A framework for monetary-policy analysis

Overview

- Basic concepts: Goals, targets, intermediate targets, indicators, operating targets, instruments
- Commitment equilibrium, discretion equilibrium, reaction functions
- Monetary policy rules: Instrument rules, targeting rules
- Linear model, simple example

Basic concepts

- Goals (price stability, high sustainable growth, full employment)
- Targets (operational goals: inflation target, zero output gap,...)
- Intermediate targets (fixed exchange rate, $k\%$ money growth)
  - Not inherent goal
  - Highly correlated with goals (targets)
  - Easier to observe and fulfill than goals (targets)
- Indicators (information variables: state of the economy, stance of monetary policy, goal fulfillment)
- Operating target (almost perfect control, short-term target)
  - Federal funds rate
  - Main refinancing rate
  - Repo rate
- Instrument (perfect control, used to implement policy)
  - Monetary base, nonborrowed monetary base
  - Simplification: Short interest rate

- Monetary policy rules
  - Targeting rule: A given condition for the target variables (or forecasts thereof) to be fulfilled (target criterion). (Optimal targeting rule is first-order condition for optimal policy.)
  - Instrument rule: The instrument is a given function of observed variables
    - Explicit instrument rule: The instrument is a given function of predetermined variables only
    - Implicit instrument rule: The instrument is a given function also of forward-looking variables
    - Simple instrument rule: The instrument is a function of only a few variables

Linear model (Lecture notes: "Optimization under commitment...")

$$\begin{bmatrix} X_{t+1} \\ Hx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1} \tag{1}$$

X_t predetermined variables (constant: $X_t = (1, ...)'$), x_t forward-looking variables, i_t instruments (control variables), ε_t iid zero-mean shocks with covariance matrix I

Exogenous variables: predetermined variables that depend only on current shocks and lagged variables of themselves (X_{1t} below)

$$\begin{bmatrix} X_{1,t+1} \\ X_{2,t+1} \\ [H_{11}\ H_{12}] \begin{bmatrix} x_{1,t+1|t} \\ x_{2,t+1|t} \end{bmatrix} \\ 0 \end{bmatrix} = \begin{bmatrix} A_{X11} & 0 & 0 & 0 \\ A_{X21} & A_{X22} & A_{X23} & A_{X24} \\ A_{x11} & A_{x12} & A_{x13} & A_{x14} \\ A_{x21} & A_{x22} & A_{x23} & A_{x24} \end{bmatrix} \begin{bmatrix} X_{1t} \\ X_{2t} \\ x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} 0 \\ B_{X2} \\ B_{x1} \\ B_{x2} \end{bmatrix} i_t + \begin{bmatrix} C_1 \\ C_2 \\ 0 \\ 0 \end{bmatrix} \varepsilon_{t+1}$$

H need not be invertible. $\begin{bmatrix} A_{x13} & A_{x14} \\ A_{x23} & A_{x24} \end{bmatrix}$ invertible; 3rd and 4th block determine x_t

Target variables: Endogenous variables that enter the loss function. For a target variable Y_{jt} with target level $\hat{Y}_{jt}$, the loss function is increasing in $\left|Y_{jt} - \hat{Y}_{jt}\right|$.
For simplicity, measure target variables as deviations from (constant) target levels

$$Y_t = D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}. \tag{2}$$

Period loss function

$$L_t = \frac{1}{2} Y_t' \Lambda Y_t \equiv \frac{1}{2} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}' W \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}, \tag{3}$$

Λ and $W \equiv D'\Lambda D$ are given symmetric positive semidefinite matrices. The elements of Λ are the weights on the target variables in the period loss function.
Intertemporal loss function in period t be

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} (1-\delta)\delta^{\tau} L_{t+\tau}, \tag{4}$$

where $0 < \delta < 1$ is a discount factor.

- "Optimization under Commitment and Discretion..."
- Optimization under commitment: The commitment equilibrium
  - Lagrange method: Lagrangian, FOCs, difference equation, solution
    - ∗ Solving a system of linear difference equations with forward-looking variables
      - · Diagonalization, Jordan decomposition
      - · Generalized Schur decomposition (Klein, Söderlind, Sims Gensys)
  - Recursive saddlepoint method (Marcet-Marimon): Formulate recursive dual problem, Bellman equation, solve
    - ∗ Linear-quadratic regulator
- Optimization under discretion: The discretion equilibrium
  - Recursive formulation, Bellman equation, iteration

- Commitment equilibrium: Optimal policy under commitment (Söderlind 99, Svensson-Woodford 05)
  CB commits *once and for all* to a reaction function that minimizes (4) in period t_0, subject to X_{t_0}, (1) and (3).
  For $t \geq t_0$:

$$x_t = F_x \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix} \tag{5}$$

$$i_t = F_i \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}, \tag{6}$$

$$\begin{bmatrix} X_{t+1} \\ \Xi_t \end{bmatrix} = M \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix} + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}. \tag{7}$$

  Ξ_t is a vector of Lagrange multipliers of lower block of (1),

$$\Xi_{t_0-1} = 0.$$

  The matrices F_x, F_i, and M depend on A, B, H, D, Λ, and δ, but are independent of C. This demonstrates the *certainty equivalence* of the commitment solution: it is independent of the covariance matrix of the shocks to X_t, CC', and the same as when that covariance matrix is zero.

– History-dependence (Woodford)

$$\Xi_t = M_{\Xi X} X_t + M_{\Xi\Xi} \Xi_{t-1} = \sum_{\tau=0}^{t-t_0} M_{\Xi\Xi}{}^{\tau} M_{\Xi X} X_{t-\tau}$$

- Commitment in a timeless perspective (Woodford, Svensson-Woodford 05). Recommitment in period t, taking into account

$$\Xi_{t-1} \neq 0.$$

Modified intertemporal loss function in period t

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} (1-\delta)\delta^\tau L_{t+\tau} + \frac{1-\delta}{\delta} \Xi'_{t-1} H x_t \tag{8}$$

- Discretion equilibrium (Söderlind 99, Svensson-Woodford 05): CB minimizes (3) *in each period* t, subject to X_t, (1),

$$x_{t+1} = G_{t+1} X_{t+1},$$

and *reoptimization* in period $t+1$.
Equilibrium reaction function under discretion

$$i_t = \hat{F}_{iX} X_t \tag{9}$$

$$x_t = \hat{F}_{xX} X_t$$

– No history-dependence (no dependence on Ξ_{t-1})
– Stabilization bias, $\hat{F}_{iX} \neq F_{iX}$, $\hat{F}_{xX} \neq F_{xX}$
– Sometimes inflation bias.

Discretion equilibrium for modified intertemporal loss function (8) results in commitment in timeless perspective (cf. recursive saddlepoint method)

- Given (explicit) reaction function (policy function)

$$i_t = F X_t \tag{10}$$

F matrix of response coefficients
Respond to predetermined variables only, else equilibrium condition (simultaneity problem)
For given F, add (10) to (1), solve system of difference equations ("A. Solving a system...":

$$\begin{bmatrix} X_{t+1} \\ H x_{t+1|t} \end{bmatrix} = \begin{bmatrix} A_{11} + B_1 F & A_{12} \\ A_{21} + B_2 F & A_{22} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}$$

where

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

This results in

$$x_t = G X_t \tag{11}$$

$$X_{t+1} = (A_{11} + A_{12} G + B_1 F) X_t + C \varepsilon_{t+1} \tag{12}$$

- "Implicit" reaction function, equilibrium relation

$$i_t = FX_t + \tilde{F}x_t \tag{13}$$

  Circularity, equilibrium condition. Implementation? Iteration? Also, not unique.
  Solve system of difference equations
  Equilibrium reaction function ($x_t = GX_t$ in equilibrium)

$$i_t = (F + \tilde{F}G)X_t \tag{14}$$

Simple example (Svensson EER 97, 99, 03 appendix) (backward-looking)

- Lags, imperfect control
  - 2-year control lag for inflation, 1-year control lag for output (VAR studies, Leeper-Sims-Zha)
- Similarity to some models used by CBs (previously)
- Simplest possible (compromise on expectations, Lucas critique)
  Accelerationist Phillips curve

$$\pi_{t+1} = \pi_t + \alpha x_t + \varepsilon_{t+1} \tag{15}$$

  Aggregate demand (output gap)

$$x_{t+1} = \beta_x x_t - \beta_r(i_t - \pi_{t+1|t} - \bar{r}) + \eta_{t+1} \tag{16}$$

  ($\pi_{t+\tau|t} \equiv \mathrm{E}_t\pi_{t+\tau}$, $\bar{r}$ average real interest rate).
  Years, $\pi_t = p_t - p_{t-1}$ inflation, x_t (log) output (gap), i_t instrument (short interest rate), and ε_t and exogenous η_t zero-mean iid shocks, variances σ_ε^2, σ_η^2. All parameters ≥ 0.
  - Note that 1-period-ahead inflation expectations are predetermined in period t

$$\pi_{t+1|t} = \pi_t + \alpha x_t \tag{17}$$

- Objectives: Loss function
  Stabilize inflation around constant inflation target π^*
  Stabilize output gap
  Period loss function

$$L(\pi_t, x_t) = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda x_t^2\right], \tag{18}$$

  $\lambda \geq 0$ (relative) weight on output-gap stabilization
  $\lambda > 0$ "flexible inflation targeting", $\lambda = 0$ "strict inflation targeting"
- State-space form

$$X_{t+1} \equiv \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ \beta_r & \beta_x + \alpha\beta_r \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ -\beta_r \end{bmatrix} (i_t - \bar{r}) + \begin{bmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \end{bmatrix}$$

$$Y_t = \begin{bmatrix} \pi_t - \pi^* \\ x_t \end{bmatrix}, \quad W = \begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix}$$

  – No forward-looking variables, no difference discretion-commitment

- Optimal policy
  – Rewrite AS (1-period-ahead expectations)

$$\begin{aligned} \pi_{t+2} &= \pi_{t+2|t+1} + \varepsilon_{t+2} \\ \pi_{t+2|t+1} &= \pi_{t+2|t} + \varepsilon_{t+1} + \alpha\eta_{t+1} \\ \pi_{t+2|t} &= \pi_{t+1|t} + \alpha x_{t+1|t} \\ x_{t+1} &= x_{t+1|t} + \eta_{t+1} \end{aligned} \tag{19}$$

Note that

$$\mathrm{E}_t\frac{1}{2}[(\pi_{t+1} - \pi^*)^2 + \lambda x_{t+1}^2] = \frac{1}{2}[(\pi_{t+1|t} - \pi^*)^2 + \lambda x_{t+1|t}^2] + \frac{1}{2}(\sigma_\varepsilon^2 + \lambda\sigma_\eta^2)$$

Consider problem

$$\min \mathrm{E}_0 \sum_{t=0}^{\infty} (1-\delta)\delta^{t+1}\frac{1}{2}[(\pi_{t+1|t} - \pi^*)^2 + \lambda x_{t+1|t}^2]$$

subject to

$$\begin{aligned} \pi_{t+2|t+1} &= \pi_{t+1|t} + \alpha x_{t+1|t} + \varepsilon_{t+1} + \alpha\eta_{t+1} \\ &\equiv \pi_{t+1|t} + \alpha x_{t+1|t} + \theta_{t+1} \end{aligned}$$

where $\theta_t \equiv \varepsilon_t + \alpha\eta_t$, and consider $x_{t+1|t}$ the control variable in period t. Then, given $\pi_{t+1|t}$, $x_{t+1|t}$ and x_t, use (16) to choose i_t according to

$$i_t = \bar{r} + \pi_{t+1|t} - \frac{1}{\beta_r}(x_{t+1|t} - \beta_x x_t).$$

– Solve by dynamic programming (problem set)

$$V(\pi_{t+1|t}) = \min_{x_{t+1|t}} \left\{ (1-\delta)\frac{1}{2}\left[\left(\pi_{t+1|t} - \pi^*\right)^2 + \lambda x^2_{t+1|t} \right] + \delta \mathrm{E}_t V(\pi_{t+2|t+1}) \right\} \quad (20)$$

subject to

$$\pi_{t+2|t+1} = \pi_{t+1|t} + \alpha x_{t+1|t} + \theta_{t+1} \quad (21)$$

– Alternative solution: Lagrange method

Introduce the variables

$$\tilde{\pi}_t \equiv \pi_{t+1|t}$$
$$\tilde{x}_t \equiv x_{t+1|t}$$

Constraint

$$\tilde{\pi}_{t+1} = \tilde{\pi}_t + \alpha \tilde{x}_t + \theta_{t+1} \quad (22)$$

Period loss function

$$\tilde{L}_t = \frac{1}{2}[(\tilde{\pi}_t - \pi^*)^2 + \lambda \tilde{x}^2_t]$$

Lagrangian

$$\mathfrak{L}_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} (1-\delta)\delta^t \{ \frac{1}{2}[(\tilde{\pi}_t - \pi^*)^2 + \lambda \tilde{x}^2_t] + \delta \varphi_{t+1}(\tilde{\pi}_{t+1} - \tilde{\pi}_t - \alpha \tilde{x}_t - \theta_{t+1}) \}, \quad (23)$$

where φ_{t+1} is the Lagrange multiplier of the constraint (22). Note that $\tilde{\pi}_t$ is predetermined in period t, and consider the first-order conditions for an optimum, with respect to $\tilde{\pi}_{t+1}$ and $\tilde{x}_t$.

They are

$$\mathrm{E}_t\tilde{\pi}_{t+1} - \pi^* + \mathrm{E}_t\varphi_{t+1} - \delta\mathrm{E}_t\varphi_{t+2} = 0 \quad (24)$$

with respect to $\tilde{\pi}_{t+1}$, and

$$\lambda\tilde{x}_t - \delta\alpha\mathrm{E}_t\varphi_{t+1} = 0 \quad (25)$$

with respect to $\tilde{x}_t$. From (25), we have

$$\mathrm{E}_t\varphi_{t+1} = \frac{\lambda}{\delta\alpha}\tilde{x}_t.$$

Using this in (24), we can write a "consolidated" (without Lagrange multipliers) first-order condition as (Targeting rule!)

$$\mathrm{E}_t\tilde{\pi}_{t+1} - \pi^* + \frac{\lambda}{\delta\alpha}(\tilde{x}_t - \delta\mathrm{E}_t\tilde{x}_{t+1}) = 0 \quad (26)$$

for $t \geq 0$.
In order to find the equilibrium, rewrite (22) as

$$\tilde{x}_t = \frac{1}{\alpha}(\mathrm{E}_t\tilde{\pi}_{t+1} - \tilde{\pi}_t) \quad (27)$$

and use this to eliminate $\tilde{x}_t$ in (26). This results in a difference equation for $\tilde{\pi}_t$,

$$\mathrm{E}_t\tilde{\pi}_{t+1} - \pi^* + \frac{\lambda}{\delta\alpha^2}[(\mathrm{E}_t\tilde{\pi}_{t+1} - \tilde{\pi}_t) - \delta(\mathrm{E}_t\tilde{\pi}_{t+2} - \mathrm{E}_t\tilde{\pi}_{t+1})] = 0.$$

For the case of flexible inflation targeting, $\lambda > 0$, rewrite the difference equation as (problem set!)

$$\mathrm{E}_t[(\tilde{\pi}_{t+2} - \pi^*) - 2a(\tilde{\pi}_{t+1} - \pi^*) + \frac{1}{\delta}(\tilde{\pi}_t - \pi^*)] = 0,$$

where

$$2a \equiv 1 + \frac{1}{\delta} + \frac{\alpha^2}{\lambda} \quad (28)$$

(since $\tilde{\pi}_t$ is given, it is natural to express the difference equation in terms of the inflation forecasts).

By standard methods, the solution to this difference equation can be shown to fulfill

$$\mathrm{E}_t\tilde{\pi}_{t+1} - \pi^* = c(\tilde{\pi}_t - \pi^*) \tag{29}$$

for $t \geq 0$ (recall that $\tilde{\pi}_t$ is given in period t). Here, the coefficient c fulfills $0 < c < 1$ and is the smaller root of the characteristic equation,

$$\mu^2 - 2a\mu + \frac{1}{\delta} = 0; \tag{30}$$

hence given by

$$c \equiv a - \sqrt{a^2 - \frac{1}{\delta}}. \tag{31}$$

Furthermore, c is an increasing function of λ, $c(\lambda)$, which fulfills $c(0) = \lim_{\lambda\to 0} c(\lambda) = 0$, $c(\infty) \equiv \lim_{\lambda\to\infty} c(\lambda) = 1$ (show!).

For the case of strict inflation targeting, $\lambda = 0$, we have $c(0) = 0$, so (29) is replaced by

$$\mathrm{E}_t\tilde{\pi}_{t+1} - \pi^* = 0$$

for $t \geq 0$.

It follows from (22) $\tilde{x}_t$ fulfills

$$\tilde{x}_t = -\frac{1-c}{\alpha}(\tilde{\pi}_t - \pi^*).$$

- Optimal reaction function

  By (16), the optimal interest setting in period t then follows

$$\begin{aligned} i_t &= \bar{r} + \pi_{t+1|t} - \frac{1}{\beta_r}x_{t+1|t} + \frac{\beta_x}{\beta_r}x_t \\ &= \bar{r} + \tilde{\pi}_t - \frac{1}{\beta_r}\tilde{x}_t + \frac{\beta_x}{\beta_r}x_t \\ &= \bar{r} + \tilde{\pi}_t + \frac{1-c}{\alpha\beta_r}(\tilde{\pi}_t - \pi^*) + \frac{\beta_x}{\beta_r}x_t \\ &= \bar{r} + \pi^* + \left(1 + \frac{1-c(\lambda)}{\alpha\beta_r}\right)(\pi_{t+1|t} - \pi^*) + \frac{\beta_x}{\beta_r}x_t & (32) \\ &\equiv \bar{\imath} + f_\pi(\lambda)(\pi_{t+1|t} - \pi^*) + \tilde{f}_x x_t & (33) \end{aligned}$$

  Express in terms of π_t and x_t:

$$\begin{aligned} i_t &= \bar{\imath} + \left(1 + \frac{1-c(\lambda)}{\alpha\beta_r}\right)(\pi_t - \pi^*) \\ &\quad + \left[\alpha\left(1 + \frac{1-c(\lambda)}{\alpha\beta_r}\right) + \frac{\beta_x}{\beta_r}\right]x_t & (34) \\ &\equiv \bar{\imath} + f_\pi(\lambda)(\pi_t - \pi^*) + f_x(\lambda)x_t & (35) \end{aligned}$$

  Definition: A(n explicit) reaction function expresses the instrument as a function of predetermined variables.

- Properties of the optimal reaction function. Dependence on λ.
  - Properties of coefficients

    Coefficients are functions of parameters of AD, AS and LF.

    Coefficients of $\pi_{t+1|t}, x_t$

$$f_\pi(\lambda) \geq 1, \frac{\partial f_\pi}{\partial \lambda} < 0, \quad \tilde{f}_x > 0, \frac{\partial \tilde{f}_x}{\partial \lambda} = 0,$$

    Coefficients of π_t, x_t,

$$f_\pi(\lambda) \geq 1, \frac{\partial f_\pi}{\partial \lambda} < 0, \quad f_x(\lambda) > 0, \frac{\partial f_x}{\partial \lambda} < 0$$

    Result: Optimal to respond to all of π_t, x_t, (or $\pi_{t+1|t}, x_t$).

    Result: Optimal response implies $i_t - \pi_t$ increasing in π_t (or $i_t - \pi_{t+1|t}$ increasing in $\pi_{t+1|t}$). (Taylor principle, Woodford)

    Result: Coefficient response to λ intricate. Not necessarily $\frac{\partial f_x}{\partial \lambda} > 0$.

  - Strict inflation targeting, $\lambda = 0 \Rightarrow c(0) = 0$

    First-order condition

$$\pi_{t+2|t} = \pi^* \tag{36}$$

    2-year-ahead inflation forecast on target (*targeting rule*)

    Optimal reaction function

$$\begin{aligned} i_t &= \bar{\imath} + \left(1 + \frac{1}{\alpha\beta_r}\right)(\pi_{t+1|t} - \pi^*) + \frac{\beta_x}{\beta_r} x_t \\ &= \bar{\imath} + \left(1 + \frac{1}{\alpha\beta_r}\right)(\pi_t - \pi^*) + \left[\alpha\left(1 + \frac{1}{\alpha\beta_r}\right) + \frac{\beta_x}{\beta_r}\right] x_t \end{aligned}$$

  - Flexible inflation targeting ($\lambda > 0$), $\lambda > 0 \Rightarrow 0 < c < 1$

$$\pi_{t+2|t} - \pi^* = \frac{\lambda}{\delta\alpha}(\delta x_{t+2|t} - x_{t+1|t})$$

    2-year-ahead inflation-forecast gap proportional to forecast of output-gap change (*targeting rule*)

– Strict output stabilization, $\lambda \to \infty \Rightarrow c(\infty) = 1$

$$x_{t+1|t} = 0$$

Optimal reaction function

$$\begin{aligned} i_t &= \bar{r} + \pi_{t+1|t} + \frac{\beta_x}{\beta_r} x_t \\ &= \bar{r} + \pi_t + \left(\alpha + \frac{\beta_x}{\beta_r} \right) x_t \end{aligned}$$

• Relative variability of π_t and x_t (problem set)

$$\begin{aligned} \pi_{t+2|t+1} - \pi^* &= c(\lambda) \left(\pi_{t+1|t} - \pi^* \right) + \varepsilon_{t+1} + \alpha \eta_{t+1} \\ \pi_{t+2} &= \pi_{t+2|t+1} + \varepsilon_{t+2} \\ x_{t+1|t} &= \frac{1}{\alpha} \left(\pi_{t+2|t} - \pi_{t+1|t} \right) \\ &= -\frac{1 - c(\lambda)}{\alpha} (\pi_{t+1|t} - \pi^*) \\ x_{t+1} &= x_{t+1|t} + \eta_{t+1} \end{aligned}$$

π_t is an AR(1) plus serially correlated (s.c.) noise.
x_t is proportional to an AR(1) (plus s.c. noise), and hence itself an AR(1) plus s.c. noise.
$\mathrm{Var}[\pi_t]$ increasing, $\mathrm{Var}[x_t]$ decreasing in λ (show!)

– Under strict inflation targeting, π_t is constant plus s.c. noise.

– Under strict output stabilization, x_t is constant plus noise. π_t has unit root.

- Sum up
  - Basic concepts: Goals, targets, intermediate targets, indicators, instruments
  - Predetermined/forward-looking variables
  - Reaction function, implicit reaction function
  - Monetary policy rules: Targeting rules, instrument rules
  - Discretion/commitment equilibria
  - Simple example
    - Endogenous predetermined variables
    - No forward-looking variables: No difference discretion-commitment
    - AS, AD, LF
    - Reasonable control lags
    - Dynamic programming (alternative, Lagrangian)
    - First-order condition
    - Optimal reaction function
    - Respond to all (relevant) predetermined variables (all determinants of forecasts of target variables)
    - Response coefficient properties, model-dependent
    - Targeting rule