- Aggregate supply/Phillips curve, model sticky prices
  - Relation between inflation and real activity
  - Inflation determined by inflation expectations, output gap/marginal cost, fluctuations in markups
  - Inflation expectations: Expectations matter, volatility, "credibility" (inflation expectations equal to inflation target)
  - Marginal cost: Activity, output gap, wages, cost of raw materials, intermediate goods, imported intermediate goods, exchange rates (depreciation), foreign price levels (inflation),...
  - Lags, shocks, imperfect information, fog of monetary policy

- Aggregate demand, real activity
  - Flexible prices: Demand equals supply, potential output, "full" employment (if distortions, not socially optimal)
    * Vertical short-run Phillips curve
    * Prices/inflation determined by demand for real balances and money supply, no role for CBs' influencing real aggregate demand
    * Exogenous money supply (growth) determines prices (inflation), nominal GDP (growth)
    * Nominal interest rates determined by real interest rates (independent of monetary policy) and expected inflation
  - Sticky prices
    * Output demand-determined in short and medium term, output deviates from potential output, output gap $\neq 0$
    * Short real interest rates affected by short nominal interest rates
    * CBs control short interest rates, affect aggregated demand/output

- Models of aggregate demand
  - Nominal aggregate demand, exogenous velocity
  - Traditional IS
  - (Fuhrer-Moore)
  - Forward-looking (microfoundations)
    - ∗ Keeping up with the Joneses, habit formation
- Role of the money demand

Aggregate demand

- Nominal aggregate demand, exogenous velocity
  Instrument: money supply (not interest rate)
  Quantity equation (definition)

$$P_t Y_t \equiv M_t V_t$$
$$p_t + y_t \equiv m_t + v_t$$

  Exogenous v_t, $p_t + y_t$ determined by m_t, separate effects on y_t and p_t determined by aggregate supply
  Problems:
  - Status of quantity equation? Definition, causal relation, long-run equilibrium relation?
  - Instrument only very narrow monetary aggregate (monetary base, nonborrowed reserves) (check Mishkin textbook)
  - Broad money supply (M2, M3) = money multiplier (endogenous) · monetary base
  - Velocity endogenous
  - Lags in effect of m, different lags for p and y
  - Frequent in earlier theoretical work, convenient
  - Not practical use (except Bundesbank money-growth target/ECB reference value)
  - Old-fashioned, outdated

Now, more realism:

- Instrument: Short nominal interest rate, i_t (Fed: Fed funds rate, ECB: Main refinancing rate)
- Effect on *real* aggregate demand, via effect on real interest rate (interest rate channel, intertemporal substitution),

$$r_t \equiv i_t - \pi_{t+1|t} \tag{1}$$

($i_t \equiv \ln(1+i_t^e)$, $r_t \equiv \ln(1+r_t^e)$, continously compounded; i_t^e, r_t^e, "effective")
Expected inflation $\pi_{t+1|t}$ sufficiently sticky, such that i_t has at least a temporary effect on r_t.

- Traditional AD (IS curve), with lag (Svensson EER 97)

$$\begin{aligned} y_{t+1} &= \beta_y y_t - \beta_r r_t + \eta_{t+1} \\ r_t &= i_t - \pi_{t+1|t} \end{aligned}$$

- [Fuhrer-Moore (AER 95)]

$$\begin{aligned} y_{t+1} &= \beta_y y_t - \beta_r r_t^{\mathrm{L}} + \eta_{t+1} \\ r_t^{\mathrm{L}} &= \frac{1}{1+D}\sum_{\tau=0}^{\infty}\left(\frac{D}{1+D}\right)^{\tau} r_{t+\tau|t} \end{aligned}$$

r_t^{L} real interest rate on long coupon bond, D is duration (measure of average maturity of coupon bond).

- Forward-looking (King and Wolman 96, McCallum and Nelson 97, Woodford 96)
Representative consumer, intertemporal CES, $\sigma > 0$

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^{\tau}[U(C_{t+\tau}) + V(\frac{M_{t+\tau}}{P_{t+\tau}})]$$

$$U(C_t) = \begin{cases} A_t \frac{C_t^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}} & \sigma \neq 1 \\ A_t \lim_{\sigma\to 1} \frac{C_t^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}} = A_t \ln C_t & \sigma = 1 \end{cases}$$

(A_t preference shock)
Euler condition, asset-pricing condition, i_t continously compounded nominal interest rate, P_t price level

$$e^{-i_t} = \delta \mathrm{E}_t \frac{U_{C,t+1}/P_{t+1}}{U_{C,t}/P_t} = \delta \mathrm{E}_t \frac{A_{t+1}C_{t+1}^{-1/\sigma}/P_{t+1}}{A_t C_t^{-1/\sigma}/P_t}$$

Loglinear approximation: $\ln \mathrm{E}_t x_t = \mathrm{E}_t \ln x_t$ (disregard Jensen's inequality)
$c_t = \ln C_t,\ p_t = \ln P_t$, $\pi_t = p_t - p_{t-1}$.

$$-i_t = \ln\delta + \mathrm{E}_t(a_{t+1} - \frac{1}{\sigma}c_{t+1} - p_{t+1} - a_t + \frac{1}{\sigma}c_t + p_t)$$

Solve for c_t

$$c_t = c_{t+1|t} - \sigma(i_t - \pi_{t+1|t}) + \sigma(a_t - a_{t+1|t} - \ln\delta)$$

Disregard investment and intermediate inputs. Identify c_t as (log) aggregate demand. Assume output demand determined, y_t (log) output,

$$y_t = c_t$$

Forward-looking aggregate demand equation

$$y_t = y_{t+1|t} - \sigma(i_t - \pi_{t+1|t}) + \sigma(a_t - a_{t+1|t} - \ln\delta) \tag{2}$$

Assume exogenous potential output, exogenous stochastic process $\{\bar{y}_t\}$. Subtract from (2)

$$y_t - \bar{y}_t = y_{t+1|t} - \bar{y}_{t+1|t} - \sigma(i_t - \pi_{t+1|t}) + \sigma(a_t - a_{t+1|t} - \ln\delta) + (\bar{y}_{t+1|t} - \bar{y}_t)$$

Introduce output gap

$$x_t \equiv y_t - \bar{y}_t \tag{3}$$

Natural real interest rate (Wicksell 1898) (exogenous stochastic process)

$$\bar{r}_t \equiv (a_t - a_{t+1|t} - \ln\delta) + \frac{1}{\sigma}(\bar{y}_{t+1|t} - \bar{y}_t) \tag{4}$$

Forward-looking output-gap equation

$$x_t = x_{t+1|t} - \sigma(r_t - \bar{r}_t) \tag{5}$$

Natural interest rate: Real interest rate corresponding to zero output gap. (Includes transient components here. Alternatively: $\bar{r}_t + \xi_t$ where $\bar{r}_t$ more persistent.)

Solve (5) forward

$$x_t = x_{t+T|t} - \sigma\sum_{\tau=0}^{T-1}(r_{t+\tau|t} - \bar{r}_{t+\tau|t}) \quad \text{(measure of monetary-policy stance).}$$

Assume $\lim_{T\to\infty} x_{t+T|t} = 0$, $\lim_{T\to\infty}\sum_{\tau=0}^{T-1}(r_{t+\tau|t} - \bar{r}_{t+\tau|t})$ exists ($r_{t+\tau|t} - \bar{r}_{t+\tau|t} \to 0$ sufficiently fast).

$$\begin{aligned} x_t &= -\sigma\rho_t \\ \rho_t &\equiv \sum_{\tau=0}^{\infty}(r_{t+\tau|t} - \bar{r}_{t+\tau|t}) \equiv \sum_{\tau=0}^{\infty}(i_{t+\tau|t} - \pi_{t+1+\tau|t} - \bar{r}_{t+\tau|t}) \end{aligned}$$

Long real (spot) rate r_t^T, maturity T. Long real natural rate, $\bar{r}_t^T$. Expectations hypothesis:

$$\exp(r_t^T T) = \mathrm{E}_t \exp(\textstyle\sum_{\tau=0}^{T-1} r_{t+\tau})$$

Loglinear approximation

$$\mathrm{E}_t \exp(\textstyle\sum_{\tau=0}^{T-1} r_{t+\tau}) = \exp(\mathrm{E}_t \textstyle\sum_{\tau=0}^{T-1} r_{t+\tau})$$

$$r_t^T = \frac{1}{T}\sum_{\tau=0}^{T-1} r_{t+\tau|t}, \qquad \bar{r}_t^T = \frac{1}{T}\sum_{\tau=0}^{T-1}\bar{r}_{t+\tau|t},$$

$$\rho_t \approx T(r_t^T - \bar{r}_t^T)$$

– Output gap depends on long real rate

– Long real rate depends on expected future short nominal rates (term structure) and inflation expectations

– Expectations matter

– CBs affect the output gap mostly by the expectations of future short nominal interest rate setting and future inflation

- Current output/consumption not forward-looking, predetermined one period

Euler condition

$$\mathrm{E}_t e^{-i_{t+1}} = \delta \mathrm{E}_t \frac{U_{C,t+2}/P_{t+2}}{U_{C,t+1}/P_{t+1}} = \delta \mathrm{E}_t \frac{A_{t+2} C_{t+2}^{-1/\sigma}/P_{t+2}}{A_{t+1} C_{t+1}^{-1/\sigma}/P_{t+1}}$$

Same derivation results in

$$x_{t+1|t} = x_{t+2|t} - \sigma(r_{t+1|t} - \bar{r}_{t+1|t})$$

$$x_{t+1|t} = -\sigma \rho_{t+1|t}$$

$$\rho_{t+1|t} = \sum_{\tau=0}^{\infty} (i_{t+1+\tau|t} - \pi_{t+2+\tau|t} - \bar{r}_{t+1+\tau|t})$$

Now $i_{t+1|t}$, $i_{t+2|t}$,..., matter, not i_t

Current output, output gap, η_t unexplained shock, $\mathrm{E}_t \eta_{t+1} = 0$

$$\begin{aligned} y_t &= y_{t|t-1} + \eta_t \\ x_t &= y_t - \bar{y}_t = y_{t|t-1} + \eta_t - \bar{y}_{t|t-1} - (\bar{y}_t - \bar{y}_{t|t-1}) \\ &= x_{t|t-1} + \eta_t - (\bar{y}_t - \bar{y}_{t|t-1}) \end{aligned}$$

- More persistence: Keeping up with the Joneses (Abel 90) (external habit formation); (internal) habit formation (Fuhrer 98). External habit formation easiest
  $\bar{C}_t$ aggregate consumption, $\gamma \geq 0$

$$U(C_t, \bar{C}_{t-1}) = \begin{cases} A_t \frac{(C_t/\bar{C}_{t-1}^{\gamma})^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}} & \sigma \neq 1 \\ A_t \ln(C_t/\bar{C}_{t-1}^{\gamma}) & \sigma = 1 \end{cases}$$

$$e^{-i_t} = \delta \mathrm{E}_t \frac{U_{C,t+1}/P_{t+1}}{U_{C,t}/P_t} = \delta \mathrm{E}_t \frac{A_{t+1}[C_{t+1}^{-1/\sigma}/\bar{C}_t^{\gamma(1-\frac{1}{\sigma})}]/P_{t+1}}{A_t[C_t^{-1/\sigma}/\bar{C}_{t-1}^{\gamma(1-\frac{1}{\sigma})}]/P_t}$$

$$\begin{aligned} -i_t &= \ln\delta + a_{t+1|t} - \frac{1}{\sigma}c_{t+1|t} - \gamma(1-\frac{1}{\sigma})\bar{c}_t - p_{t+1|t} \\ &\quad - a_t + \frac{1}{\sigma}c_t + \gamma(1-\frac{1}{\sigma})\bar{c}_{t-1} + p_t \\ c_t &= -\gamma(1-\sigma)\bar{c}_t + \gamma(1-\sigma)\bar{c}_{t-1} + c_{t+1|t} \\ &\quad - \sigma(i_t - \pi_{t+1|t}) + \sigma(a_t - a_{t+1|t} - \ln\delta) \end{aligned}$$

  In equilibrium, $y_t = c_t = \bar{c}_t$ (disregard investment, intermediate inputs)

$$y_t = \beta y_{t-1} + (1-\beta)y_{t+1|t} - \tilde{\sigma}[i_t - \pi_{t+1|t} - (a_t - a_{t+1|t} - \ln\delta)]$$

$$\beta \equiv \frac{\gamma(1-\sigma)}{1+\gamma(1-\sigma)} \qquad \tilde{\sigma} \equiv \frac{\sigma}{1+\gamma(1-\sigma)}$$

- Credit channel (Bernanke and Gertler JEP 95, Walsh chap. 7, Bernanke-Gertler-Gilchrist 99)
  Parallel to interest rate/intertemporal substitution channel
  – Bank-lending channel

$$i \uparrow, M \downarrow \Rightarrow \text{Bank deposits} \downarrow \Rightarrow \text{Bank lending} \downarrow$$
$$\Rightarrow \text{Loan interest rates} \uparrow \Rightarrow y \downarrow$$

  – Balance-sheet channel, financial accelerator

$$i \uparrow \Rightarrow \text{Collateral} \downarrow, \text{External-finance premium} \uparrow \Rightarrow y \downarrow$$

- Summary of interest rate/credit channel

$$i \uparrow \Rightarrow r \uparrow \Rightarrow y \downarrow \Rightarrow \pi \downarrow$$

- The role of money
  - Suppose money additively separable

$$u(C, \frac{M}{P}) = U(C) + V(\frac{M}{P})$$

$$\frac{V'(\frac{M}{P})}{U'(C)} = \frac{i_t^e}{1+i_t^e} = 1 - \frac{1}{1+i_t^e} = 1 - e^{-i_t}$$

Just add money demand equation

$$\frac{M_t^d}{P_t} = L(C_t, i_t)$$

CB: Supply whatever money demanded at chosen i_t, money supply perfectly elastic

$$M_t^s = M_t^d$$

Use (narrow) money as an instrument to achieve given i_t (operating target).
Money no other role in the transmission mechanism
Svensson EER 97, 99, Rudebusch-Svensson 02 on money growth targeting vs. inflation targeting

- Suppose not additively separable, $\frac{\partial^2 u}{\partial C \partial \frac{M}{P}} \neq 0$,

$$u(C, \frac{M}{P})$$

Real balances affect U_C and aggregate demand
For reasonable parameters, quantitatively very small effect, disregard (Woodford 03-2, McCallum, Nelson)

- P^* model, price gap, real money gap, alternative Phillips curve, no micro foundations
Real money gap has some predictive power for future inflation, less than the output gap (Gerlach and Svensson 03).
- Nominal money growth has little predictive power for inflation (cf. Eurosystem monetary pillar)