Preliminary and incomplete. Comments welcome.

Unemployment Gaps: A Note on Blanchard and Galí (2010)*

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Abstract

The relevant unemployment gap as a target variable for monetary policy turns out to be the gap between the unemployment rate and the rational-expectations steady-state unemployment rate, the long-run sustainable rate of unemployment (LSRU). The relevant gap as an indicator of inflation pressure is different and is the gap to the short-run NAIRU, defined as the unemployment rate that keeps inflation constant. The short-run NAIRU is state-dependent and is increasing in past levels of and changes in the unemployment rate. However, the short-run NAIRU is an ambigious and less usefule concept in more general models, whereas LSRUremains a well-defined and useful concept.

1 The short-run NAIRU – depends on lagged unemployment

Blanchard and Galí (2010) (BG) present a New Keynesian model with unemployment. Under some simplifying assumptions, they present a simplified Phillips curve (their equation (33)) that can, with a slight change in notation and assumptions, be written as the backward-looking Phillips curve [explain how]

$$\pi_t - \pi^* = -\kappa(u_t - u) + \kappa_1(u_{t-1} - u) - \kappa_2 \hat{a}_t, \tag{1}$$

where π_t denotes inflation; π^* denotes the inflation target; u_t denotes the unemployment rate, u denotes the rational-expectations steady-state unemployment rate, the long-run sustainable rate of unemployment (LSRU); $\hat{a}_t$ denotes an exogenous productivity parameter representing exogenous technology ($\hat{}$ denotes the log deviation from the rational-expectations steady state); and the

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coefficients satisfy $\kappa > \kappa_1 > 0$, $\kappa_2 > 0$, and[1]

$$\begin{aligned}\kappa_1 &\equiv (1-\delta)(1-x)\kappa, \\ \kappa_2 &\equiv \Psi\gamma.\end{aligned}$$

From (1) I now derive an expression for the short-run NAIRU, denoted $\bar{u}_t$ and defined as the unemployment rate at which inflation is constant.[2] From (1) it follows that

$$\Delta\pi_t = -\kappa\Delta u_t + \kappa_1\Delta u_{t-1} - \kappa_2\Delta\hat{a}_t,$$

where $\Delta v_t \equiv v_t - v_{t-1}$ for any variable v_t. Solving for the unemployment rate $\bar{u}_t$ that sets $\Delta\pi_t = 0$ gives the following expression for the short-run NAIRU,

$$\bar{u}_t \equiv u_{t-1} + \frac{\kappa_1}{\kappa}\Delta u_{t-1} - \frac{\kappa_2}{\kappa}\Delta\hat{a}_t. \tag{2}$$

The short-run NAIRU is not a constant. It is state-dependent and increasing in the level of and the change in the lagged unemployment rate and decreasing in the change in (log) productivity.

From (1) and (2) follows that the change in inflation is negatively proportional to the gap between the unemployment rate and the short-run NAIRU,

$$\Delta\pi_t = -\kappa(u_t - \bar{u}_t). \tag{3}$$

Indeed, this relation follows from the definition of the short-run NAIRU.

The fact that the short-run NAIRU according to (2) depends on the level of and change in the lagged unemployment rate and that the unemployment rate depends on monetary policy means that monetary policy affects the short-run NAIRU. A policy that leads to higher unemployment than the LSRU also all else equal leads to a higher short-run NAIRU than the LSRU and all else equal to a worse tradeoff between inflation and unemployment, in the sense that all else equal actual unemployment then has to be higher in order to reduce inflation.[3] This is apparent from (3), where for a given change in inflation the unemployment rate must be higher for a higher short-run NAIRU. It is also apparent from (1), where a higher lagged unemployment rate all else equal gives rise to higher inflation.

[1] The coefficients κ, δ, x, Ψ, and γ are explained in BG. I have modifed their notation and assumptions by adding an assumption that Calvo-style price setters index prices to the inflation target π^* when not setting optimal prices. Setting $\pi^* = 0$ and $u_t - u^* = \hat{u}_t$ results in equation (33) of BG (in BG $\hat{}$ denotes the log deviation from steady state except for the unemployment rate where it denotes the gap between the unemployment rate and the steady-state rate).

[2] As is well known, the NAIRU, the Non-Accelerating Inflation Rate of Unemployment, is a misnomer and should be called the NIIRU, the Non-Increasing Inflation Rate of Unemployment, or the CIRU, the Constant-Inflation Rate of Unemployment.

[3] "All else equal" here means an unchanged productivity path.

2 The long-run sustainble rate of unemployment rate – the target unemployment rate

Under some specified assumptions BG derive a second-order approximation to the welfare of the representative household (equation (37) in BG). Introducing the assumption of indexing to an inflation target mentioned above, the corresponding intertemporal loss function can be written

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} L_{t+\tau}, \tag{4}$$

where the period loss function is

$$L_t = (\pi_t - \pi^*)^2 + \lambda_u (u_t - u)^2, \tag{5}$$

where I introduce the notation λ_u for Blanchard-Galí's α_u. The relevant target level for the unemployment rate is the constrained-efficient unemployment rate. Under the assumptions of BG, this constrained-efficient unemployment rate equals the LSRU, u.

Thus, from the point of view of monetary policy, the relevant period loss function involves the squared inflation gap and the squared unemployment gap, where the relevant unemployment gap is the gap between the unemployment rate and the LSRU, $u_t - u$.

[Extend further on the constrained-efficient unemployment rate and the LSRU. Why the appropriate target level is the LSRU also if the constrained-efficient unemployment rate is lower than the LRSU by a constant. Refer to optimization under both commitment and discretion.]

3 Flexible inflation targeting

Flexible inflation targeting aims at stabilizing inflation around the inflation target and resource utilization around a sustainable level (Svensson (2011)). From (5) follows that the relevant measure of resource utilization as a target variable, that is, as an argument of the intertemporal loss function, is the gap between the unemployment rate and the LSRU, $u_t - u$.

It is important to distinguish between measures of resource utilization for the purpose of being a target variable and being an indicator of inflation pressure. Different purposes result in different measures. From (3) follows that the relevant measure of resource utilization as an indicator of inflation pressure, and indicator of the change in inflation, is (by definition) the gap between the unemployment rate and the short-run NAIRU, $u_t - \bar{u}_t$, where the short-run NAIRU, $\bar{u}_t$, fluctuates over time and depend on level and change of the lagged unemployment rate and the change in the productivity according to (2).

The fact that the appropriate unemployment gap in the loss function and in the Phillips curve are different illustrates that "the divine coincidence" does not hold here, as BG emphasizes. Instead, there is a tradeoff between stabilizing inflation and stabilizing unemployment.

In particular, if the period loss function would be taken to include the unemployment gap to the short-run NAIRU,

$$L_t = (\pi_t - \pi^*)^2 + \lambda_u (u_t - \bar{u}_t)^2, \tag{6}$$

by (3) this is equivalent to having a loss function in terms of the squared inflation gap and the squared change in inflation,

$$L_t = (\pi_t - \pi^*)^2 + \frac{\lambda_u}{\kappa^2}(\pi_t - \pi_{t-1})^2. \tag{7}$$

This effectively introducing "inflation smoothening" in monetary policy. From the analysis of BG follows that there is no theoretical support for such a loss function (except the very special case when the model is so special that the short-run NAIRU and LSRU are the same). The relevant period loss function is instead (5). The relevant target unemployment rate is thus the LSRU, not the short-run NAIRU.

4 A more general Phillips curve and the aggregate-demand relation

The simple Phillips curve (1) was derived by BG under some simplifying assumptions. Without these assumptions, they derive the more general Phillips curve

$$\pi_t - \pi^* = \beta \mathrm{E}_t(\pi_{t+1} - \pi^*) - \kappa_0(u_t - u) + \kappa_L(u_{t-1} - u) + \kappa_F \mathrm{E}_t(u_{t+1} - u) - \kappa_a \hat{a}_t, \tag{8}$$

where

$$\begin{aligned} \kappa_0 &\equiv \lambda h_0/(1-u), \\ \kappa_L &\equiv -\lambda h_L/(1-u), \\ \kappa_F &\equiv \lambda \Phi \gamma, \end{aligned}$$

and where λ, h_0, and h_L and Φ are specified in BG.

The aggregate-demand relation, how the policy rate i_t affects unemployment, can be derived in the following way: The Euler condition relating consumption $\hat{c}_t$ ($\hat{}$ denotes the log deviation from the rational-expectations steady state), the policy rate i_t, and the rate of time preference $\rho \equiv -\ln \beta$ (equation (28) in BG); the relation between consumption and employment $\hat{n}_t$ (the first equation in section IV of BG); and the relation between unemployment and employment (in the third line of page 15 of BG) are

$$\begin{aligned} \hat{c}_t &= \mathrm{E}_t \hat{c}_{t+1} - (i_t - \mathrm{E}_t \pi_{t+1} - \rho), && (9) \\ \hat{c}_t &= \hat{a}_t + \xi_0 \hat{n}_t + \xi_1 \hat{n}_{t-1}, && (10) \\ u_t - u &= -(1-u)\hat{n}_t, && (11) \end{aligned}$$

where ξ_0 and ξ_1 are specified in BG. Combination of (10) and (11) gives the relation between consumption and unemployment,

$$\hat{c}_t = \hat{a}_t - \xi_0 (u_t - u)/(1-u) - \xi_1 (u_{t-1} - u)/(1-u).$$

Using this in (9) gives

$$\hat{a}_t - \xi_0 (u_t - u)/(1-u) - \xi_1 (u_{t-1} - u)/(1-u) =$$

$$\mathrm{E}_t[\hat{a}_{t+1} - \xi_0 (u_{t+1} - u)/(1-u) - \xi_1 (u_t - u)/(1-u)] - (i_t - \mathrm{E}_t \pi_{t+1} - \rho),$$

which can be rewritten as

$$-(\xi_0 + \xi_1)(u_t - u)/(1-u) =$$

$$\xi_1 (u_{t-1} - u)/(1-u) - \xi_0 \mathrm{E}_t (u_{t+1} - u)/(1-u) - (i_t - \mathrm{E}_t \pi_{t+1} - \rho) + \mathrm{E}_t \hat{a}_{t+1} - \hat{a}_t,$$

and finally as

$$(\xi_0 + \xi_1)(u_t - u) =$$

$$-\xi_1 (u_{t-1} - u) + \xi_0 \mathrm{E}_t (u_{t+1} - u) + (1-u)(i_t - \mathrm{E}_t \pi_{t+1} - \rho) + (1-u)(\mathrm{E}_t \hat{a}_{t+1} - \hat{a}_t). \tag{12}$$

The economy is then summarized by the Phillips curve (8) (the relation between inflation, unemployment, and productivity), the aggregate-demand relation (12) (the relation between unemployment, the real policy rate, $i_t - \mathrm{E}_t \pi_{t+1}$, and productivity), and the central bank's loss function (4) and (5).

5 Short-run NAIRU with the more general Phillips curve?

What is the short-run NAIRU that can be defined from the more general Phillips curve (8)? That is not so obvious. Proceeding as above for the simple Phillips curve (1), the change in inflation is given by

$$\Delta\pi_t = \beta\Delta \mathrm{E}_t\pi_{t+1} - \kappa_0\Delta u_t + \kappa_L\Delta u_{t-1} + \kappa_F\Delta \mathrm{E}_t u_{t+1} - \kappa_a\Delta\hat{a}_t.$$

As for the simple Phillips curve, the change in inflation depends on Δu_t, Δu_{t-1}, and $\Delta\hat{a}_t$, the changes in unemployment, lagged unemployment, and productivity. However, it now also depends on $\Delta \mathrm{E}_t\pi_{t+1} \equiv \mathrm{E}_t\pi_{t+1} - \mathrm{E}_{t-1}\pi_t$ and $\Delta \mathrm{E}_t u_{t+1} \equiv \mathrm{E}_t u_{t+1} - \mathrm{E}_{t-1}u_t$, the changes in expectations of future inflation and unemployment. This makes it difficult to define the short-run NAIRU in an unambiguous way.

This difficulty illustrates the finding in Rogerson (1997) that the short-run NAIRU is an ambiguous equilibrium concept, in contrast to the LSRU, which Rogerson finds to be a "useful" (p. 82) theoretical concept "that has a very clear and precise meaning in dynamic economic theory" (p. 85).

It follows that one has to distinguish the unemployment gap relevant as a target variable in loss function (5), the gap to the LSRU, from the way unemployment affects inflation in the Phillips curve (8).

References

Blanchard, Olivier, and Jordi Galí (2010), "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics* 2, 1–30.

Rogerson, Richard (1997), "Theory Ahead of Language in the Economics of Unemployment," *Journal of Economic Perspectives* 11, 73–92.

Svensson, Lars E.O. (2011), "Inflation Targeting," in Friedman, Benjamin M., and Michael Woodford (eds.), *Handbook of Monetary Economics, Volume 3b*, chap. 22, Elsevier, 1237–1302.