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# Comment on Michael Woodford, Imperfect Common Knowledge and the Effects of Monetary Policy*

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The general idea of this paper, Woodford [1], is to abandon the Phelps-Lucas model for the Phelps-Woodford model. We know that the Phelps-Lucas idea is that real effects of nominal disturbances arise because of assumed imperfect information about these disturbances. But the standard Lucas formulation of this results in too little persistence of these real effects. Mike suggests a more elaborate interpretation of the Phelps idea, such that prices depend on "higher-order" estimates of nominal disturbances. If these higher-order estimates are updated more slowly than first-order estimates, more persistent real effects arise. Mike shows that this idea can generate impulse responses of inflation and output with considerable persistence.

The model has a continuum of firms in monopolistic competition. Each firm $i$ will in equilibrium set its (log) price $p_t(i)$ according to

$$p_t(i) = p_{t|t}(i) + \xi y_{t|t}(i) \tag{2.6}$$

where $p_{t|t}(i)$ denotes the firm's estimate in period $t$ of the (log) price level $p_t$, $y_{t|t}(i)$ denotes the firm's estimate of the output gap $y_t$, and $\xi$ $(0 < \xi < 1)$ is a parameter indicating the degree of strategic complementarity between firms' pricing decisions (a lower $\xi$ corresponds to a higher degree of complementarity). Averaging over firms leads to the aggregate price equation

$$p_t = p_{t|t} + \xi y_{t|t}, \tag{2.6a}$$

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where $p_{t|t}$ and $y_{t|t}$ denote average estimates over firms of the price level and the output gap, respectively.

Mike assumes that monetary policy corresponds to nominal-GDP-growth targeting (adjusted for the growth of potential output). More precisely, he assumes that (1) (adjusted) nominal GDP growth is exogenous and (2) nominal GDP (adjusted for potential output) is the crucial variable that firms observes (with noise). This is modeled as nominal GDP (adjusted for potential output), $p_t + y_t$, being equal to an exogenous stochastic process with a unit root, $q_t$,

$$p_t + y_t = q_t. \tag{2.6b}$$

The process $\Delta q_t$ can the be interpreted as corresponding to either an exogenous stochastic target for nominal GDP growth or unavoidable deviations from a constant growth target. Combining this with (2.6a) leads to

$$\begin{aligned} p_t &= \xi q_{t|t} + (1-\xi) p_{t|t} && (2.7) \\ &= \sum_{k=1}^{\infty} \xi (1-\xi)^{k-1} q_t^{(k)}, && (2.8) \end{aligned}$$

where $q_t^{(k)}$ denotes the $k$-order estimate of nominal GDP according to the notation $q_t^{(k)} \equiv q_{t|t}^{(k-1)} \equiv \int \mathrm{E}_t^i q_t^{(k-1)} di$, $k \geq 1$, $q_t^{(0)} \equiv q_t$.

Mike first assumes that nominal GDP growth follows an AR(1) process,

$$\Delta q_t = (1-\rho) g + \rho \Delta q_{t-1} + u_t, \tag{2.10}$$

where the parameter $\rho$ fulfills $0 \leq \rho < 1$ and $u_t$ is an iid disturbance.

He further assumes that each firm receives a noisy observation of current nominal GDP,

$$z_t(i) = q_t + v_t(i), \tag{2.12}$$

where $v_t(i)$ is an iid measurement error. With impressive analysis, precisely what one would expect from Mike, he then derives expressions for the equilibrium Kalman-filter estimates $q_{t|t}$ and $p_{t|t}$.

In order to get more specific results and in order to discuss the impulse responses of a permanent increase in nominal GDP, he assumes the special case of $q_t$ being a random walk with drift, $\rho = 0$ and $\Delta q_t = g + u_t$. He can then derive the dynamics for the output gap and the price level, which obey

$$y_t = \nu (y_{t-1} + u_t), \tag{3.8}$$

with the price level given by (2.6b). Here the degree of persistence of output, $\nu$, fulfills $0 < \nu = 1 - \hat{k} < 1$, where $\hat{k}$ is the Kalman gain, given by $0 < \hat{k} = (\sqrt{\gamma^2 + 4\gamma} - \gamma)/2 < 1$, where $\gamma = \xi\sigma_u^2/\sigma_v^2 > 1$. Since $\nu$ is decreasing in the parameter $\gamma$, it follows that an increase in the degree of complementarity (a fall in $\xi$) and a fall in the signal-to-noise ratio $\sigma_u^2/\sigma_v^2$ both increase $\nu$ and thus imply more persistent output response to nominal GDP disturbances.

Mike then picks reasonable parameter values and shows that reasonable impulse responses result. In particular, when nominal GDP growth is an AR(1) process, the impulse responses of inflation and output have the attractive and realistic feature that the inflation response peaks later than the output response.

My first comment is that the results are very sensitive to assumptions (1) and (2) mentioned above, that nominal GDP is exogenous and the variable that is observed with noise by the firms. Exogenous nominal GDP corresponds to an assumption of successful nominal-GDP-growth targeting. One reason for this assumption is that it allows Mike to discuss the response to one-time increases in nominal GDP and compare to previous literature which has often discussed that case. However, no real-world central bank has attempted nominal GDP targeting, and it is of some interest to see to what extent the results depend on the nature of monetary policy assumed.

Furthermore, nominal GDP growth (even if not adjusted for potential output) is arguably not the variable that is most easily observed by firms. For one thing, nominal GDP is only published quarterly, with a lag and with considerable noise, as evidenced by substantial later revisions. Media attention also seems to be less on nominal GDP growth than on real growth. Furthermore, the CPI is published monthly in most countries, with a shorter lag, and without subsequent revisions (which of course does not mean that the observations contain no noise). Media attention to CPI numbers seems to me to be normally larger than that to nominal GDP (more so for countries where central banks have explicit CPI inflation targets, where actual inflation is constantly compared to the target).

A simple way to demonstrate the sensitivity of the results to changes in the above assumptions is to assume a more general "targeting rule", where monetary policy is assumed to achieve the equality of a linear combination of the price level and the output gap, $p_t + \lambda y_t$ for some coefficient $\lambda \geq 0$, with the exogenous unit-root disturbance $q_t$,

$$p_t + \lambda y_t = q_t. \tag{2.6c}$$

This exogenous disturbance $q_t$ is then assumed to be observed with noise, as in (2.12). When

$\lambda = 1$, this would corresponds to Mike's case of nominal-GDP-growth targeting. When $\lambda = 0$, it would correspond to strict inflation targeting, where inflation becomes exogenous and the price level is the variable observed with noise.

Using (2.6c) in (2.6a) gives

$$p_t = \tilde{\xi} q_{t|t} + (1 - \tilde{\xi}) p_{t|t}, \tag{2.6d}$$

which is similar to (2.7) except that $\xi$ is replaced by $\tilde{\xi} \equiv \xi/\lambda$. Thus, we see that, even if $\xi < 1$, for sufficiently small $\lambda < 1$ we have $\tilde{\xi} > 1$, and for $\lambda \to 0$ we have $\tilde{\xi} \to \infty$. In particular, for $\lambda = \xi < 1$, we have $\bar{\xi} = 1$, and $p_t = q_{t|t}$. Then, no higher-order estimates of $q_t$ matter, and the degree of persistence $\nu$ will be at its minimum. Furthermore, for $\lambda < \xi$, we have $\bar{\xi} > 1$, and the expression (2.8) does not converge. For this case, we do not know whether an equilibrium exists, and if it does, what its properties are.

This simple modification of (2.6) shows that Mike's results are very sensitive to assumptions about monetary policy and the information used by the firms, and relatively small variations in these assumptions can have large consequences for the results, leading to doubts about the robustness of the results. More generally, both (2.6) and (2.6c) are drastically simplified assumptions about the outcome of monetary policy and the information structure of the economy, and more relevant analysis of the outcome under various kinds of optimizing policy and alternative information structures is called for.

My second comments refers to the realism of higher-order estimates. The higher-order estimates are very sophisticated estimates. For instance, the second order estimate $q_t^{(2)} \equiv q_{t|t|t}$ is given by

$$q_{t|t|t} = \int \mathrm{E}_t^i q_{t|t} di.$$

Here, $q_{t|t}$ is the average of firms' (first-order) estimates of nominal GDP, $\mathrm{E}_t^i q_{t|t}$ is firm $i$'s estimate of the average of other firms' estimates of nominal GDP conditional on its specific information, and $q_{t|t|t}$ is average over all firms of such firm-specific estimates. Clearly, this is a sophisticated concept, and most readers probably need to concentrate a bit to grasp the meaning of it. And this is only order two. In Mike's graphs, you have to go order four or higher to find substantial persistence. In this paper, people are supposed not to be aware of all available information, but they are still supposed to be aware of such sophisticated concepts.

It is possible that people with bounded rationality and limited computational capacity may apply some simplifications in considering higher-order estimates. One possible simplification is not to update estimates above a certain order, and thus to set such estimates equal to a

constant. This would increase the persistence of the real effects of nominal disturbances. Another possible simplification is to set estimates above a certain order equal to estimates of that certain order. This would reduce the persistence of the real effects of nominal disturbances. Both these alternative simplification may seem equally plausible *a priori*, but they move Mike's results in opposite directions.

Of course one would like to see some direct empirical evidence on the relevance and nature of higher-order estimates. In principle, one could find such evidence. Some central banks collect survey data on inflation expectations. One could of course expand such surveys and ask people: (1) What to you think inflation will be next year? (2) What do you think other people think inflation will be? (3) What do you think other people think other people think inflation will be? This gets us to third-order inflation expectations. It remains to be seen whether answers to questions (2) and (3) would be different from those to question (1); if not, bad news for Mike!

My third comment refers to the existence and possible use of additional information. There is certainly a lot of relevant information available that one has to assume that people for some reason decide not to use. There is information directly about the price level $p_t$ (or about industry price indices). In Mike's model, they could be modeled as another signal,

$$z_p(t) = p_t + v_p(t),$$

the result of which would reduce the role of higher-order estimates of $q_t$ and thereby reduce the degree of persistence in the real effects of nominal disturbances.

All over the world, there is a strong trend towards more transparent monetary policy, with inflation-targeting central banks in an increasing number of countries publishing *Inflation Report*s of increasing quality (the trend seems to be stronger outside the US). These inflation reports publish price indices, estimates of nominal and real disturbances, the central bank's judgments, etc., which all contribute to better and more accessible information. For many central banks, part of the transparent monetary policy is to teach the general public how monetary policy works and to make it aware of what information to look for in order to best predict future monetary-policy actions. Within Mike's model, this might show up as the measurement error variance falling, which improves the signal-to-noise ratio and reduces the degree of persistence in the real effects of nominal disturbances. More generally, this points to the available information and the awareness of the general public being endogenous to the monetary-policy regime, which poses interesting and important challenges for future research.

In summary, I find the idea of bringing in higher-order estimates very neat and intriguing, but I remain doubtful of how realistic it is.

## References

[1] Woodford, Michael (2001), "Imperfect Common Knowledge and the Effects of Monetary Policy," presented at the conference on *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Columbia University, NY, October 5-6, 2001.