Eco 504, part 2, Spring 2007 — 504_P3_S07.tex
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Problem Set 3

1. Consider the Lucas-type Phillips curve

$$y_t = \alpha(\pi_t - \pi_{t|t-1}) + \varepsilon_t,$$

where ε_t is iid with mean zero. Assume that the central bank uses π_t as a control variable. Consider the intertemporal loss function

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} (1-\delta)\delta^{\tau} L_{t+\tau},$$

where the period loss is

$$L_t = \frac{1}{2}(\pi_t^2 + \lambda y_t^2), \qquad \lambda > 0.$$

Rewrite the system on state-space form,

$$\begin{aligned} \begin{bmatrix} X_{t+1} \\ H x_{t+1|t} \end{bmatrix} &= A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + B i_t + C \varepsilon_{t+1}, \\ Y_t &= D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}, \\ L_t &= \frac{1}{2} Y_t' \Lambda Y_t. \end{aligned}$$

Identify the predetermined variables X_t, the forward-looking variables x_t, the instrument(s) i_t, and the target variables Y_t, as well as the matrices A, B, C, D, H, and Λ.

2. Consider the forward-looking model

$$\begin{aligned} \pi_t - \mathrm{E}[\pi_t] &= \delta(\pi_{t+1|t} - \mathrm{E}[\pi_t]) + \kappa x_t + \varepsilon_t, \\ x_t &= x_{t+1|t} - \sigma(i_t - \pi_{t+1|t} - \bar{r} - \eta_t), \end{aligned}$$

where ε_t and η_t are zero-mean and iid. Consider the first-order condition for an optimum (with the standard intertemporal loss function, with period loss $L_t = \frac{1}{2}[(\pi_t - \pi^*) + \lambda x_t^2]$, $\pi^* > 0$, $\lambda \geq 0$) under commitment in period t_0,

$$\begin{aligned} \pi_t - \pi^* - \Xi_t + \Xi_{t-1} &= 0, \qquad t \geq t_0. \\ \Xi_{t_0-1} &= 0. \end{aligned}$$

(a) How is the (log) price level p_t related to Ξ_t? (Recall that $\pi_t \equiv p_t - p_{t-1}$.)

(c) In the standard solution under commitment (start from the solution derived in class), is the price level (i) stationary, (ii) trend-stationary, or (iii) nonstationary?

(d) In an equilibrium under discretion in this model, is the price-level (i) stationary, (ii) trend-stationary, or (iii) nonstationary?

3. Assume that minimization of a central-bank loss function results in

$$\pi_{t+1} - \pi^* = c\,(\pi_t - \pi^*) + \varepsilon_{t+1},$$

$$x_t = \frac{1}{\alpha}\left(\pi_{t+1|t} - \pi_t\right),$$

where π_t is inflation, x_t is the output gap, $0 \leq c \leq 1$, $\alpha > 0$, and ε_t is a zero-mean iid shock with variance $\sigma^2 > 0$.

(a) Derive expressions of the unconditional variances, $\text{Var}[\pi_t]$ and $\text{Var}[x_t]$, as functions of c.

(b) Examine how the variances vary when c goes from zero to one.

(c) Plot (approximatly) $\text{Var}[x_t]$ against $\text{Var}[\pi_t]$ when c varies from zero to one.