} \beta^\tau x'_{t+\tau} \Lambda x_{t+\tau}$$

Admissible class of policy rules

$$u_t = f(y_t, y_{t-1}, ..., u_{t-1}, u_{t-2}, ..)$$

True model

$$\begin{aligned} x_{t+1} &= [A(L) + \tilde{A}(L)]x_t + [B_1(L) + \tilde{B}_1(L)]u_t + [B_2(L) + \tilde{B}_2(L)]\varepsilon_t \\ y_t &= [C(L) + \tilde{C}(L)]x_t + [D(L) + \tilde{D}(L)]\varepsilon_t \end{aligned}$$

Restrictions $\mathcal{R}$ on $\tilde{A}(L)$, $\tilde{B}_1(L)]$, $\tilde{B}_2(L)$, $\tilde{C}(L)$, $\tilde{D}(L)$

$\mathcal{R}$ stochastic or deterministic
Bayesian strategy ($\mathcal{R}$ stochastic)

$$\min_{\{u_t = f(\cdot)\}} \mathrm{E}_{\mathcal{R}} L_t$$

Minimax strategy ($\mathcal{R}$ deterministic)

$$\min_{\{u_t = f(\cdot)\}} \max_{\mathcal{R}} L_t$$

Rewrite

$$\begin{aligned} x_{t+1} &= A(L)x_t + B_1(L)u_t + w_t \\ y_t &= C(L)x_t + s_t \end{aligned}$$

Model errors

$$\begin{aligned} w_t &\equiv \tilde{A}(L)x_t + \tilde{B}_1(L)u_t + [B_2(L) + \tilde{B}_2(L)]\varepsilon_t \\ s_t &\equiv \tilde{C}(L)x_t + [D(L) + \tilde{D}(L)]\varepsilon_t \end{aligned}$$

Hansen-Sargent approach: Consider all errors that satisfy

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t (w'_t \Phi_1 w_t + s'_t \Phi_2 s_t) \leq \eta$$

No other structure on errors
η size of uncertainty
$\eta \to \infty$, infinite uncertainty, "extremely robust" policy minimizes so-called H_∞ norm of closed-loop system transforming ε_t into target variables (Hansen-Sargent)

Onatski-Williams: Rudebusch-Svensson 99 model as example.
Original specification and estimates

$$\pi_{t+1} = \underset{(.08)}{.70}\, \pi_t - \underset{(.10)}{.10}\, \pi_{t-1} + \underset{(.10)}{.28}\, \pi_{t-2} + \underset{(.08)}{.12}\, \pi_{t-3} + \underset{(.03)}{.14}\, y_t + \varepsilon_{t+1},$$

$$\text{SE} = 1.009, \quad \text{DW} = 1.99,$$

$$y_{t+1} = \underset{(.08)}{1.16}\, y_t - \underset{(.08)}{.25}\, y_{t-1} - \underset{(.03)}{.10}\, (\bar{\imath}_t - \bar{\pi}_t) + \eta_{t+1},$$

$$\text{SE} = 0.819, \quad \text{DW} = 2.05.$$

OW slightly different estimates
Period loss

$$\bar{\pi}_t^2 + y_t^2 + 0.5(i_t - i_{t-1})^2$$

Computes extremely robust policy for Taylor-type restricted class of instrument rules

$$i_t = g_\pi \bar{\pi}_{t-1} + g_y y_{t-2}$$

$$i_t = 3.10 \bar{\pi}_{t-1} + 1.4 y_{t-2}.$$

But small change in model coefficients (within 20% confidence ellipsoid) makes model unstable.
Conclusion: Structure of model uncertainty must be specified for progress. “Modeling Model Uncertainty.”

Model uncertainty in different ways
Reference model’s errors distributed lags of model variables.
Parametric method: Low order polynomial, Bayesian estimation, resulting in probability distribution of possible models. Bayesian optimal policy.
Nonparametric method: Calibrate size of uncertainty in frequency domain. At each frequency, half of draws from posterior distribution in chosen set. Robust policy.
Tradeoff between performance at different frequencies. Aggressive policy rules better performance at low frequencies, worse at high. Reference model focus on business cycles rather than low-frequency properties.
Many detailed results.
Specific to the example (RS model)?
Main conclusion: Model uncertainty must be modeled.