The transmission mechanism

How the monetary-policy instrument affects the economy and the target variables

Variables

t = ..., −1, 0, 1, ... denotes years
it Federal funds rate during t = ..., −1, 0, 1, ..., %/yr (instrument)
(one-year nominal interest rate from year t to year t + 1)
Yt real GDP during year t (measured in physical units)
Y real potential GDP (constant for simplicity, could be growing at exogenous rate)
yt output gap during year t, %
yt = 100Yt − Y
Output gap measure of aggregate demand, business-cycle indicator
y > 0 boom, yt < 0 recession

Ut unemployment rate in year t, %
U natural/potential unemployment rate, NAIRU (non-accelerating inflation rate of unemployment), %
(constant for simplicity)
Ut unemployment gap, %
Ut = Ut − U
Okun’s Law (approximation)
yt = −γut, γ ≈ 2.5 ± 0.5

Pt price level in year t (CPI)
πt inflation in year t (from year t − 1 to year t), %/yr
πt = 100Pt − Pt−1
Pt−1
rt real interest rate during period t, %
(one year real interest rate from year t to year t + 1)
rt = it − Etπt+1,
where Et denotes expectations in year t (mean of uncertain πt+1 given the information available in
year t)
(real interest rate between year t and t + 1 = nominal interest rate between year t and t + 1 −
expected inflation between year t and t + 1)
Instead, use simplification (pseudo real interest rate)
r = it − πt
Empirical regularities (simplifications)

- Inflation in year $t+1$ depends (approximately) positively on inflation in year $t$ and the output gap in year $t$
  
  Simplest representation

$$\pi_{t+1} = \pi_t + \alpha y_t + \varepsilon_{t+1}$$  

(1)

$\alpha > 0$ (positive constant)

$\varepsilon_{t+1}$ random shock/disturbance in year $t+1$ (other determinants of inflation), not known in year $t$, zero on average: $E_t \varepsilon_{t+1} = 0$

- Phillips curve, aggregate-supply relation
- Simplest explanation: Prices of goods and services set one year in advance, depend positively on expected inflation and current business cycle. Expected inflation depends positively on current inflation.
- For zero shock, increasing/constant/decreasing inflation if positive/zero/negative output gap.

$$\pi_{t+1} - \pi_t = \alpha y_t$$

- Output (gap) in year $t+1$ depends positively on output (gap) in year $t$ and negatively on real interest rate in year $t$

$$y_{t+1} = \beta_y y_t - \beta_r (i_t - \pi_t - \bar{r}) + \eta_{t+1}$$  

(2)

$\beta_y, \beta_r, \bar{r}$ constants, $0 < \beta_y < 1$, $\beta_r > 0$, $\bar{r} > 0$

$\eta_{t+1}$ random shock in year $t+1$ (other determinants of output gap), not known in year $t$, zero on average, $E_t \eta_{t+1} = 0$

- Aggregate-demand relation, IS curve
- Simplest explanation: Output in year $t+1$ is determined by aggregate demand in year $t+1$.

  Aggregate demand is the sum of demand for consumption and investment. Demand for consumption and investment in year $t+1$ depends positively on output (income) in period $t$. Demand for consumption and investment depend negatively on the real interest rate in year $t$.

  A higher real interest rate stimulates saving and reduces consumption, increases the cost of borrowing and reduces demand for consumer durables, housing, and investment. In an open economy, a higher interest rate tends to attracts capital from abroad and appreciates the currency, makes exports and import substitutes less competitive and thereby less in demand.
For zero shock and zero output gap in year $t$, increasing/constant/decreasing output gap in year $t+1$ for real interest rate in year $t$ below/equal to/above average real interest rate.

Equations (1) and (2) provide simplest possible model of the transmission mechanism

$$i_t \uparrow \Rightarrow y_{t+1} \downarrow \Rightarrow \pi_{t+2} \downarrow$$

- 1-yr lag for effect on output gap, 2-yr lag for effect on inflation
- Current monetary policy ($i_t$) has no effect on the output gap this year ($y_t$) and inflation this year ($\pi_t$) and next year ($\pi_{t+1}$)
- Shocks in years $t+1$ and $t+2$ not known in year $t$ ⇒ Uncertainty about future inflation and output gap
- Simplification: lags and model coefficients known
- Effects of $\pi_t$ and $y_t$ for given $i_t$

$$\pi_t \uparrow \Rightarrow \pi_{t+1} \uparrow, y_{t+1} \uparrow \Rightarrow \pi_{t+2} \uparrow$$

$$y_t \uparrow \Rightarrow \pi_{t+1} \uparrow, y_{t+1} \uparrow \Rightarrow \pi_{t+2} \uparrow$$
• Long run-equilibrium for zero shocks
  Can we find a “steady state”: Constant interest rate, output gap, inflation: \( i, y, \pi \)?
  
  - Phillips curve:
    \[
    \pi = \pi + \alpha y \\
    y = 0
    \]
    \( Y = \bar{Y} \) (independent of \( i \), monetary policy)
    \( \pi \) not determined (any constant \( \pi \) possible, will depend on monetary policy)
  - Aggregate-demand relation:
    \[
    y = \beta_i y - \beta_r (i - \pi - \bar{r}) \\
    0 = -\beta_r (i - \pi - \bar{r}) \\
    i = \bar{r} + \pi
    \]
    For any constant \( \pi, i = \bar{r} + \pi \)

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Objectives for monetary policy

- Flexible inflation targeting: Stabilize inflation around an inflation target with some weight on stabilizing the output gap

- Target variables: \( \pi_t \) and \( y_t \)
  Target levels: Constant inflation target, \( \pi^* > 0 \). Output-gap target: 0 (Output target: \( \bar{Y} \))
  Period loss function
  \[
  L_t = (\pi_t - \pi^*)^2 + \lambda y_t^2 \geq 0
  \]
  \( \pi^* > 0 \), constant inflation target
  \( \lambda > 0 \), constant relative weight on output gap stability relative to inflation stability
  \( L_t = 0 \) for \( \pi_t = \pi^* \) and \( y_t = 0 \)

Intertemporal loss function in year \( t \)

\[
E_t \sum_{t=0}^{\infty} \delta^t L_{t+\tau} = E_t[L_t + \delta L_{t+1} + \delta^2 L_{t+2} + \delta^3 L_{t+3} + ...] \tag{3}
\]
\( \delta \) given discount factor \((0 < \delta \leq 1)\), expresses how the central bank weighs the future vs. the present
Sum of discounted future period losses

- Policy problem in year \( t \): Set the instrument rate \( i_t \) so as to minimize (3) subject to (1), (2), and given \( \pi_t \) and \( y_t \).
Optimal policy under strict inflation targeting

- Simple case: Strict inflation targeting, $\lambda = 0$
  
  $$L_t = (\pi_t - \pi^*)^2$$

Simple case: Suppose all shocks are zero (no uncertainty, perfect control)

Optimal policy in year $t$? How to set $i_t$?

- $y_t, \pi_t$ given
  
  $$\pi_{t+1} = \pi_t + \alpha y_t$$

- $\pi_{t+1}$ given, independent of $i_t$

- $\pi_{t+2}$ depends on $i_t$.

  $$L_{t+2} = (\pi_{t+2} - \pi^*)^2 \begin{cases} \geq 0 & (\pi_{t+2} \neq \pi^*) \\
= 0 & (\pi_{t+2} = \pi^*) \end{cases}$$

Minimum loss for $\pi_{t+2} = \pi^*$. Hence, set $i_t$ such that

  $$\pi_{t+2} = \pi^*$$

Policy rule, optimal “targeting rule,” rule for target variables (only $\pi_{t+2}$ here)

- Express $\pi_{t+2}$ as a function of $i_t, \pi_t$ and $y_t$; set $\pi_{t+2} = \pi^*$; and solve for $i_t$

  $$\pi_{t+2} = \pi_{t+1} + \alpha y_{t+1}$$
  
  $$= (\pi_t + \alpha y_t) + \alpha[\beta_y y_t - \beta_r (i_t - \pi_t - \bar{r})]$$
  
  $$= \pi_t + \alpha(1 + \beta_y) y_t - \alpha \beta_r (i_t - \pi_t - \bar{r})$$

  $$\pi_t + \alpha(1 + \beta_y) y_t - \alpha \beta_r (i_t - \pi_t - \bar{r}) = \pi^*$$

  $i_t - \pi_t - \bar{r} = \frac{1}{\alpha \beta_r}(\pi_t - \pi^*) + \frac{1 + \beta_y}{\beta_r} y_t$

  $$i_t = \bar{r} + \pi_t + \frac{1}{\alpha \beta_r}(\pi_t - \pi^*) + \frac{1 + \beta_y}{\beta_r} y_t$$
\[ i_t = \bar{r} + \pi^* + (1 + \frac{1}{\alpha \beta_r})(\pi_t - \pi^*) + \frac{1 + \beta_y}{\beta_r}y_t \]

- Optimal policy function/reaction function/instrument rule: Says how to set \( i_t \) as a function of current “state variables” \( \pi_t \) and \( y_t \) (“state variables” describe state of the economy)

\[ i_t = \bar{r} + \pi^* + f_\pi(\pi_t - \pi^*) + f_y y_t \quad (6) \]

Response coefficients \( f_\pi \) and \( f_y \)

\[ f_\pi \equiv 1 + \frac{1}{\alpha \beta_r} > 1 \]

\[ f_y \equiv \frac{1 + \beta_y}{\beta_r} > 0 \]

In this case similar to form as well-known Taylor rule (1993)

\[ i_t = \bar{r} + \pi^* + 1.5(\pi_t - \pi^*) + 0.5y_t = 4 + 1.5(\pi_t - 2) + 0.5y_t \]

- Properties of optimal reaction function:
- \( \pi_t = \pi^* \) and \( y_t = 0 \) implies \( i_t = \bar{r} + \pi^* \) (steady state)
- \( i_t \) increasing in \( \pi_t - \pi^* \) and \( y_t \) (\( f_\pi > 0, f_y > 0 \))
- \( i_t \) increasing more than one-to-one in \( \pi_t - \pi^* \) (\( f_\pi > 1 \)), “Taylor principle”

\[ r_t = i_t - \pi_t = \bar{r} + \frac{1}{\alpha \beta_r}(\pi_t - \pi^*) + f_y y_t \]

\[ r_t \] increasing in \( \pi_t - \pi^* \)

- Response coefficients depends on the model parameters \((\alpha, \beta_y, \beta_r, \bar{r})\)

- Derive targeting rule (4) in different way.

Equation (5) specifies \( \pi_{t+2} \) as a function \( h(\pi_t, y_t, i_t) \) of \( \pi_t, y_t, i_t \)

\[ \pi_{t+2} = h(\pi_t, y_t, i_t) \equiv (1 + \alpha \beta_r)\pi_t + \alpha(1 + \beta_y) y_t - \alpha \beta_r(i_t - \bar{r}) \quad (7) \]

Write the loss \( L_{t+2} \) as a function of \( \pi_t, y_t, i_t \)

\[ L_{t+2} = (\pi_{t+2} - \pi^*)^2 = [h(\pi_t, y_t, i_t) - \pi^*]^2 \]

In order to find a minimum of \( L_{t+2} \) with respect to \( i_t \), set the derivative \( \frac{\partial L_{t+2}}{\partial i_t} \) equal to zero.

First-order condition for minimum of \( L_{t+2} \)

\[ 0 = \frac{\partial L_{t+2}}{\partial i_t} = 2[h(\pi_t, y_t, i_t) - \pi^*] \frac{\partial h}{\partial i_t} = 2(\pi_{t+2} - \pi^*)(-\alpha \beta_r) \]

\( \alpha \beta_r \neq 0 \Rightarrow \pi_{t+2} = \pi^*. \) Hence, (4) follows. The optimal targeting rule is actually a first-order condition for a minimum of the loss function.

- The optimal reaction function is found by combining the targeting rule, (4), with the target variable as a function of \( i_t \) and the state variables, (7), and solving for \( i_t \).