

Gaps*

Lars E.O. Svensson
Sveriges Riksbank and Stockholm University

First draft: December 2011
This draft: February 25, 2012

Start from a Cobb-Douglas production function,

$$Y_t = A_t H_t^\alpha K_t^{1-\alpha}, \quad (1)$$

where Y_t is output, A_t is total factor productivity (TFP), H_t is (total) hours (worked), K_t is capital services, and the coefficient α satisfies $0 < \alpha < 1$. Furthermore,

$$\begin{aligned} H_t &\equiv L_t M_t, \\ L_t &\equiv \bar{L}_t (1 - u_t), \\ \bar{L}_t &\equiv G_t N_t, \\ K_t &\equiv Z_t \bar{K}_t, \end{aligned}$$

where L_t is employment, M_t is average hours (worked), \bar{L}_t is the labor force, u_t is the unemployment rate, G_t is the (labor-force) participation rate, N_t is the population size, Z_t is the capital utilization rate, and \bar{K}_t is the capital stock. (This is inspired by the OECD approach, see for instance Beffy, Ollivaud, Richardson, and Sédillot (2006).)

Let lower-case letters denote logarithms. Then we can write the production function as

$$\begin{aligned} y_t &= a_t + \alpha h_t + (1 - \alpha) k_t \\ &= [a_t + (1 - \alpha)(k_t - h_t)] + h_t, \end{aligned}$$

where

$$y_t - h_t = a_t + (1 - \alpha)(k_t - h_t)$$

is labor(-hours) productivity.

*The views expressed here are my own and are not necessarily shared by the other members of the Riksbank's Executive Board or the Riksbank's staff.

Let $*$ denote “potential” levels or rates. Then the employment gap, $l_t - l_t^*$, satisfies

$$\begin{aligned} l_t - l_t^* &\equiv \bar{l}_t - \bar{l}_t^* + [\ln(1 - u_t) - \ln(1 - u_t^*)] \\ &= (\bar{l}_t - \bar{l}_t^*) - (u_t - u_t^*) \\ &\equiv (g_t - g_t^*) - (u_t - u_t^*), \end{aligned}$$

where $\bar{l}_t - \bar{l}_t^*$ is the labor-force gap, the approximation (for small u_t) $[\ln(1 - u_t) - \ln(1 - u_t^*)] = -(u_t - u_t^*)$ is used, $u_t - u_t^*$ is the unemployment gap, and $g_t - g_t^*$ is the participation gap. The hours gap, $h_t - h_t^*$, satisfies

$$h_t - h_t^* = (l_t - l_t^*) + (m_t - m_t^*),$$

where $m_t - m_t^*$ is the average-hours gap. The output gap, $y_t - y_t^*$, satisfies

$$\begin{aligned} y_t - y_t^* &= (a_t - a_t^*) + \alpha(h_t - h_t^*) + (1 - \alpha)(k_t - k_t^*) \\ &= \{[a_t + (1 - \alpha)(k_t - h_t)] - [a_t^* + (1 - \alpha)(k_t^* - h_t^*)]\} + (h_t - h_t^*), \end{aligned}$$

where $a_t - a_t^*$ is the TFP gap, $k_t - k_t^*$ is the capital-services gap, and $[a_t + (1 - \alpha)(k_t - h_t)] - [a_t^* + (1 - \alpha)(k_t^* - h_t^*)]$ is the labor-productivity gap. The labor productivity gap satisfies

$$[a_t + (1 - \alpha)(k_t - h_t)] - [a_t^* + (1 - \alpha)(k_t^* - h_t^*)] = (a_t - a_t^*) + (1 - \alpha)(k_t - k_t^*) - (1 - \alpha)(h_t - h_t^*).$$

Note that we can write

$$\begin{aligned} l_t - l_t^* &\equiv (\bar{l}_t - \bar{l}_t^*) - (u_t - u_t^*) \\ &\equiv (g_t - g_t^*) - (u_t - u_t^*) \\ &\equiv -\{u_t - [u_t^* + (\bar{l}_t - \bar{l}_t^*)]\} \\ &\equiv -\{u_t - [u_t^* + (g_t - g_t^*)]\}. \end{aligned}$$

Thus, stabilizing employment around potential employment is equivalent to stabilizing unemployment around the potential rate corrected with the labor-force gap or (equivalently) the participation gap.

References

Beffy, Pierre-Olivier, Patrice Ollivaud, Pete Richardson, and Franck Sédillot (2006), “New OECD Methods for Supply-side and Medium-term Assessments: A Capital Services Approach,” OECD Economics Department Working Paper No. 482, OECD.