Problem Set 2

1. Consider a representative consumer with utility function

$$\mathrm{E}_t \sum_{s=t}^{\infty} \delta^{s-t} u(C_s, m_s, L_s)$$

where $C_s \geq 0$ is aggregate consumption, $m_s \geq 0$ is real balances, and $L_s \geq 0$ is labor in period s. Let

$$u(C, m, L) \equiv U(C) + \tilde{U}(m) - V(L),$$

where $U_C > 0$, $U_{CC} < 0$, $V_L > 0$, $V_{LL} > 0$, and let

$$C_s = \left(\int_{j=0}^{1} C_{js}^{1-\frac{1}{\theta_s}} dj \right)^{\frac{\theta_s}{\theta_s - 1}}, \qquad \theta_s > 1,$$

where C_{js} denotes differentiated consumption goods of variety j, $0 \leq j \leq 1$, in period s.

Assume that goods of each variety j is produced by firm j ($0 \leq j \leq 1$), such that $L_{js} = \Phi(Y_{js})$ denotes the labor input required to produce the quantity Y_{js} of goods of variety j, where $\Phi' > 0$ and $\Phi'' > 0$. Assume that the labor market is perfectly competitive but that each firm operates under monopolistic competition in the market for its variety j.

In particular, assume that $\theta_s > 1$ is an iid disturbance with mean $\bar{\theta} > 1$.

(a) Compare with the derivation of the Calvo-type Phillips curve in class and explain what z_τ, $\bar{z}$, $h_\tau(z_\tau)$, $f_\tau(z_\tau)$, and $g_\tau(z_\tau)$ are in this case.

(b) Define potential output $\bar{Y}_t$ as the flex-price output level when $\theta_t = \bar{\theta}$. Which equation determines potential output?

(c) Complete the derivation of the Phillips curve in terms of the output gap and the disturbance θ_t.

(d) Define potential output $\bar{Y}_t$ instead as the flex-price output level when θ_t varies, that is, as a function of θ_t. Which equation determines potential output? Does the Phillips curve in terms of the output gap still depend on the disturbance θ_t?

2. Assume that a central bank has the period loss function

$$L_t = \frac{1}{2}[(\pi_t - \pi^*)^2 + \lambda(y_t - y^*)^2],$$

where π_t is inflation, π^* is a constant inflation target, y_t is (log) output, y^* is a constant output target, and $\lambda > 0$ is a constant. Suppose the economy has a Lucas-type Phillips curve,

$$y_t = \bar{y} + \alpha(\pi_t - \pi_{t|t-1}) + \varepsilon_t,$$

where $\bar{y}$ is (log) potential output (assumed constant), $\alpha > 0$ is a constant, $\pi_{t|t-1} \equiv \mathrm{E}_{t-1}\pi_t$ (expectations in period $t-1$ of inflation in period t), and ε_t is a zero-mean iid disturbance in period t (not known in period $t-1$). Suppose that the central bank can choose y_t in each period t after it has observed the realization of ε_t.

(a) Assume that the central bank minimizes the loss function under discretion. That is, the central bank chooses y_t each period so as to minimize the period loss function, taking expectations $\pi_{t|t-1}$ as given. Find the equilibrium π_t, y_t, L_t, $\mathrm{E}_{t-1}L_t$ as a function of ε_t.

(b) Assume that the central bank minimizes the loss function under commitment. More precisely, assume that the central bank commits to a linear policy rule, $y_t = a + b\varepsilon_t$, and chooses the response coefficients so as to minimize $\mathrm{E}_{t-1}L_t$. Find the equilibrium π_t, y_t, L_t, and $\mathrm{E}_{t-1}L_t$ as a function of ε_t.

(c) Assume that the central bank minimizes the loss function under commitment. More precisely, assume that the central bank solves the problem

$$\min_{\pi_t^e, y_t, \pi_t} \mathrm{E}_{t-1}L_t$$

subject to the constraints

$$\begin{aligned} y_t &= \bar{y} + \alpha(\pi_t - \pi_t^e) + \varepsilon_t, \\ \pi_t^e &= \mathrm{E}_{t-1}\pi_t, \end{aligned}$$

where π_t and y_t may depend on ε_t but π_t^e does not. Find the equilibrium π_t, y_t, L_t, and $\mathrm{E}_{t-1}L_t$ as a function of ε_t.

(d) Suppose $y^* > \bar{y}$. Compare $\mathrm{E}[\pi_t]$ and $\mathrm{E}[L_t]$ under discretion and commitment.

3. Consider the model

$$\begin{aligned} \pi_t &= \frac{1}{\alpha}\mathrm{E}_t\pi_{t+1} - \frac{\beta}{\alpha}x_t, \\ x_t &= \gamma x_{t-1} + \varepsilon_t. \end{aligned}$$

where α, β, and γ are constants and ε_t is an iid shock with zero mean.

(a) Specify the vector $y_t \equiv (y_{1t}, y_{2t})'$ where y_{1t} is predetermined and y_{2t} nonpredetermined and write the above model as

$$\begin{bmatrix} y_{1,t+1} \\ \mathrm{E}_t y_{2,t+1} \end{bmatrix} = M y_t + N \varepsilon_{t+1}.$$

(b) What are the eigenvalues and eigenvectors of the matrix M?

(c) What conditions must α, β, and γ satisfy for a unique solution of the model?

(d) Form the matrix Q where the columns are the eigenvectors of M. Rewrite the model in terms of the new vector of variables

$$\tilde{y}_t \equiv Q^{-1} y_t$$

and find the solution for $\tilde{y}_t$.

(e) Find the solution for y_t from the relation $y_t \equiv Q \tilde{y}_t$.

(f) Suppose $\alpha = 1.25$, $\beta = 1$, and $\gamma = 0.7$. What are the eigenvalues and eigenvectors in this case?

(g) The Schur decomposition of a square matrix M is a real or complex unitary matrix Q and an upper triangular matrix T such

$$M = UTU'.$$

The diagonal elements of T are the eigenvalues of M. (For a complex matrix, U' denotes the transpose of the complex conjugate of the matrix U. A unitary matrix U is a matrix that satisfies $U' = U^{-1}$, so $U'U = UU' = I$.)

For the parameters in (f), what is the Schur decomposition of M? (You may use Matlab for this question.)