Discussion of Hansen and Sargent

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- Representation of robust control

$$\begin{bmatrix} z_{t+1} \\ \mathrm{E}_t x_{t+1} \end{bmatrix} = A \begin{bmatrix} z_t \\ x_t \end{bmatrix} + Bu_t + Cw_{t+1} + C \begin{bmatrix} \eta_{t+1} \\ 0 \end{bmatrix}$$

$$d_t = H \begin{bmatrix} z_t \\ x_t \end{bmatrix} + Ju_t$$

z_t predetermined, x_t forward-looking, u_t control, w_{t+1} distortion,
η_t iid shock (mean zero, covariance I)
d_t targets
$\{A, B, C\}$ model approximation
$\{\beta, H, J\}$ preferences
θ cost to distortions

$$\max_{\{u_t\}_{t=0}^{\infty}} \min_{\{w_{t+1}\}_{t=0}^{\infty}} -\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (d_t' d_t - \theta w_{t+1}' w_{t+1}) \qquad (1)$$

- Solution

Control

$$u_t = -F_1 z_t - \Phi_1 \mu_{xt} \equiv F_1^t z^t$$

Worst shock

$$w_{t+1} = -F_2 z_t - \Phi_2 \mu_{xt} \equiv F_2^t z^t$$

Lagrange multipliers for forward-looking equations

$$\mu_{xt} = S z_{t-1} + \Sigma \mu_{x,t-1} \equiv S^{t-1} z^{t-1}$$

Forward-looking variables

$$x_t = G z_t + \Gamma \mu_{xt} \equiv G^t z^t$$

Predetermined variables

$$\begin{aligned} z_{t+1} = & (A_{11} + A_{12}G - B_1 F_1 - C_1 F_2) z_t \\ & + (A_{12}\Gamma - B_1 \Phi_1 - C_1 \Phi_2) \mu_{xt} + C_1 \eta_{t+1} \end{aligned}$$

- Alternative representation of robust control

  - Feasible set of distortions

$$\max_{\{u_t\}_{t=0}^{\infty}} \min_{\{w_{t+1}\}_{t=0}^{\infty}} -\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t d_t' d_t$$

subject to

$$\sum_{t=0}^{\infty} \beta^t w_{t+1}' w_{t+1} \leq \Omega \tag{2}$$

Interpret θ in (1) as Lagrange multiplier of (2)

  * How to determine Ω or θ?
  * Breakdown level of θ?

– Feasible set of models

$$\begin{bmatrix} z_{t+1} \\ \mathrm{E}_t x_{t+1} \end{bmatrix} = A_{t+1} \begin{bmatrix} z_t \\ x_t \end{bmatrix} + B_{t+1} u_t + C_{t+1} \begin{bmatrix} \eta_{t+1} \\ 0 \end{bmatrix}$$

$$m_{t+1} \equiv \{A_{t+1}, B_{t+1}, C_{t+1}\} \in M$$

$$\max_{\{u_t\}_{t=0}^{\infty}} \min_{\{m_{t+1}\}_{t=0}^{\infty}} -\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t d_t' d_t$$

subject to $m_{t+1} \in M$.

* Giannoni (2000), Forward-looking model
* Svensson (2000), "Robust Control Made Simple," Backward-looking model
  - Worst model often boundary of M
  - Assumptions about M crucial: Priors matter
  - Bayesian probability on boundary: Results determined by very unlikely models

– Relations M, Ω, θ?

– Reasonableness vs. analytical convenience?

- Filtering, partial information
  Observable variables

$$Z_t = D_1 \begin{bmatrix} z_t \\ x_t \end{bmatrix} + D_2 \begin{bmatrix} z_{t|t} \\ x_{t|t} \end{bmatrix} + D_3 u_t$$

  - Filtering and robust control (HS)
  - Filtering and forward-looking variables (CLP, SW)

- Example (Woodford)
  - Too complex, simplify

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa(o_t - o_t^n) + w_{t+1} + \eta_t$$

o_t control

$$Z_t = \begin{bmatrix} \pi_t \\ o_t^n + v_t \end{bmatrix}$$

  - Numerical example
    - Breakdown point $\theta \approx 22.75$. Interpretation?
    - No filtering?
    - Robust policy very similar to standard policy

- Additional comments
  - Organization of paper?
  - Why commitment only?
    * No commitment mechanism
    * Examine discretion equilibria
  - How to model model uncertainty? Bayesian vs robust
    * Arguments against robust
      · Priors still matter
      · "Unlikely" models may determine outcome
      · Learning, updating?
    * Arguments against Bayesian
      · Know too little?