Specify equilibrium under perfect foresight in model in L2

Assume M_0 and B_0 given.

Determine $\{C_t, G_t, Y_t, M_t, B_t, T_t, r_t, i_t, P_t\}_{t=1}^{\infty}$ that fulfill

- First-order conditions ("asset-pricing" conditions)
- Transversality condition
- Budget constraints
- Market equilibrium conditions

Assume exogenous (exchange economy) constant output,

$$Y_t = Y. \tag{1}$$

Assume exogenous constant government consumption (determined by government),

$$G_t = G < Y. \tag{2}$$

Assume exogenous money supply (determined by government/central bank, $B_t^c = M_t$, CB delivers z_t to fiscal authority), constant growth

$$M_t = M_0 \mu^t, \qquad \mu > \beta, \quad 0 < \beta < 1 \tag{3}$$

Assume government (fiscal authority) chooses $\{T_t, B_t\}_{t=1}^{\infty}$ so as to fulfill its intertemporal budget constraint ($B_t = B_t^g - B_t^c$)
("Ricardian policy" in Fiscal Theory of the Price Level)

Household

Goods market equilibrium:

$$C_t = C = Y - G. \tag{4}$$

Real balances, $m_t = \frac{M_t}{P_t}$, equation (L2.7), use (4):

$$\frac{1}{P_t} = \frac{\beta u_C\left(C, m_{t+1}\right)/P_{t+1}}{u_C\left(C, m_t\right) - u_{M/P}\left(C, m_t\right)}. \tag{L2.7}$$

Use (3):

$$[u_C\left(C, m_t\right) - u_{M/P}\left(C, m_t\right)]m_t = \frac{\beta}{\mu}u_C\left(C, m_{t+1}\right)m_{t+1}. \tag{5}$$

From analysis in L2, constant solution to (5),

$$\frac{u_{M/P}(C,m)}{u_C(C,m)} = 1 - \frac{\beta}{\mu} > 0, \quad < 1$$

$$m_t = m > 0. \tag{6}$$

Hence, $\{P_t\}_{t=1}^{\infty}$ determined by

$$P_t = \frac{M_t}{m} = \frac{M_0}{m}\mu^t. \tag{7}$$

(Interpret (L2.7) as asset-pricing equation for real value of money, $\frac{1}{P_t}$.)

Real interest rate:

$$\frac{1}{1+r_t} = \frac{\beta u_C\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)}{u_C\left(C_{t,}\frac{M_t}{P_t}\right)}. \tag{L2.4}$$

Use (4) and (6) in (L2.4):

$$1 + r_t = 1 + r = \frac{1}{\beta} > 1. \tag{8}$$

Nominal interest rate:

$$\frac{1}{1+i_t} = \frac{\beta u_C\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)/P_{t+1}}{u_C\left(C_{t,}\frac{M_t}{P_t}\right)/P_t}. \tag{L2.5}$$

Use (4), (6), (7) and (8) in (L2.5):

$$1 + i = \frac{\mu}{\beta} \; > \; 1 \tag{9}$$

$$\frac{i}{1+i} = 1 - \frac{\beta}{\mu} > 0, \quad < 1$$

Government budget constraint

Period t, nominal

$$P_t G_t + W_t = \frac{1}{1+i_t} W_{t+1} + \frac{i_t}{1+i_t} M_t + P_t T_t \tag{L2.18}$$

(where $W_t \equiv M_{t-1} + B_{t-1}$).

Intertemporal, real

$$\sum_{t=1}^{\infty} d_{1,t} G_t = \sum_{t=1}^{\infty} d_{1,t} \left(\frac{i_t}{1+i_t} m_t + T_t \right) - \frac{M_0 + B_0}{P_1} \tag{10}$$

(where we use $W_1 = M_0 + B_0$ and
assume $\lim_{T\to\infty} d_{1,T+1} \frac{W_{T+1}}{P_{T+1}} = \lim_{T\to\infty} d_{1,T} \frac{1}{1+i_T} \frac{W_{T+1}}{P_T} = 0$)

Use (2), (6), (8) and (9) in (10):

$$\frac{1+r}{r} G = \frac{1+r}{r} \frac{i}{1+i} m + \sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^{t-1} T_t - \frac{M_0 + B_0}{P_1}.$$

Any sequence of taxes $\{T_t\}_{t=1}^{\infty}$ with present value fulfilling

$$\sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^{t-1} T_t = \frac{1+r}{r} \left(G - \frac{i}{1+i} m \right) + \frac{M_0 + B_0}{P_1}$$

is consistent with equilibrium (recall: lumpsum taxes).
("Ricardian equivalence")

For any such sequence of taxes, $\{T_t\}_{t=1}^{\infty}$, the sequence of bond issues, $\{B_t\}_{t=1}^{\infty}$, is then given by

$$\frac{1}{1+i_t} B_t = P_t G - P_t T_t - M_t + M_{t-1} + B_{t-1},$$

in this special case:

$$\frac{1}{1+i} B_t = \frac{M_0}{m} \mu^t (G - T_t - \frac{\mu - 1}{\mu} m) + B_{t-1}.$$

Household budget constraint

For any such sequences of taxes and bond issues, the household budget constraint is fulfilled:

$$\sum_{t=1}^{\infty} d_{1,t} \left(C_t + \frac{i_t}{1+i_t} m_t \right) = \sum_{t=1}^{\infty} d_{1,t} (Y_t - T_t) + \frac{W_1}{P_1}. \tag{L2.15}$$

In this special case:

$$\frac{1+r}{r} \left(C + \frac{i}{1+i} m \right) = \frac{1+r}{r} Y - \sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^{t-1} T_t + \frac{M_0 + B_0}{P_1}.$$

Transversality condition fulfilled (equality above).

- Determination of P_1

  (Above: $P_1 = M_1/m$, where $M_t = M_0\mu^t$ is exogenous.)

  Equilibrium condition for real balances (demand for real balances)

$$\frac{u_m(C, \frac{M_1^d}{P_1})}{u_C(C, \frac{M_1^d}{P_1})} = \frac{i_1}{1+i_1}$$

  Equilibrium condition for nominal interest rate (Fisher equation, inflation expectations, expected future money growth)

$$\frac{1}{1+i_t} = \frac{1}{1+r_t}\frac{1}{\frac{P_{t+1}}{P_t}}$$

  Exogenous supply of money, M_1^s

  Money equilibrium condition

$$M_1^d = M_1^s$$

- Different from Fiscal Theory of Price Level (Cochrane, Leeper, Sims, Woodford). There:
  - Money supply M_1^s endogenous
  - Sequence of taxes $\{T_t\}_{t=1}^{\infty}$ exogenous ("non-Ricardian policy")
  - P_1 adjusts to fulfill government's intertemporal budget constraint, (10) (by affecting real value of nominal liabilities, $W_1 = M_0 + B_0$).

- Determination of the price level in the FTPL

  Assume $\{T_t\}$ exogenous. Assume, for simplicity, $M_t = M_1\mu^{t-1}$ for $t \geq 2$.

  Then it remains to determine endogenous M_1 and P_1.

  Given M_1, we have $m_t = m$ $(t \geq 1)$, $P_1 = \frac{M_1}{m}$, $P_t = \frac{M_t}{m} = \frac{M_1}{m}\mu^{t-1} = P_1\mu^{t-1}$ $(t \geq 1)$,

$$1 + i_t = 1 + i = (1+r)\mu = \frac{\mu}{\beta}, \qquad 1 + r_t = 1 + r = \frac{1}{\beta}.$$

  Government budget constraint (now equilibrium condition), determines P_1 and thereby M_1,

$$\frac{1+r}{r}G = \frac{1+r}{r}\frac{i}{1+i}m + \sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1}T_t - \frac{M_0+B_0}{P_1}.$$

  Solve for P_1,

$$P_1 = \frac{M_0+B_0}{\frac{1+r}{r}\frac{i}{1+i}m + \sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1}T_t - \frac{1+r}{r}G}.$$

  For such an equilibrium to exist, we must assume that M_0, B_0, G and $\{T_t\}$ satisfy.

$$\frac{M_0+B_0}{\frac{1+r}{r}\frac{i}{1+i}m + \sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1}T_t - \frac{1+r}{r}G} > 0.$$

  We can examine how P_1 depends on the exogenous variables (changes in G, $\{T_t\}$, $M_0 + B_0$).

Equilibrium under uncertainty (see Woodford 03 chap. 2 for details)

Assume M_0 and A_1 given (A_1 may depend on P_1).

Determine stochastic processes $\{C_t, G_t, Y_t, M_t, A_{t+1}, T_t, Q_{t,t+1}, P_t\}_{t=1}^{\infty}$ that fulfill

- First-order conditions ("asset-pricing" conditions)
- Transversality condition
- Budget constraints
- Market equilibrium conditions

3. Empirical evidence on money and the real economy

(Walsh 03 chap. 1)

- Long-run correlations between money, inflation and output
- Short-run correlations
- VAR analysis and impulse responses

Long-run correlations Two primary conclusions

1. Money growth and inflation highly correlated ($\rho \geq .9$)
   Friedman: "Inflation is always and everywhere a monetary phenomenon"
   Often misunderstood (cf. ECB rhetoric, see Issing, Gaspar, Angeloni and Tristani 01), Nelson 03:
   - Correlation, no causality implied
   - Causality depends on nature of monetary policy
     - Monetary targeting:
       Exogenous money growth, endogenous inflation, "money growth causes inflation"
     - Inflation targeting:
       Exogenous inflation, endogenous money, "inflation causes money growth"
     - Exchange-rate targeting:
       Both money and inflation endogenous
   - Gali 01: Correlation higher between inflation and nominal interest rates, and inflation and exchange rate depreciation
     "Inflation is always and everywhere a monetary/interest-rate/
     exchange-rate phenomenon"
   - Correlation high between money growth and other nominal variables

2. Money growth and output growth uncorrelated
   "Vertical long-run Phillips curve"
   Not quite as robust as correlation between money growth and inflation
   Correlation inflation and growth: Zero or slightly negative.
   Barro: For high inflation, negative correlation.

Short-run correlations more complex

- See figure 1.1 in Walsh 03.

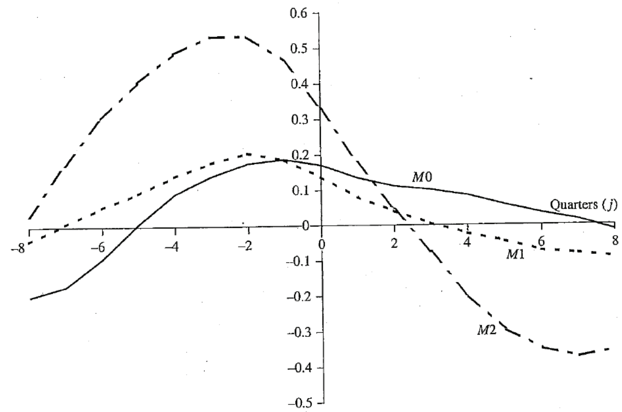


Figure 1.1
Dynamic Correlations, GDP_t and M_{t+j}: 1967:1–2000:4

- Evidence of Friedman and Schwartz 63 of money and output, figure 1.3 in Walsh 98(03)
  - Money seems to lead output (before 1982...)

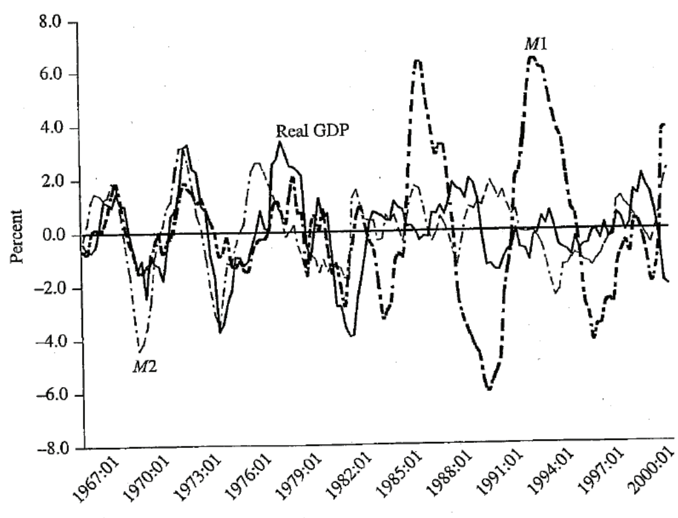


Figure 1.3
Detrended Money and Real GDP

- Debate on causality (both money and output endogenous)
- Granger causality (Sims 72)
  X Granger causes Y if and only if lagged values of X have marginal predictive power in forecasting Y
  Different from causality. Rational expectations! (Necessary condition for causality, not sufficient.)
- Large literature on monetary indicators forecasting output
  Mixed results. Short interest rate better predictive power.
- Output and unanticipated money (Barro, Mishkin)
- Money and inflation? Recent literature on the predictive power of money-growth on inflation (Eurosystem's 1st pillar) (Estrella-Mishkin, Stock-Watson, Gerlach-Svensson, Nicoletti Altamari)
  Money growth poor marginal predictor of future inflation at short and medium horizons (up to 2-3 years)
  Little or no marginal predictive power beyond current inflation and the output gap
- Money arguably less important in monetary policy (broad money not instrument, endogenous, role in transmission mechanism)

VAR analysis and impulse responses

- VAR, reduced form, Z_t k-dimensional vector of variables,
  $$Z_t = B_1 Z_{t-1} + ... + B_q Z_{t-q} + u_t, \tag{11}$$
  u_t serially uncorrelated, $\mathrm{E}u_t = 0$, $\mathrm{E}u_t u_t' = V$
  OLS and fitted residuals give estimates of $B_1, ..., B_q$, and V.
  Underlying structural model
  $$A_0 Z_t = A_1 Z_{t-1} + ... + A_q Z_{t-q} + e_t$$
  where A_0 is invertible and e_t are fundamental economic shocks (structural shocks), $\mathrm{E}e_t e_t' = D$, D diagonal. We have
  $$B_l = A_0^{-1} A_l,\ l = 1, ..., q; \qquad u_t = A_0^{-1} e_t,$$
  $$V = A_0^{-1} D (A_0^{-1})'$$
  In order to identify A_0, A_1, ..., A_q, and D, additional assumptions have to be made (usually zero or linear restrictions on the elements of A_0).
- Impulse responses to shock j , $1 \leq j \leq k$:
  Solution $\{Z_t^j\}_{t=0}^{\infty}$ to (11) for $u_0 = A_0^{-1} e^j$, where e^j is k-vector with $e_j^j = 1$, $e_i^j = 0$ for $i \neq j$; $u_t = 0$ for $t \neq 0$; $Z_t = 0$ for $t < 0$.
  $$Z_t^j = \mathrm{E}[Z_t | e_0 = e^j].$$
- Large literature on impulse responses to monetary-policy shocks (Christiano, Eichenbaum, and Evans 99)

- Definition of monetary-policy shock

$$i_t = f(\Omega_t) + \varepsilon_t$$

i_t monetary-policy instrument (federal funds rate, nonborrowed reserves, monetary base,...) (one of the variables in Z_t)
Ω_t information available to the central bank
$f(\cdot)$ central-bank reaction function, systematic part of policy
ε_t monetary-policy shock

CEE 99, interpretations of monetary-policy shocks:

1. "Exogenous shocks to the preferences of the FOMC" (weight on output-gap stabilization relative inflation stabilization)
   [But this should affect reaction function!]
2. "Strategic considerations of the FOMC", costs of disappointing private sector expectations (?).
3. "Technical factors, measurement errors"
   Bernanke-Mihov: Model of reserves market: Extract MP shock among shocks to demand for total reserves and borrowed reserves
4. [Random mistakes]

Inherent problem: Omitted variables, info/expectations about future variables

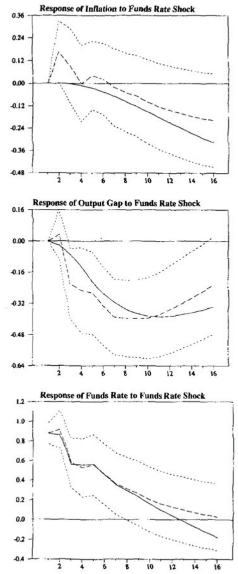
Debate Rudebusch, Sims on identification of monetary-policy shocks.

My view: Measures of monetary-policy shocks and "monetary policy stance" requires structure (model of transmission mechanism, objectives, instruments,...). Also, structure gives systematic part of policy (Rudebusch-Svensson 99)

- General agreement (results robust across large set of identification schemes)
  A contractionary monetary policy shock (for instance, a shock to the federal funds rate) leads to
  - A rise in short term interest rates
  - A fall in output, employment, profits and various monetary aggregates
  - A slow fall in the price level (or a gradual fall in inflation)
  - A modest fall in various measures of wages
- Conventional wisdom among central bankers
  Tighter monetary policy leads to
  - a gradual reduction in the output gap (output less potential output)
  - a gradual fall in inflation, slower/longer lag than for the output gap

  "Monetary-policy actions affect output in about one year and inflation in about two years"

- Example of impulse responses to shock (two std) to federal funds rate (Rudebusch-Svensson 99, figure 1)



- VAR analysis is silent on effects of systematic monetary policy
  - Impulse responses show transmission of shocks under given monetary policy (reaction function)
  - Impulse responses depend on reaction function
  - The reaction function is not structural but reduced form (follows from CB objectives, transmission mechanism)
  - Different reaction functions (one equation in the VAR) may affect other equations in the VAR (Lucas critique)
- To analyze the effect of different (systematic) monetary policies (reaction functions), structural models and structural assumptions are needed

  Next, the transmission mechanism of monetary policy