Eco 504, Macroeconomic Theory II
Final exam, Part 2, Monetary Theory and Policy,
with solutions to problems 1 and 2

Answer all questions. You have 120 minutes to complete the exam, and the questions are worth a total of 120 points. Please allocate your time accordingly.

Short-answer questions (10 points each). Give short answers to these questions (not longer than half a page). Note any important qualifications that you consider necessary.

1. How can multiple equilibria in the form of hyper-inflation or hyper-deflation be avoided in Cagan-type model?
2. What is meant by the Taylor (a) rule, (b) principle, and (c) curve?
3. What is meant by Friedman's (a) optimum quantity of money, (b) $k\%$?
4. Define predetermined variables, forward-looking variables, instruments, and target variables.
5. What is (a) an instrument rule, (b) a targeting rule.
6. Under strict inflation targeting, can the central bank let the instrument rate respond to inflation only?

Problems (60 points).

1. (30 points) Consider an economy with a Phillips curve of the form

$$\pi_t = \delta\pi_{t+1|t} + \kappa x_t + \varepsilon_t, \tag{0.1}$$

where π_t is inflation in period t, x_t is the output gap in period t, ε_t is an i.i.d. shock with zero mean and variance σ_ε^2, $\pi_{t+1|t} \equiv \mathrm{E}_t\pi_{t+1}$, δ is the discount factor ($0 < \delta < 1$), and $\kappa > 0$ is a constant. Assume that the central bank has an intertemporal loss function in period t,

$$\mathrm{E}_t \textstyle\sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau},$$

with a period loss function given by

$$L_t = \frac{1}{2}[\pi_t^2 + \lambda(x_t - x^*)^2],$$

where $\lambda \geq 0$ and $x^* \geq 0$. Assume that the central bank can control x_t and optimizes its loss function under discretion.

(a) Derive a consolidated first-order condition for the policy under discretion.

(b) Solve for the equilibrium dynamics of π_t and x_t as functions of ε_t.

(c) Solve for the unconditional variances $\mathrm{Var}[\pi_t]$ and $\mathrm{Var}[x_t]$. How do they depend on λ?

(d) How does this solution under discretion differ from the solution under commitment? (You don't need to derive the commitment solution.)

Solution: Note that this Phillips curve differs from the one used in Lecture Notes 7 in that the long-run Phillips curve is not vertical (non-optimizing firms keep their price constant, not indexed to average inflation).

(a) In equilibrium under discretion, π_t and x_t will be linear functions of ε_t. Hence, $\pi_{t+1|t}$ will be constant and equal to the unconditional mean $\pi \equiv \mathrm{E}_t[\pi]$ (to be determined). Then the Phillips curve can be written

$$\pi_t = \delta\pi + \kappa x_t + \varepsilon_t. \tag{0.2}$$

Period-by-period optimization of the period loss function subject to (0.2) gives the first-order condition

$$\pi_t + \frac{\lambda}{\kappa}(x_t - x^*) = 0. \tag{0.3}$$

(b) Taking the unconditional mean of (0.2) and (0.3) results in

$$(1-\delta)\pi = \kappa x, \tag{0.4}$$

$$\pi + \frac{\lambda}{\kappa}(x - x^*) = 0, \tag{0.5}$$

where $x \equiv \mathrm{E}[x_t]$. Solving these two equations for π and x gives

$$x = \frac{(1-\delta)\lambda}{(1-\delta)\lambda + \kappa^2} x^*,$$

$$\pi = \frac{\lambda\kappa}{(1-\delta)\lambda + \kappa^2} x^*.$$

Define the deviations from the unconditional means, $\tilde{\pi}_t \equiv \pi_t - \pi$ and $\tilde{x}_t \equiv x_t - x$. Subtracting (0.4) from (0.2) and (0.5) from (0.3) gives

$$\tilde{\pi}_t = \kappa\tilde{x}_t + \varepsilon_t,$$

$$\tilde{\pi}_t + \frac{\lambda}{\kappa}\tilde{x}_t = 0.$$

Solving for $\tilde{x}_t$ and $\tilde{\pi}_t$ gives

$$\tilde{x}_t = -\frac{\kappa}{\lambda + \kappa^2}\varepsilon_t, \tag{0.6}$$

$$\tilde{\pi}_t = \frac{\lambda}{\lambda + \kappa^2}\varepsilon_t.$$

Alternatively, we can use (0.3) to eliminate π_t from (0.1), rewrite the difference equation as

$$\tilde{x}_t = \frac{\delta\lambda}{\lambda + \kappa^2}\tilde{x}_{t+1|t} - \frac{\kappa}{\lambda + \kappa^2}\varepsilon_t,$$

and solve forward to get (0.6).

(c) It follows that

$$\begin{aligned}
\mathrm{Var}[x_t] &\equiv \mathrm{Var}[\tilde{x}_t] = \left(\frac{\kappa}{\lambda+\kappa^2}\right)^2 \sigma_\varepsilon^2, \\
\mathrm{Var}[\pi_t] &\equiv \mathrm{Var}[\tilde{\pi}_t] = \left(\frac{\lambda}{\lambda+\kappa^2}\right)^2 \sigma_\varepsilon^2.
\end{aligned}$$

We see that $\mathrm{Var}[x_t]$ is decreasing in λ (from $\sigma_\varepsilon^2/\kappa^2$ to 0 when λ increases from 0 to ∞) and $\mathrm{Var}[\pi_t]$ is increasing in λ (from 0 to σ_ε^2 when λ increases from 0 to ∞).

(d) Under discretion there is an *average inflation bias* when $x^* > 0$; under commitment (in a timeless perspective) the first-order condition is

$$\pi_t + \frac{\lambda}{\kappa}(x_t - x_{t-1}) = 0,$$

and there is no inflation bias. Under discretion, there is no *history-dependence*; under commitment the solution in period t depends on x_{t-1} or, equivalently, on the Lagrange multiplier Ξ_{t-1} or lagged shocks ε_{t-1}, ε_{t-2}, ... Under discretion there is *stabilization bias* in the sense that the response to the shock ε_t is different than under commitment.

2. (30 points) Consider an economy with flexible prices and the Fisher equation

$$i_t = \bar{r}_t + \pi_{t+1|t},$$

where i_t is the nominal interest rate between period t and $t+1$, π_t is inflation in period t, and $\bar{r}_t$ is the real interest rate in period t. The real interest rate is assumed to be an exogenous stationary stochastic process that is realized and observed at the beginning of each period. Interpret Wicksell's (1898) suggestion for a central-bank instrument rule as implying an implicit instrument rule of the form

$$i_t = a_t + b\pi_t, \tag{0.7}$$

where b is a constant and a_t is a timevarying intercept.

(a) How shall a_t and b be chosen in order to result in an equilibrium with a constant zero inflation rate?

(b) How shall a_t and b be chosen in order to result in an equilibrium with a constant positive inflation rate $\pi^* > 0$?

(c) In what sense is the implicit instrument rule in this case an "out of equilibrium" commitment?

Solution:

(a) Combining the Fisher equation and the instrument rule for $b \neq 0$ gives

$$\pi_t = \frac{1}{b}(\bar{r}_t - a_t) + \frac{1}{b}\pi_{t+1|t}. \tag{0.8}$$

For $a_t = \bar{r}_t$ and $b = 1$, we get

$$\pi_t = \pi_{t+1|t}.$$

This is the equation for a martingale and is consistent with *any* constant or martingale inflation, for instance a standard random walk,

$$\pi_{t+1} = \pi_t + \varepsilon_{t+1},$$

where ε_t is a zero-mean shock.

For $a_t = \bar{r}_t$ and $b = 0$, we get

$$0 = \pi_{t+1|t}.$$

This does not determine π_t, only $\pi_{t+1|t}$. Any $\pi_t = \varepsilon_t$ with $\mathrm{E}[\varepsilon_t] = 0$ is a possible equilibrium.

In order for the instrument rule to result in a *unique* equilibrium, we need to choose some fixed $b > 1$. Then, if $\pi_{t+\tau|t}$ for $\tau \geq 1$ is bounded, we can solve (0.8) forward to get

$$\pi_t = \frac{1}{b} \textstyle\sum_{\tau=0}^{\infty} b^{-\tau} \mathrm{E}_t(\bar{r}_{t+\tau} - a_{t+\tau}), \tag{0.9}$$

and choosing $a_t = \bar{r}_t$ will result in the unique equilibrium $\pi_t = 0$ for all t.

(b) For a unique equilibrium $\pi_t = \pi^* > 0$, for a fixed $b > 1$, guess that $a_t = \bar{r}_t - c$, where c is a constant to be determined. Using this in (0.9) results in

$$\pi_t = \frac{1}{b} \frac{1}{1 - 1/b} c = \frac{c}{b-1}.$$

Setting $\pi_t = \pi^*$ gives

$$c = (b-1)\pi^*.$$

The argument in (a) and (b) above goes through for $|b| > 1$, so we could also have $b < -1$. However, a negative b is not consistent with Wicksell's verbal description of the rule.l

3. (c) In equilibrium, we have $\pi_t = \pi^*$, so the equilibrium interest rate is given by

$$i_t = a_t + b\pi_t = \bar{r}_t + \pi^* + b(\pi_t - \pi^*) = \bar{r}_t + \pi^*.$$

Hence, only $i_t = \bar{r}_t + \pi^*$ is observed for all t. However, for the existence of a unique equilibrium, expectations must be formed under the assumption that the central bank sets the interest rate equal to $\bar{r}_t + \pi^* + b(\pi_t - \pi^*)$ if π_t would ever deviate from π^*. In this sense, the instrument rule is an out-of-equilibrium commitment, stating what the central bank would do if the π_t would deviate from the equilibrium level π^*.