Optimal Monetary Policy under Learning and Model Uncertainty

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Markov Jump-Linear-Quadratic (MJLQ) model of an economy
with forward-looking variables
Private sector (PS) and central bank (CB)

$$X_{t+1} = A_{11j_{t+1}} X_t + A_{12j_{t+1}} x_t + B_{1j_{t+1}} i_t + C_{1j_{t+1}} \varepsilon_{t+1}, \tag{1}$$

$$\mathrm{E}_t H_{j_{t+1}} x_{t+1} = A_{21j_t} X_t + A_{22j_t} x_t + B_{2j_t} i_t + C_{2j_t} \varepsilon_t, \tag{2}$$

$\varepsilon_t \sim N(0, I_{n_\varepsilon})$.
Model modes $j_t \in N_j \equiv \{1, 2, ..., n_j\}$
Markov process with the transition matrix $P \equiv [P_{jk}]$.
j_t, ε_t independently distributed
$C_{1j}\varepsilon_t$, $C_{2k}\varepsilon_t$ independent for all $j, k \in N_j$
$p_{t|t}$ perceived (estimated) probability distribution over modes in period t
Prediction equation

$$p_{t+1|t} = P' p_{t|t}. \tag{3}$$

CB intertemporal loss function

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L(X_{t+\tau}, x_{t+\tau}, i_{t+\tau}, j_{t+\tau}) \qquad (0 < \delta < 1) \tag{4}$$

Period loss

$$L(X_t, x_t, i_t, j_t) \equiv \frac{1}{2} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}' W_{j_t} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}, \tag{5}$$

W_j $(j \in N_j)$ positive semidefinite.
Special case of no forward-looking variables (no x_t, no (2))

General and tractable way of handling model uncertainty
Large variety of uncertainty configurations. Approximate most relevant kinds of model uncertainty

- i.i.d. and serially correlated random model coefficients (generalized Brainard-type uncertainty)
- Different structural models
  - Different variables, different number of leads and lags
  - Particular variable predetermined in one model and forward-looking in another
  - Backward- and forward-looking models
  - Backward- and forward-looking private-sector expectations
- Regime switches: shifting coefficients, means, variances
- Model misspecification
- Ambiguity aversion, robust control ($P \in \mathcal{P}$)
- Different forms of CB judgment (for instance, perceived uncertainty)
- Etc., etc.
- Approximate all relevant uncertainty situations for a policymaker? (Aside from dimensional and computational limitations)

CB optimizes under commitment in a timeless perspective

Three cases

1. **Optimal policy with no learning (NL)** (Svensson-Williams 05)
   Naive updating equation
$$p_{t+1|t+1} = P' p_{t|t}. \tag{6}$$
2. **Adaptive optimal policy (AOP)**
   Policy as in NL, Bayesian updating of $p_{t+1|t+1}$ each period
   No experimentation
3. **Bayesian optimal policy (BOP)**
   Optimal policy taking Bayesian updating into account
   Optimal experimentation

1. Optimal policy with no learning (NL)

Replace equation (2) by the two equivalent equations,

$$\mathrm{E}_t H_{j_{t+1}} x_{t+1} = z_t, \tag{7}$$
$$0 = A_{21j_t} X_t + A_{22j_t} x_t - z_t + B_{2j_t} i_t + C_{2j_t} \varepsilon_t, \tag{8}$$

z_t n_x-vector of additional forward-looking variables

Practical way of keeping track of the expectations term on the left side of (2).

Use (8) to solve x_t as a function of X_t, z_t, i_t, j_t, and ε_t

$$x_t = \tilde{x}(X_t, z_t, i_t, j_t, \varepsilon_t) \equiv A_{22j_t}^{-1}(z_t - A_{21j_t} X_t - B_{2j_t} i_t - C_{2j_t} \varepsilon_t). \tag{9}$$

For given j_t, linear in X_t, z_t, i_t, and ε_t.

Apply recursive saddlepoint (RSP) method (Marcet-Marimon)

Dual period loss function

$$\mathrm{E}_t \tilde{L}(\tilde{X}_t, z_t, i_t, \gamma_t, j_t, \varepsilon_t) \equiv \int \sum_j p_{jt|t} \tilde{L}(\tilde{X}_t, z_t, i_t, \gamma_t, j, \varepsilon_t) \varphi(\varepsilon_t) d\varepsilon_t,$$

$\tilde{X}_t \equiv (X_t', \Xi_{t-1}')'$ $(n_X + n_x)$-vector of extended predetermined variables

γ_t n_x-vector of Lagrange multipliers

$\varphi(\cdot)$ denotes a generic probability density function

$$\tilde{L}(\tilde{X}_t, z_t, i_t, \gamma_t, j_t, \varepsilon_t) \equiv L[X_t, \tilde{x}(X_t, z_t, i_t, j_t, \varepsilon_t), i_t, j_t] - \gamma_t' z_t + \Xi_{t-1}' \frac{1}{\delta} H_{j_t} \tilde{x}(X_t, z_t, i_t, j_t, \varepsilon_t).$$

Dual optimization problem

$$\tilde{V}(s_t) = \max_{\gamma_t} \min_{(z_t,i_t)} \mathrm{E}_t[\tilde{L}(\tilde{X}_t, z_t, i_t, \gamma_t, j_t, \varepsilon_t) + \delta \tilde{V}(s_{t+1})] \tag{10}$$

$s_t \equiv (\tilde{X}_t', p_{t|t}')'$ (partial) state of the economy
Transition equation for s_{t+1},

$$\begin{aligned} s_{t+1} &\equiv \begin{bmatrix} X_{t+1} \\ \Xi_t \\ p_{t+1|t+1} \end{bmatrix} = g(s_t, z_t, i_t, \gamma_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}) \\ &\equiv \begin{bmatrix} A_{11 j_{t+1}} X_t + A_{12 j_{t+1}} \tilde{x}(X_t, z_t, i_t, j, \varepsilon_t) + B_{1 j_{t+1}} i_t + C_{1 j_{t+1}} \varepsilon_{t+1} \\ \gamma_t \\ P' p_{t|t} \end{bmatrix}. \end{aligned} \tag{11}$$

Solution quasi-linear

$$z_t = z(s_t) \equiv F_z(p_{t|t})\tilde{X}_t, \tag{12}$$

$$i_t = i(s_t) \equiv F_i(p_{t|t})\tilde{X}_t, \tag{13}$$

$$\gamma_t = \gamma(s_t) \equiv F_\gamma(p_{t|t})\tilde{X}_t, \tag{14}$$

$$x_t = x(s_t, j_t, \varepsilon_t) \equiv \tilde{x}(X_t, z(s_t), i(s_t), j_t, \varepsilon_t) \equiv F_x(p_{t|t})_{j_t} \begin{bmatrix} \tilde{X}_t \\ \varepsilon_t \end{bmatrix}. \tag{15}$$

x_t quasi-linear in $\tilde{X}_t$ and ε_t

Value function for dual problem $\tilde{V}(s_t)$ quasi-quadratic

$$\tilde{V}(s_t) \equiv \frac{1}{2}\tilde{X}_t' \tilde{V}_{\tilde{X}\tilde{X}}(p_{t|t})\tilde{X}_t + \tilde{w}(p_{t|t}).$$

Equilibrium transition equation

$$s_{t+1} = \bar{g}(s_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}) \equiv g(s_t, z(s_t), i(s_t), \gamma(s_t), j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}).$$

Value function for original problem

$$\begin{aligned} V(s_t) &\equiv \tilde{V}(s_t) - \Xi_{t-1}' \frac{1}{\delta} \int \sum_j p_{jt|t} H_j x(s_t, j, \varepsilon_t) \varphi(\varepsilon_t) d\varepsilon_t \\ &= \tilde{V}(s_t) - \Xi_{t-1}' \frac{1}{\delta} \sum_j p_{jt|t} H_j x(s_t, j, 0) \\ &\equiv \frac{1}{2}\tilde{X}_t' V_{\tilde{X}\tilde{X}}(p_{t|t})\tilde{X}_t + \tilde{w}(p_{t|t}). \end{aligned} \tag{16}$$

2. Adaptive optimal policy (AOP)

Policy as under NL (disregarding Bayesian updating), $i_t = i(s_t)$, $z_t = z(s_t)$, $\gamma_t = \gamma(s_t)$

Sequence of information revelation:

(1) CB and PS enter period t with the prior $p_{t|t-1}$. Know X_{t-1}, $x_{t-1} = x(s_{t-1}, j_{t-1}, \varepsilon_{t-1})$, $z_{t-1} = z(s_{t-1})$, $i_{t-1} = i(s_{t-1})$, and $\Xi_{t-1} = \gamma(s_{t-1})$ from the previous period.

(2) In beginning of period t, mode j_t and shocks ε_t realized. Predetermined variables X_t realized according to (1).

(3) CB and PS observe X_t, but not j_t or ε_t. They then know $\tilde{X}_t \equiv (X_t', \Xi_{t-1}')'$.

(4) CB and PS update prior $p_{t|t-1}$ to the posterior $p_{t|t}$

$$p_{jt|t} = \frac{\varphi(X_t|j_t = j, X_{t-1}, x_{t-1}, i_{t-1}, p_{t|t-1})}{\varphi(X_t|X_{t-1}, x_{t-1}, i_{t-1}, p_{t|t-1})} p_{jt|t-1} \qquad (j \in N_j), \tag{17}$$

Then CB and PS know $s_t = (\tilde{X}_t', p_{t|t}')'$.

(5) CB solves dual optimization problem, determines $i_t = i(s_t)$, $\gamma_t = \gamma(s_t)$, announces and implements instrument setting i_t.

(6) Private-sector (and CB) form expectations,

$$z_t = \mathrm{E}_t H_{j_{t+1}} x_{t+1} \equiv \mathrm{E}[H_{j_{t+1}} x_{t+1} \mid s_t] \equiv z(s_t).$$

(7) After $z_t = z(s_t)$ determined, x_t determined as a function of X_t, z_t, i_t, j_t, and ε_t by (9).

(8) CB and PS use the observed x_t to update $p_{t|t}$ to new posterior $p_{t|t}^+$

$$p_{jt|t}^+ = \frac{\varphi(x_t|j_t = j, X_t, z_t, i_t, p_{t|t})}{\varphi(x_t|X_t, z_t, i_t, p_{t|t})} p_{jt|t} \qquad (j \in N_j). \tag{18}$$

(9) CB and PS leave period t and enter period $t+1$ with prior $p_{t+1|t}$ given by prediction equation

$$p_{t+1|t} = P' p_{t|t}^+. \tag{19}$$

Then the sequence of the nine steps above repeats itself.

Details:

$$\varphi(X_t|j_t = j, X_{t-1}, x_{t-1}, i_{t-1}, p_{t|t-1}) \equiv \psi(X_t - A_{11j}X_{t-1} - A_{12j}x_{t-1} - B_{1j}i_{t-1}; C_{1j}C'_{1j}), \tag{20}$$

$$\psi(\varepsilon; \Sigma_{\varepsilon\varepsilon}) \equiv \frac{1}{\sqrt{(2\pi)^{n_\varepsilon}\,|\Sigma_{\varepsilon\varepsilon}|}} \exp\left(-\frac{1}{2}\varepsilon'\Sigma_{\varepsilon\varepsilon}^{-1}\varepsilon\right)$$

$$\varphi(X_t|X_{t-1}, x_{t-1}, i_{t-1}, p_{t|t-1}) \equiv \sum_j p_{jt|t-1}\psi(X_t - A_{11j}X_{t-1} + A_{12j}x_{t-1} + B_{1j}i_{t-1}; C_{1j}C'_{1j}). \tag{21}$$

$$\varphi(x_t|j_t = k, X_t, z_t, i_t, p_{t|t}) \equiv \psi[z_t - A_{21k}X_t - A_{22k}x_t - B_{2k}i_t; C_{2k}C'_{2k}], \tag{22}$$

$$\varphi(x_t|X_t, z_t, i_t, p_{t|t}) \equiv \sum_k p_{kt|t}\psi[z_t - A_{21k}X_t - A_{22k}x_t - B_{2k}i_t; C_{2k}C'_{2k}]. \tag{23}$$

Updating equation (18)

$$\begin{aligned} p^+_{t|t} &= Q^+(s_t, z_t, i_t, j_t, \varepsilon_t) \\ &\equiv [Q^+_1(s_t, z_t, i_t, j_t, \varepsilon_t), ..., Q^+_{n_j}(s_t, z_t, i_t, j_t, \varepsilon_t)]', \end{aligned} \tag{24}$$

$$Q^+_k(s_t, z_t, i_t, j_t, \varepsilon_t) \equiv \frac{\psi[Z_k(X_t, z_t, i_t, j_t, \varepsilon_t); C_{2k}C'_{2k}]}{\sum_k p_{kt|t}\psi[Z_k(X_t, z_t, i_t, j_t, \varepsilon_t); C_{2k}C'_{2k}]} p_{kt|t} \qquad (k \in N_j) \tag{25}$$

$$Z_k(X_t, z_t, i_t, j_t, \varepsilon_t) \equiv z_t - A_{21k}X_t - A_{22k}\tilde{x}(X_t, z_t, i_t, j_t, \varepsilon_t) - B_{2k}i_t,$$

Transition equation for $p_{t+1|t+1}$

$$p_{t+1|t+1} = Q(s_t, z_t, i_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}). \tag{26}$$

Equilibrium transition equation

$$
\begin{aligned}
s_{t+1} &\equiv \begin{bmatrix} X_{t+1} \\ \Xi_t \\ p_{t+1|t+1} \end{bmatrix} = \bar{g}(s_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}) \\
&\equiv \begin{bmatrix} A_{11j_{t+1}} X_t + A_{12j_{t+1}} x(s_t, j_t, \varepsilon_t) + B_{1j_{t+1}} i(s_t) + C_{1j_{t+1}} \varepsilon_{t+1} \\ \gamma(s_t) \\ Q(s_t, z(s_t), i(s_t), j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}) \end{bmatrix},
\end{aligned}
$$

Value function for AOP case.

$$
\begin{aligned}
\bar{V}(s_t) &= \mathrm{E}_t\{L[X_t, x(s_t, j, \varepsilon_t), i(s_t), j_t] + \delta \bar{V}[\bar{g}(s_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1})]\} \\
&= \int \sum_j p_{jt|t} \left\{ \begin{array}{l} L[X_t, x(s_t, j, \varepsilon_t), i(s_t), j] \\ + \delta \sum_k P_{jk} \bar{V}[\bar{g}(s_t, j, \varepsilon_t, k, \varepsilon_{t+1})] \end{array} \right\} \varphi(\varepsilon_t) \varphi(\varepsilon_{t+1}) d\varepsilon_t d\varepsilon_{t+1}. \quad (27)
\end{aligned}
$$

Ex post,

$$
p_{t+1|t+1} = Q(s_t, z_t, i_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1})
$$

is random variable, depends on j_{t+1} and ε_{t+1}. Note that

$$
\mathrm{E}_t p_{t+1|t+1} = \mathrm{E}_t Q(s_t, z(s_t), i(s_t), j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}) = p_{t+1|t} = P' p_{t|t}.
$$

Introducing Bayesian updating of $p_{t+1|t+1}$ is mean-preserving spread of $p_{t+1|t+1}$.
If $V(s_t)$ concave in $p_{t|t}$, lower loss under AOP.
Perceived vs. true transition equation for $p_{t+1|t+1}$.

3. Bayesian optimal policy (BOP)

Transition equation (true and perceived)

$$
\begin{aligned}
s_{t+1} &\equiv \begin{bmatrix} X_{t+1} \\ \Xi_t \\ p_{t+1|t+1} \end{bmatrix} = g(s_t, z_t, i_t, \gamma_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}) \\
&\equiv \begin{bmatrix} A_{11j_{t+1}} X_t + A_{12j_{t+1}} \tilde{x}(s_t, z_t, i_t, j_t, \varepsilon_t) + B_{1j_{t+1}} i_t + C_{1j_{t+1}} \varepsilon_{t+1} \\ \gamma_t \\ Q(s_t, z_t, i_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}) \end{bmatrix}. \quad (28)
\end{aligned}
$$

Dual optimization problem: (10) with the above transition equation.
Solution to dual optimization problem

$$
\begin{aligned}
z_t &= z(s_t) \equiv F_z(\tilde{X}_t, p_{t|t}), && (29) \\
i_t &= i(s_t) \equiv F_i(\tilde{X}_t, p_{t|t}), && (30) \\
\gamma_t &= \gamma(s_t) \equiv F_\gamma(\tilde{X}_t, p_{t|t}), && (31) \\
x_t &= x(s_t, j_t, \varepsilon_t) \equiv \tilde{x}(X_t, z(s_t), i(s_t), j_t, \varepsilon_t) \equiv F_x(\tilde{X}_t, p_{t|t}, j_t, \varepsilon_t). && (32)
\end{aligned}
$$

Because of the nonlinearity of (26) and (28), solution no longer quasi-linear.
Dual value function $\tilde{V}(s_t)$ no longer quasi-quadratic.
Value function for original problem $V(s_t)$ no longer quasi-quadratic.
Optimal experimentation incorporated.

Numerical results so far

- Miranda-Fackler collocation methods, CompEcon toolbox
- Only backward-looking case so far
- Under NL, $V(s_t)$ concave in $p_{t|t}$ (proof?)
- AOP lower loss than NL
- Small difference between BOP and AOP; almost within numerical errors
- Ethical and other issues with BOP relative to AOP: Perhaps not much of a practical problem?
- More simulations and cases needed
- Programs for BOP with forward-looking variables to be completed

Applying the methods of Miranda and Fackler

$s_t \equiv (\tilde{X}_t', p_{t|t}')'$ state, $a_t \equiv (z_t', i_t', \gamma_t')'$ action

Transition equation

$$s_{t+1} = g(s_t, a_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}).$$

(differs from the standard case in its dependence on ε_t and j_t beyond s_t)

Reward function $\tilde{f}(s_t, a_t)$

$$\tilde{f}(s_t, a_t) = \int \sum_j p_{jt|t} \tilde{L}(\tilde{X}_t, z_t, i_t, \gamma_t, j, \varepsilon_t) \varphi(\varepsilon_t)\, d\varepsilon_t,$$

Bellman equation

$$\tilde{V}(s_t) = \max_{\gamma_t} \min_{(z_t, i_t)} \left\{ \tilde{f}(s_t, a_t) + \delta \mathrm{E}_t \tilde{V}[g(s_t, a_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}) \right\},$$

$$\mathrm{E}_t \tilde{V}[g(s_t, a_t, j_t, \varepsilon_t, j_{t+1}, \varepsilon_{t+1}) \equiv$$

$$\equiv \int \sum\nolimits_{j,k} P_{jk} p_{jt|t} \tilde{V}[g(s_t, a_t, j, \varepsilon_t, k, \varepsilon_{t+1})] \varphi(\varepsilon_t) \varphi(\varepsilon_{t+1}) d\varepsilon_t d\varepsilon_{t+1}.$$

In equilibrium, z_t, i_t, and γ_t are functions of s_t; $x_t = \tilde{x}(X_t, z_t, i_t, j_t, \varepsilon_t)$ is also a function of j_t, and ε_t.

Without forward-looking variables: $s_t \equiv (X_t', p_{t|t}')'$, $a_t \equiv i_t$, transition equation

$$s_{t+1} = g(s_t, a_t, j_{t+1}, \varepsilon_{t+1}),$$

reward function

$$f(s_t, a_t) = \sum_j p_{jt|t} L(X_t, i_t, j).$$

Simple example: $n_X = 1$, $n_x = 0$, $n_i = 1$, $n_j = 2$
$s_t \equiv (X_t, q_{t|t})$ *state* ($q_t \equiv p_{1t}$), $a_t \equiv i_t$ *action*.
State transition function $g(s_t, a_t, j_{t+1}, \varepsilon_{t+1})$,

$$s_{t+1} = g(s_t, a_t, j_{t+1}, \varepsilon_{t+1}).$$

$f(s_t, a_t) \equiv \frac{1}{2} X_t^2$ *reward function*. Bellman equation

$$V(s_t) = \min_{a_t} \left\{ f(s_t, a_t) + \delta \mathrm{E}_t V[g(s_t, a_t, j_{t+1}, \varepsilon_{t+1})] \right\}.$$

$\{\varepsilon_l, w_l\}$ be the quadrature nodes and probabilities of a discrete approximation of normal shock ε_{t+1}

$$r(s_t) \equiv q_{t+1|t} \equiv (1 - P_{22}) + (P_{11} + P_{22} - 1) q_{t|t}.$$

Approximation of the Bellman equation ($j_{t+1} \in \{1, 2\}$)

$$V(s_t) = \min_{a_t} \left\{ f(s_t, a_t) + \delta \sum_l w_l \{ r(s_t) V[g(s_t, a_t, 1, \varepsilon_l)] + [1 - r(s_t)] V[g(s_t, a_t, 2, \varepsilon_l)] \} \right\}.$$

Function approximation by collocation methods:
$\{\varphi_j(s)\}_{j=1}^n$ be a set of basis functions. Use the approximation

$$V(s) \approx \sum_{j=1}^{n} c_j \varphi_j(s).$$

Fix the collocation nodes $\{s_i\}_{i=1}^n$. Nonlinear system of n equations for the determination of the n basis coefficients $\{c_j\}$,

$$\sum_{j=1}^{n} c_j \varphi_j(s_i) = \min_a \left\{ f(s_i, a) + \delta \sum_l w_l \left\{ \begin{array}{l} r(s_i) \sum_j^n c_j \varphi_j[g(s_i, a, 1, \varepsilon_l)] \\ +[1 - r(s_i)] \sum_j^n c_j \varphi_j[g(s_i, a, 2, \varepsilon_l)] \end{array} \right\} \right\}$$

for $i = 1, ..., n$. Similar to Miranda and Fackler, except $r(s_i)$ (corresponding to $\sum_{j_{t+1}}$)
Collocation equation

$$\Phi c = v(c),$$

collocation matrix Φ

$$\Phi_{ij} \equiv \varphi_j(s_i),$$

$v : R^n \to R^n$, typical ith element satisfies

$$v_i(c) \equiv \min_a \left\{ f(s_i, a) + \delta \sum_l w_l \sum_j^n c_j \left\{ \begin{array}{l} r(s_i) \varphi_j[g(s_i, a, 1, \varepsilon_l)] \\ +[1 - r(s_i)] \varphi_j[g(s_i, a, 2, \varepsilon_l)] \end{array} \right\} \right\}.$$

Typical ijth element of the Jacobian v' of the collocation function satisfies

$$v'_{ij}(c) \equiv \delta \sum_l w_l \{ r(s_i) \varphi_j[g(s_i, a, 1, \varepsilon_l)] + (1 - r(s_i)) \varphi_j[g(s_i, a, 2, \varepsilon_l)] \}.$$

Main challenge in implementing the collocation method is in coding the routine `vmax` that solves the minimization problem of the Bellman equation,

$$[\mathtt{v}, \mathtt{a}, \mathtt{vjac}] = \mathtt{vmax}(\mathtt{s}, \mathtt{c}),$$

`s` $n \times 1$ vector of collocation nodes, `c` $n \times 1$ vector of basis coefficients, `v` $n \times 1$ vector of optimal values at the collocation nodes, `a` $n \times 1$ vector of optimal action values at the nodes, `vjac` is the $n \times n$ Jacobian of the collocation function at `c`.
The optimization routines of Miranda and Fackler use first- and second-order derivatives of the state transition function with respect to the action. Solves first-order conditions with Newton method.

Numerical example

One predetermined variable, no forward-looking variables, two modes

Loss function

$$L_t = \frac{1}{2} X_t^2, \quad \delta = 0.95$$

Transition equation

$$X_{t+1} = A_k X_t + B_k i_t + C_k \varepsilon_{t+1}$$

$$A_1 = A_2 = 1, \quad C_1 = C_2 = 1,$$

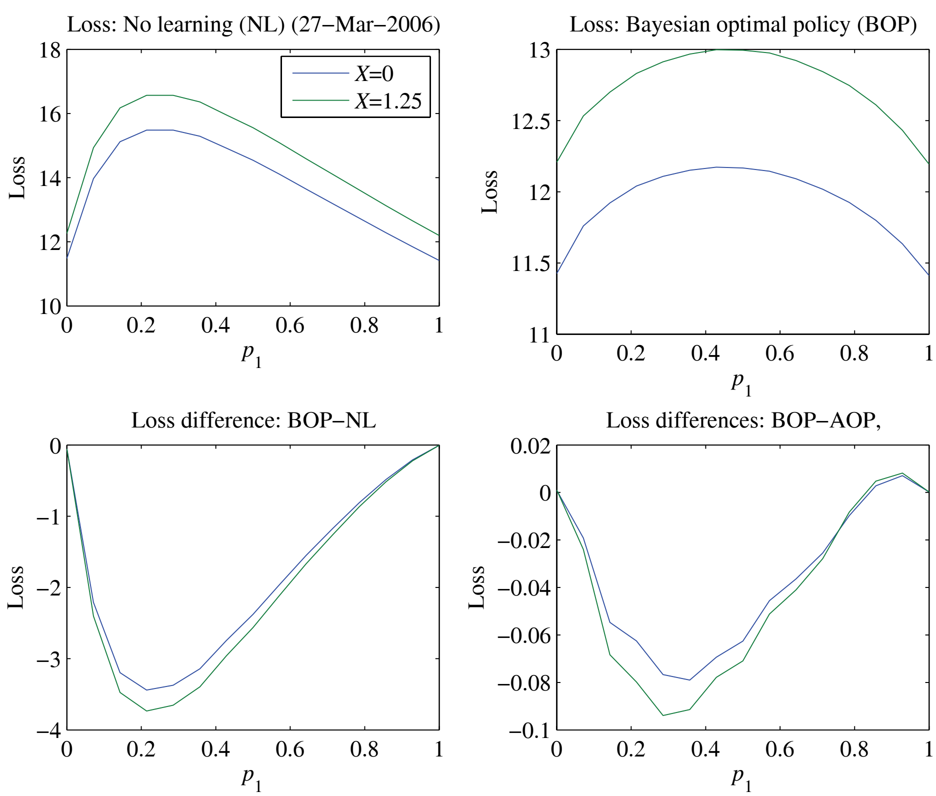
Instrument more effective in mode 1

$$B_1 = -1.5, \quad B_2 = -0.5$$

Highly persistent modes

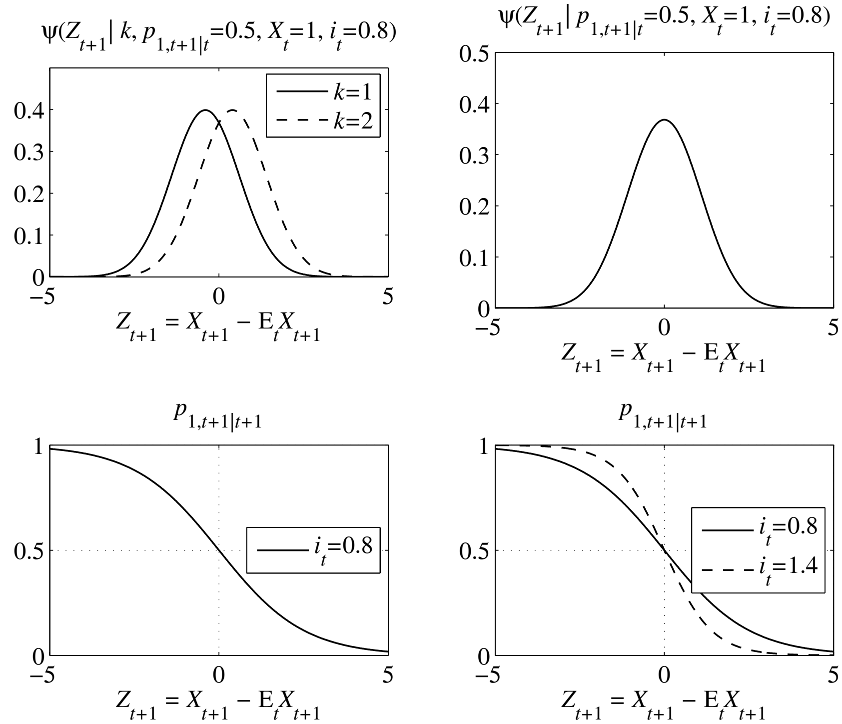
$$P = \begin{bmatrix} 0.999 & 0.001 \\ 0.001 & 0.999 \end{bmatrix}$$

Loss functions



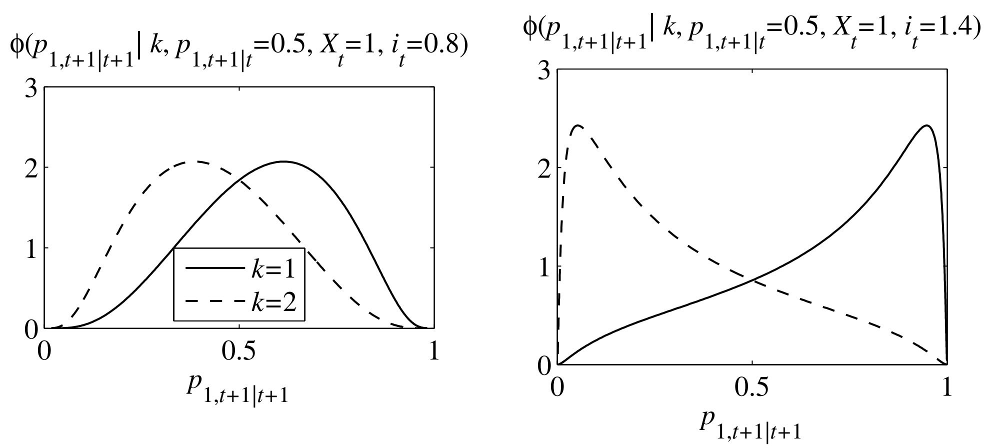
Bayesian uppdating of $p_{t+1|t+1}$

Larger magnitude of i_t, larger difference in X_{t+1} between modes

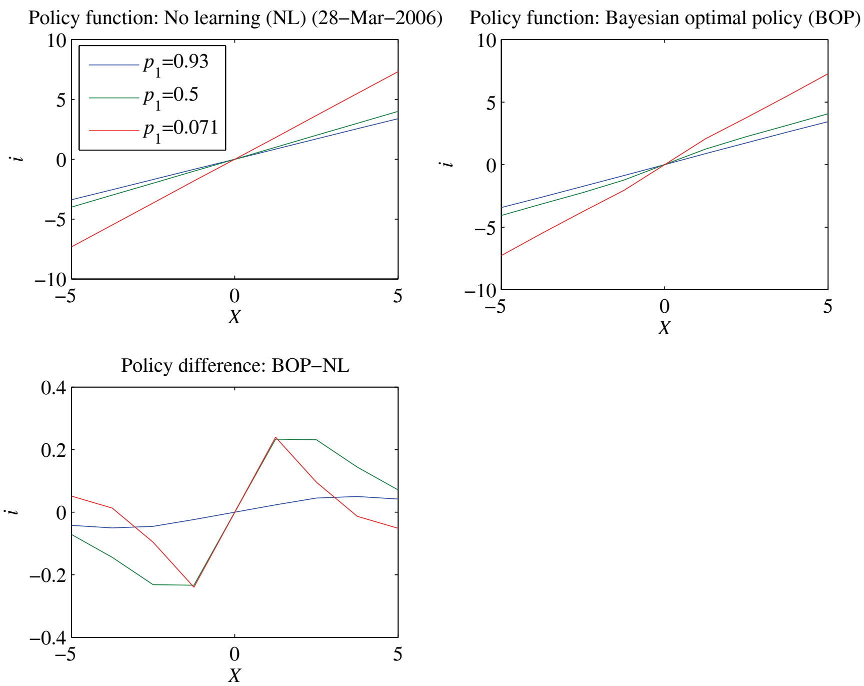


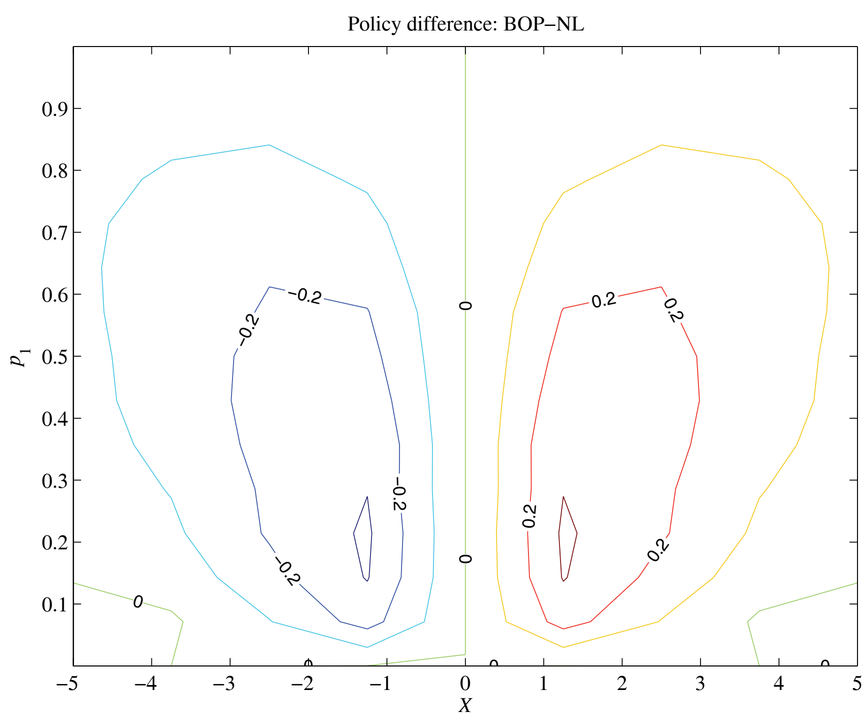
Probability distribution of $p_{t+1|t+1}$, dependence on magnitude of i_t

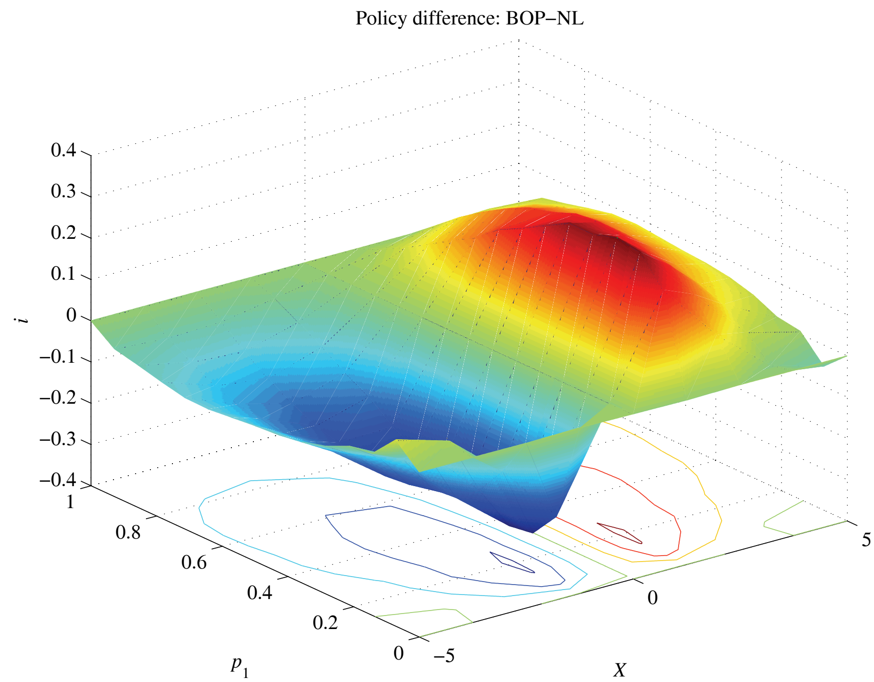
Larger magnitude of i_t, larger difference in X_{t+1} between modes



Policy difference: Under BOP, increased magnitude of i_t






Policy difference: BOP−NL
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
i
1
0.8
0.6
0.4
0.2
0
p_1
−5
0
5
X