Preliminary and incomplete. Comments are welcome.

A Simple Frictionless Model of User Cost and User-Cost-to-Income and User-Cost-to-Rent Ratios*

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Abstract

A simple frictionless model of user cost and user-cost-to-income and user-cost-to-rent ratios. Also user cost with loan-to-value restriction.

Contents

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1 Introduction

This note uses a simplified setup (without transactions costs and any taxes on property, capital gains, and imputed rental income) to explain why the user cost is the owner-occupied-housing equivalent to the rent of rental housing. It also explains why the user-cost-to-income and user-cost-to-rent ratios—rather than price-to-income and price-to-rent ratios—are relevant indicators of the valuation of owner-occupied housing.[1] [2]

Conceptually, for owner-occupied housing, it is crucial to distinguish the price of the dwelling, an asset, from the user cost of housing (services), the price of the housing services that the dwelling delivers. The user cost of housing is the cost of living in owner-occupied housing; the price of the dwelling is not the cost of living in owner-occupied housing (Poterba, 1984; Himmelberg et al., 2005; Mulheirn, 2019).

Furthermore, conceptually, an owner-occupying household is both a landlord/investor and a renter. As a renter, the household pays an imputed rent, the user cost, to itself as the landlord/investor. As a landlord/investor, the household owns the house, rents it out to itself, and receives the user cost and the capital gains on the property as imputed income. In a perfect equilibrium, the household as renter pays the same user cost as the rent for an equivalent rented dwelling, and the household as landlord/investor earns the same rate of return as that on an alternative investment.

In this note, first, the budget constraint for a household with rented housing is presented. Second, the budget constraint for a household with owner-occupied housing is introduced and rewritten in terms of the user cost of housing (services). By comparing these budget constraints, it is apparent that—in a well-functioning market for rental and owner-occupied housing—user cost and rents for similar dwellings will be approximately equal. Put differently, in such a well-functioning housing market, house prices are consistent with fundamentals and housing is correctly valued if user cost and rents for similar dwellings are approximately equal. Third, from the zero-profit condition for a landlord/investor in perfect competition, one can in a different way show that in a well-functioning market the user cost of housing equals the rent for similar dwellings. The separation of the role of the homeowner into a renter and a landlord is also explicitly shown.

Furthermore, under the simple assumption that household preferences are Cobb-Douglas, the total user cost (the user cost times the units of housing) will be a constant preferred share of total non-housing and housing consumption. If total consumption is a stable share of disposable income—that is, if the saving rate is approximately constant—the equilibrium user-cost-to-income ratio will

[1] The user cost of housing is discussed by, for example, Prescott (1997), Dougherty and Van Order (1982), Poterba (1984), Office of Policy Development and Research (2000), Himmelberg et al. (2005), Poterba and Sinai (2008), Díaz and Luengo-Prado (2008), Verbrugge (2008), Garner and Verbrugge (2009a,b), Englund (2011, 2020), Haffner and Heylen (2011), Muellbauer (2012), Diewert (2013, appendix), Fox and Tulip (2014), Duca et al. (2016, 2021a,b), Hansson (2019), Mulheirn (2019), and Svensson (2019, 2020, 2023).

[2] Díaz and Luengo-Prado (2008) derive general expressions for the user cost in an optimizing life-cycle model with endogenous tenure choice and households facing house-price risk and idiosyncratic uninsurable earnings risk, transactions costs, LTV constraints, and different taxes on owner-occupied and rented homes. They examine the bias caused by using the rental-equivalence approach to estimate user costs. The rental-equivalence approach is to use rental market value for homes as an estimate of the user cost.

be a stable over time. Put differently, in a well-functioning housing market with such household preferences, correctly valued housing and house prices being consistent with fundamentals imply that the user cost of housing is an approximately stable share of income and correspond to a preferred share of total consumption.

In addition, it is shown that the equilibrium price-to-income and price-to-rent ratios are inversely proportional to the user cost of capital and very sensitive to changes in real interest rate. A trend in the real interest rate results in an opposite trend in the price-to-income and price-to-rent ratios.

2 Rented housing and rents

Consider for simplicity a situation with no uncertainty and perfect foresight. Consider the nominal budget constraint for (the beginning of) period $t \geq 1$ of a representative household that rents housing (buys housing services) and does not own any housing. We can think of the period as a year. The household's budget constraint can be written

$$P_t^c c_t + R_t h_t + A_t = W_t + Y_t, \; t \geq 1, \tag{2.1}$$

where

$$W_t \equiv (1 + i_{t-1}) A_{t-1}, \; t \geq 1. \tag{2.2}$$

Here, in (2.1), P_t^c denotes the (nominal) price of *non-housing* consumption (the CPI excluding consumption of housing services) in period t, c_t denotes the non-housing consumption, R_t denotes the rent per unit of housing (for example, per standardized dwelling or per sqm), and h_t denotes the number of units of housing and housing-services consumption. One unit of a dwelling is assumed to deliver one unit of housing services each period.

Furthermore, $A_t \gtrless 0$ denotes the value of net (financial) assets (financial assets minus liabilities) acquired in period t to be carried into period $t+1$, W_t denotes (net) wealth in the beginning of period t, before transactions in period t, and Y_t denotes the (after-tax, earned) income in period t. Wealth in the beginning of period t is defined in (2.2), where i_{t-1} denotes the (after-tax) nominal interest rate between period $t-1$ and period t (assumed to apply to both financial assets and liabilities). In period 1, i_0, and A_0 are given.

Using (2.1) and (2.2) to eliminate A_t, the household's budget constraint can be written

$$P_t^c c_t + R_t h_t + \frac{W_{t+1}}{1 + i_t} = W_t + Y_t, \; t \geq 1, \tag{2.3}$$

where wealth and net assets satisfy (2.2).

3 Owner-occupied housing and the user cost of housing

Consider now the nominal budget constraint for (the beginning of) period $t \geq 1$ for a household with owner-occupied housing. The household is not subject to credit constraints beyond the budget constraint. The budget constraint can then then be written as

$$P_t^c c_t + P_t h_t - D_t = W_t + Y_t, \; t \geq 1, \tag{3.1}$$

where

$$W_t \equiv (P_t - M_{t-1})h_{t-1} - (1 + i_{t-1})D_{t-1}, \; t \geq 1. \tag{3.2}$$

Here, in (3.1), P_t denotes the dwelling price (the price of a standardized house or apartment, or the price per square meter of houses or apartments, including the value of the land the dwelling sits. Furthermore, $D_t \gtrless 0$ denotes the net debt (financial liabilities minus financial assets) acquired in period t and carried into period $t+1$. Thus, net debt D_t in (3.1) is just the negative of net (financial) assets A_t in (2.1). I could equally well have expressed (2.1) and (2.2) in terms of net debt instead of net assets.

Furthermore, wealth in the beginning of period t is now given by (3.2). The first term on the right-hand side is the value in period t of the housing carried over from period $t-1$. Here, M_{t-1} denotes the operating, maintenance, repair, and depreciation (OMRD) costs. The OMRD cost for period $t-1$ is paid at the beginning of period t. The second term on the right-hand side is the negative of the net debt from period $t-1$ including interest, where i_{t-1} denotes the (after-tax) nominal interest rate between period $t-1$ and period t (assumed to apply to both financial assets and liabilities). In period 1, P_0, M_0, h_0, i_0, and D_0 are all given.

By adding and subtracting the term $(M_t - P_{t+1})h_t/(1 + i_t)$, the left-hand side of (3.1) can be written

$$\begin{aligned} P_t^c c_t + P_t h_t - D_t &= P_t^c c_t + \left(P_t + \frac{M_t - P_{t+1}}{1 + i_t}\right) h_t + \left(\frac{(P_{t+1} - M_t)h_t}{1 + i_t} - D_t\right) \\ &\equiv P_t^c c_t + \mathrm{UC}_t \, h_t + \frac{W_{t+1}}{1 + i_t}, \end{aligned} \tag{3.3}$$

where I have used (3.2). Furthermore, UC_t denotes the *user cost of housing* (services)—the cost and price of a unit of using housing *services*—and is defined as

$$\mathrm{UC}_t \equiv P_t + \frac{M_t - P_{t+1}}{1 + i_t} = \frac{M_t + i_t P_t - (P_{t+1} - P_t)}{1 + i_t} \tag{3.4}$$

Combining (3.1) and (3.3), we can write the household's period-t budget constraint as

$$P_t^c c_t + \mathrm{UC}_t \, h_t + \frac{W_{t+1}}{1 + i_t} = W_t + Y_t, \; t \geq 1. \tag{3.5}$$

In (3.4), the user cost is (the present value in the beginning of period t of) the cost of living in one unit of housing during period t. The middle expression in (3.4) shows that it equals the cost P_t of buying a unit of housing in the beginning of period t, paying the OMRD cost of M_t at the end of period t/beginning of period $t+1$, and selling the housing at the price of P_{t+1} in the beginning of period $t+1$. The present value in period t of receiving $P_{t+1} - M_t$ in period $t+1$ is $(P_{t+1} - M_t)/(1 + i_t)$. This is subtracted from the price P_t to give the user cost of housing services.[3]

[3] The above can be seen as a variant of the derivation in Dougherty and Van Order (1982). They derive an expression like (3.4) by taking the first-order conditions for a maximum of utility subject to a budget constraint similar to (3.1) and (3.2), whereas here I just rewrite the budget constraint as (3.3) and (3.4) and note the analogy to the renter's budget constraint (2.3). In particular, Dougherty and Van Order note that "[The user cost expression] is an appropriate measure of housing cost on the grounds that it is a measure of the dollar value of the bribe necessary to get homeowners to give up one unit of housing."

4 User costs and rents

By comparing the budget constraint for a renting household, (2.1), with the budget constraint for an owner-occupying household in terms of user cost, (3.5), we see that for a household to be indifferent between renting and owning a similar dwelling, the user cost and rent must be equal,

$$\mathrm{UC}_t = R_t. \tag{4.1}$$

That is, the user-cost-to-rent (UCTR) ratio satisfies

$$\mathrm{UCTR}_t \equiv \frac{\mathrm{UC}_t}{R_t} = 1. \tag{4.2}$$

In an equilibrium in a well-functioning market for rental and owner-occupied housing with some realistic transactions costs, we would then expect the user-cost-to-rent (UCTR) ratio for similar rented and owner-occupied dwellings to be *approximately* equal.

We probably find it obvious that the natural metric for assessing the affordability of rental housing is the rent and the rent relative to income. By comparing the budget constraints (2.1) and (3.5), it should be equally obvious that the natural metric to assess the affordability of owner-occupied housing is the user cost and the user cost relative to income.

4.1 The problem of a landlord/investor

Alternatively, consider the problem of a landlord/investor, who purchases 1 unit of housing at the beginning of period t at the price of P_t and rents it out at the rent R_t, paid at the beginning of period t. Furthermore, in the beginning of period $t+1$ the landlord pays the OMRD cost M_t. Then the landlord sells the housing unit at the price P_{t+1}.

Under perfect competition among landlords/investors and the resulting zero-profit condition, the present value of these cash flows must be zero in equilibrium,

$$-P_t + R_t + \frac{-M_t + P_{t+1}}{1 + i_t} = 0. \tag{4.3}$$

It follows that R_t satisfies

$$R_t = P_t - \frac{-M_t + P_{t+1}}{1 + i_t} = \frac{M_t + i_t P_t - (P_{t+1} - P_t)}{1 + i_t} = \mathrm{UC}_t, \tag{4.4}$$

where I have used (3.4). Thus, (4.1) is confirmed.

4.1.1 Gross and net rental yield

The concepts gross and net rental yield are popular in the property investment literature (for example, Rohde, 2022). The Gross Rental Yield (GRY) is defined as the rent-to-price ratio, that is, the reciprocal of the price-to-rent ratio,

$$\mathrm{GRY}_t \equiv \frac{(1 + i_t) R_t}{P_t}, \tag{4.5}$$

with the modification that the rent is multiplied by $1+i_t$ to correspond to the end-of-period value of the rent.

It follows from (4.3) and (4.5) that the GRY satisfies

$$\mathrm{GRY}_t = i_t + \frac{M_t}{P_t} - \frac{P_{t+1}-P_t}{P_t}, \tag{4.6}$$

The Net Rental Yield (NRY) is defined as the ratio of the rent net of OMRD costs to the price,

$$\mathrm{NRY}_t \equiv \frac{(1+i_t)R_t - M_t}{P_t}. \tag{4.7}$$

It follows from (4.5)–(4.7) that the NRY satisfies

$$\mathrm{NRY}_t = i_t - \frac{P_{t+1}-P_t}{P_t}, \tag{4.8}$$

that is, the NRY plus the (expected) capital gains equal the interest rate,

$$\mathrm{NRY}_t + \frac{P_{t+1}-P_t}{P_t} = i_t. \tag{4.9}$$

4.2 Separating the owner-occupier into renter and landlord/investor

The role of an owner-occupying household can conceptually be separated into a renter and a landlord/(investor). Too see this, start from the budget constraint and wealth definition of an owner-occupying household, (3.1) and (3.2). Add and subtract the term $R_t h_t$ (where R_t equals the user cost according to (4.4)) and then split the budget constraint, the debt, and the wealth into a renter (R) and landlord(/investor) (L) part, according to

$$P^c_t c_t + R_t h_t - D^R_t = W^R_t + Y_t, \tag{4.10}$$

$$P_t h_t - R_t h_t - D^L_t = W^L_t, \tag{4.11}$$

where

$$D_t = D^R_t + D^L_t, \tag{4.12}$$

$$W_t = W^R_t + W^L_t, \tag{4.13}$$

$$W^R_t \equiv -(1+i_{t-1})D^R_{t-1}, \tag{4.14}$$

$$W^L_t \equiv (P_t - M_{t-1})h_{t-1} - (1+i_{t-1})D^L_{t-1}. \tag{4.15}$$

Thus, according to (4.10) and (4.14), the renter does not own any housing and pays rent (equal to the user cost) to the landlord. According to (4.11) and (4.15), the landlord owns the housing, receives rent from the renter, does not consume anything, and is in this sense a pure investor.

Furthermore, from (4.3), we have $(P_t - R_t)h_t = (P_{t+1} - M_t)h_t/(1+i_t)$. Using this on the left hand side of (4.11) together with (4.15), we can write (4.11) as

$$\frac{W^L_{t+1}}{1+i_t} = W^L_t. \tag{4.16}$$

That is, $W^L_{t+1} = (1+i_t)W^L_t$, and the rate of return on the landlord's wealth equals the interest rate i_t.

5 The user cost of capital

The right-hand side of (3.4) shows that the user cost can be written as the present value of the OMRD cost plus the foregone interest on the housing purchase minus the capital gains. For small interest rates (or short periods, for example, with monthly interest payments), we can approximate the user cost as

$$\mathrm{UC}_t \approx M_t + i_t P_t - (P_{t+1} - P_t). \tag{5.1}$$

Furthermore, we can define the user cost per SEK of housing "capital", the *user cost of capital* (UCC), as

$$\mathrm{UCC}_t \equiv \frac{\mathrm{UC}_t}{P_t}. \tag{5.2}$$

It follows from (5.1) and (5.2) that

$$\mathrm{UCC}_t \approx m_t + i_t - \pi^h_{t+1}, \tag{5.3}$$

where $m_t \equiv M_t/P_t$ denotes the ratio of the OMRD cost to the house price (the OMRD rate) and $\pi^h_{t+1} \equiv (P_{t+1} - P_t)/P_t$ denotes the rate of nominal housing appreciation.

Let me now for simplicity replace the approximation sign by the equality sign, thus assuming that the interest rate is small and/or the period is short and that the approximation is acceptable. Furthermore, by subtracting and adding the rate of CPI inflation, $\pi_{t+1} \equiv (\mathrm{CPI}_{t+1} - \mathrm{CPI}_t)/\mathrm{CPI}_t$, we get

$$\mathrm{UCC}_t = m_t + r_t - \tilde{\pi}^h_{t+1}, \tag{5.4}$$

where $r_t \equiv i_t - \pi_{t+1}$ denotes the real interest rate and $\tilde{\pi}^h_{t+1} \equiv \pi^h_{t+1} - \pi_{t+1}$ denotes the real rate of housing appreciation (the real rate of housing capital gains).

Give this, the user cost can conveniently be expresses as

$$\mathrm{UC}_t = \mathrm{UCC}_t\, P_t = (m_t + r_t - \tilde{\pi}^h_{t+1}) P_t \tag{5.5}$$

Let the price-to-income and price-to-rent ratios, PTI_t and PTR_t, be defined as

$$\mathrm{PTI}_t \equiv \frac{P_t}{\mathrm{DI}_t} \quad \text{and} \quad \mathrm{PTR}_t \equiv \frac{P_t}{R_t}, \tag{5.6}$$

where DI_t denotes the representative owner-occupying household's disposable income. Define the user-cost-to-(disposable-)income (UCTI) ratio as

$$\mathrm{UCTI}_t \equiv \frac{\mathrm{UC}_t}{\mathrm{DI}_t}. \tag{5.7}$$

Then, given the user cost of capital and the PTI and PTR ratios, the UCTI and UCTR ratios can conveniently be calculated as

$$\mathrm{UCTI}_t = \mathrm{UCC}_t\, \mathrm{PTI}_t \quad \text{and} \quad \mathrm{UCTR}_t = \mathrm{UCC}_t\, \mathrm{PTR}_t. \tag{5.8}$$

6 Cobb-Douglas preferences and the user-cost-to-income ratio

Assume for simplicity that the representative household has Cobb-Douglas preferences. More precisely, assume that the household in period 1 has an intertemporal utility function of the form

$$\sum_{t=1}^{\infty} \beta^{t-1} U(c_t, h_t). \tag{6.1}$$

Here, β is a subjective discount factor that satisfies $0 < \beta < 1$ and the utility function $U(c_t, h_t)$ satisfies

$$U(c_t, h_t) = \bar{U}(u(c_t, h_t)), \tag{6.2}$$

where $\bar{U}(\cdot)$ denotes an increasing concave function (with $U' > 0$, $U'' < 0$), and the (sub)utility function $u(c_t, h_t)$ is a Cobb-Douglas utility function,

$$u(c_t, h_t) = c_t^{1-\alpha} h_t^{\alpha}, \tag{6.3}$$

where the constant α satisfies $0 < \alpha < 1$.

The function $\bar{U}(\cdot)$ together with the discount factor represents the intertemporal preferences for consumption in different periods, and the Cobb-Douglas function represent the atemporal preferences for housing and non-housing consumption within a period.

For a Cobb-Douglas utility function, a standard result is that the spending shares of the consumed goods and services in total consumption are constant. In the present case, maximizing the utility function (6.1)–(6.3) subject to the budget constraint for an owner-occupying household (3.5) can be shown to result in the first-order conditions

$$P_t^c c_t = (1-\alpha) C_t, \tag{6.4}$$

$$\mathrm{UC}_t\, h_t = \alpha\, C_t, \tag{6.5}$$

where

$$C_t \equiv P_t^c c_t + \mathrm{UC}_t\, h_t \tag{6.6}$$

denotes the household's total (nominal) consumption during period t. That is, the share of housing consumption in total consumption is by (6.5) equal to the constant α.[4]

Equations (6.4)–(6.6) are here equilibrium relations that refer to the representative owner-occupying household in the economy. We can then multiply both sides of (6.5) by the number of owner-occupying households to get

$$\mathrm{UC}_t\, H_t = \alpha\, C_t^{\mathrm{O}}, \tag{6.7}$$

where H_t denotes the aggregate owner-occupied housing stock and C_t^{O} denotes the aggregate consumption of owner-occupying households.

[4] We note that, for a renter with the budget constraint (2.3), the first-order condition (6.5) would be replaced by $R_t h_t = \alpha\, C_t$.

The owner-occupying households' aggregate consumption and aggregate disposable income satisfy

$$C_t^{\mathrm{O}} = (1 - s_t^{\mathrm{O}})\mathrm{DI}_t^{\mathrm{O}}, \tag{6.8}$$

where s_t^{O} and $\mathrm{DI}_t^{\mathrm{O}}$ denote, respectively, the aggregate saving rate and disposable income of owner-occupying households.

We can then use (6.5) and (6.8) to substitute for C_t^{O} and write

$$\mathrm{UC}_t\, H_t = \alpha(1 - s_t^{\mathrm{O}})\mathrm{DI}_t^{\mathrm{O}}. \tag{6.9}$$

Furthermore, let

$$\gamma_t \equiv \mathrm{DI}_t^{\mathrm{O}}/(N_t\, \mathrm{DI}_t) \tag{6.10}$$

denote the share of owner-occupiers' disposable income in the total aggregate disposable income of the economy, $N_t\mathrm{DI}_t$, where N_t denotes the population in the economy and DI_t the disposable income per capita. Then we can write

$$\mathrm{UC}_t\, H_t = \alpha(1 - s_t^{\mathrm{O}})\gamma_t N_t\, \mathrm{DI}_t. \tag{6.11}$$

It follows from (6.11) that the household's UCTI ratio will satisfy

$$\mathrm{UCTI}_t \equiv \frac{\mathrm{UC}_t}{\mathrm{DI}_t} = \alpha\, \gamma_t\, (1 - s_t^{\mathrm{O}})(N_t/H_t). \tag{6.12}$$

With data on the components of the right-hand side of (6.12), the latter will be a time-varying benchmark for the equilibrium UCTI ratio. Absent such data, we may simply assume that there is no trend in the benchmark and that the benchmark is relatively stable over time. Under this assumption, the equilibrium UCTI ratio should be relatively stable over time.

Figure 6.1: Population per owner-occupied dwelling, household consumption rate (1 – saving rate), and household saving rate.

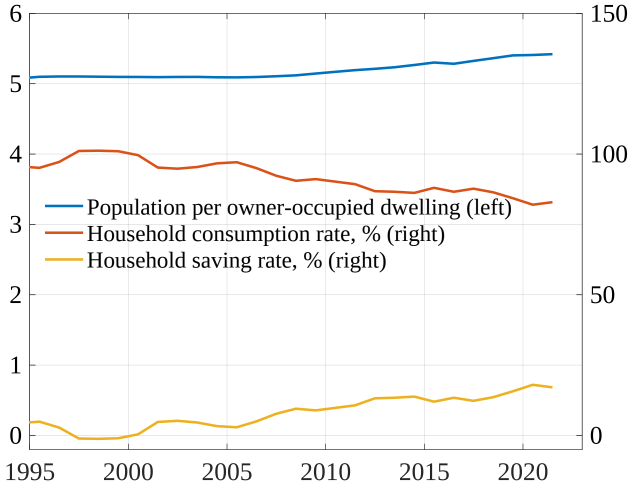
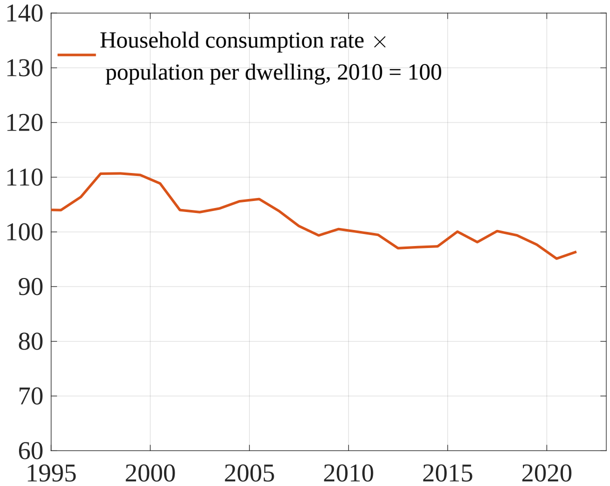


Figure 6.2: The product of the household consumption rate and population per owner-occupied dwelling. Index, 2010 = 100.



Source and note: Statistics Sweden (2022a,b,c) and own calculations. Note that the saving and consumption rates are the aggregate rates of all households, not those of owner-occupying households.

Figure 6.1 shows data on two of the factors determining the benchmark of the right-hand side of (6.12). The first is the population per owner-occupied dwelling (N_t/H_t). The second is one minus the household saving rate, what can be called the consumption rate. However, note that the saving rate here is the aggregate saving rate of *all* households, not the saving rate of only *owner-occupying* households. The aggregate saving rate is thus used as a proxy for the owner-occupying households' saving rate.

Figure 6.2 shows (the proxy for) the product of these two factors, $(1 - s_t^{\mathrm{O}})(N_t/H_t)$, indexed to 100 for 2010. We see that the product is relatively stable from 2008, except in 2019–2020, when the saving rate rises and the product drops. However, the high saving rate during the 2020 and 2021 is certainly the reflection of a drop in nonhousing consumption—in particular, a drop in various services, due to the restrictions during the corona crisis. As for housing consumption and demand, it is likely to have increased, in the sense of a shift toward larger dwellings and from apartments to houses (Sveriges Riksbank, 2021). This can be represented by an increase in a time-varying share of housing consumption, α_t.[5] Furthermore, there may have been a rise in γ_t, the share of owner-occupying households in total disposable income, given that disposable income of owner-occupiers with safer jobs may have been less affected by the corona crisis than the average household. Altogether, the benchmark may very well have risen during the corona crisis.

6.1 The equilibrium price-to-income and price-to-rent ratios

What are the equilibrium PTI and PTR ratios in this case? We have from (5.8)

$$\mathrm{PTI}_t = \frac{\mathrm{UCTI}_t}{\mathrm{UCC}_t} = \frac{\alpha\gamma_t(1 - s_t^{\mathrm{O}})(N_t/H_t)}{\mathrm{UCC}_t} = \frac{\alpha\gamma_t(1 - s_t^{\mathrm{O}})(N_t/H_t)}{m_t + r_t - \tilde{\pi}_{t+1}^h}, \tag{6.13}$$

$$\mathrm{PTR}_t = \frac{\mathrm{UCTR}_t}{\mathrm{UCC}_t} = \frac{1}{\mathrm{UCC}_t} = \frac{1}{m_t + r_t - \tilde{\pi}_{t+1}^h}. \tag{6.14}$$

Here, (6.13) follows from (5.4) and (6.12), that is, from the Cobb-Douglas assumption. In contrast, (6.14) just follows from (4.2) and (5.4) and does not need the Cobb-Douglas assumption.

We see that, even if the numerators of the right-most expressions above are constant or relatively stable, the equilibrium PTI and PTR ratios will vary substantially negatively with variation in the UCC. In particular, if there is a negative trend in the UCC, there will be a positive trend in the PTI and PTR ratios.

However, Duca et al. (2021a,b) argue that the "asset pricing approach from finance" in the form of a simple user-cost-rent arbitrage and house prices being the present value of futur rents is misleading for *actual* house prices in the short run, because of inertia, transactions costs, and imperfect markets. It results in an equality between rent and user cost and thereby (6.14). This equation suggests that the elasticity of the PTR ratio with respect to the UCC would be -1. However, according to Duca et al. (2016), it is about -0.15 for the US. That is, inertia, transactions cost, credit constraints, and other market imperfections make house prices less sensitive to the UCC and mortgage rates than suggested by the simple frictionless model.

[5] The model can trivially be extended to a time-varying housing consumption share, α_t.

7 The present value of rented and owner-occupied housing

7.1 Rental surplus and user-cost surplus

Introduce the rental surplus, RS_t, of the landlord/investor, defined by

$$\mathrm{RS}_t \equiv R_t - \frac{M_t}{1+i_t}. \tag{7.1}$$

It equals the rent during period t minus the present value of the OMRD costs for that period.

We can then write (4.4) as

$$P_t = \mathrm{RS}_t + \frac{P_{t+1}}{1+i_t} = \sum_{\tau=t}^{\infty} \hat{D}_{\tau,t}\,\mathrm{RS}_\tau, \tag{7.2}$$

where $\hat{D}_{\tau,t}$ denotes the nominal discount factor in period t of nominal payments in period $\tau \geq t$ and is defined as

$$\hat{D}_{\tau,t} \equiv 1 \text{ for } \tau = t, \quad \hat{D}_{\tau,t} \equiv \prod_{k=t}^{\tau-1}(1+i_k)^{-1} \text{ for } \tau \geq t+1, \quad t \geq 1. \tag{7.3}$$

That is, "correctly" valued rental housing equals the present value of current and future rental surpluses.

Similarly, we can define the user-cost surplus, UCS_t, of the owner-occupier as

$$\mathrm{UCS}_t \equiv \mathrm{UC}_t - \frac{M_t}{1+i_t}. \tag{7.4}$$

It equals the user cost during period t minus the present value of the OMRD costs for that period.

We can then write (3.4) as

$$P_t = \mathrm{UCS}_t + \frac{P_{t+1}}{1+i_t} = \sum_{\tau=t}^{\infty} \hat{D}_{\tau,t}\,\mathrm{UCS}_\tau. \tag{7.5}$$

That is, correctly valued owner-occupied housing equals the present value of current and future user-cost surpluses.

7.2 Alternative present-value expressions

Make the simplifying assumption that the OMRD costs during period t are proportional to the value of the house at the beginning of period t, that is, that the OMRD rate is constant, $m_t = m$,

$$M_t = mP_t. \tag{7.6}$$

Then we can write (4.4) as

$$P_t = R_t - \frac{mP_t}{1+i_t} + \frac{P_{t+1}}{1+i_t}, \tag{7.7}$$

which can be rewritten as

$$P_t = \frac{(1+i_t)R_t + P_{t+1}}{1+i_t+m} = \frac{1}{1+i_t+m}\sum_{\tau=t}^{\infty} \tilde{D}_{\tau,t}\,(1+i_\tau)R_\tau. \tag{7.8}$$

Here $\tilde{D}_{\tau,t}$ denotes the modified nominal discount factor in period t of nominal payments in period $\tau \geq t$ that is defined as

$$\tilde{D}_{\tau,t} \equiv 1 \text{ for } \tau = t, \quad \tilde{D}_{\tau,t} \equiv \prod_{k=t}^{\tau-1}(1+i_k+m)^{-1} \text{ for } \tau \geq t+1. \;\; t \geq 1. \tag{7.9}$$

Similarly, under the assumption (7.6), we can rewrite (3.4) as

$$P_t = \frac{(1+i_t)\mathrm{UC}_t + P_{t+1}}{1+i_t+m} = \frac{1}{1+i_t+m}\sum_{\tau=t}^{\infty}\tilde{D}_{\tau,t}\,(1+i_\tau)\mathrm{UC}_\tau \tag{7.10}$$

Under the assumption that the interest rate, rent, and user cost are constant ($i_t = i, R_t = R, \mathrm{UC}_t = \mathrm{UC}$), the house price is also constant ($P_t = P$) and satisfies, respectively,

$$P = \frac{(1+i)R}{i+m} \text{ and} \tag{7.11}$$

$$P = \frac{(1+i)\mathrm{UC}}{i+m}. \tag{7.12}$$

7.3 Negative mortgage rate, positive house price

From (7.11) and (7.12), we can also see that mortgage rates can be negative and house prices positive in a steady state ($i < 0, P > 0$) as long as interest rate and the OMRD rate satisfy

$$1+i > 0 \text{ and } i+m > 0. \tag{7.13}$$

Because $m < 1$, the condition is simply

$$m > -i. \tag{7.14}$$

8 The user cost with a loan-to-value restriction

Consider the optimization problem of choosing $\{c_t, h_t\}_{t=1}^{\infty}$ so as to maximize the intertemporal utility function

$$\sum_{t=1}^{\infty}\beta^{t-1}U(c_t, h_t) \tag{8.1}$$

subject to budget and LTV constraints,

$$P_t^c c_t + P_t h_t - D_t = (P_t - M_{t-1})h_{t-1} - (1+i_{t-1})D_{t-1} + Y_t, \tag{8.2}$$

$$D_t \leq \ell_t P_t h_t, \;\; t \geq 1, \tag{8.3}$$

where $\ell_t \geq 0$ is the maximum LTV ratio.

The corresponding Lagrangian is

$$L = \sum_{t=1}^{\infty} \beta^{t-1} U(c_t, h_t) \tag{8.4}$$
$$+ \sum_{t=1}^{\infty} \lambda_t[(P_t - M_{t-1})h_{t-1} - (1 + i_{t-1})D_{t-1} + Y_t - P_t^c c_t - P_t h_t + D_t] \tag{8.5}$$
$$+ \sum_{t=1}^{\infty} \mu_t[\ell_t P_t h_t - D_t]. \tag{8.6}$$

The first-order conditions are

$$\frac{\partial L}{\partial c_t} = \beta^{t-1} U_{ct} - \lambda_t P_t^c = 0, \tag{8.7}$$
$$\frac{\partial L}{\partial h_t} = \beta^{t-1} U_{ht} + \lambda_{t+1}(P_{t+1} - M_{t)}) - \lambda_t P_t + \mu_t \ell_t P_t = 0, \tag{8.8}$$
$$\frac{\partial L}{\partial D_t} = - \lambda_{t+1}(1 + i_t) + \lambda_t - \mu_t = 0, \tag{8.9}$$
$$\mu_t(\ell_t P_t h_t - D_t) = 0, \tag{8.10}$$
$$\mu_t \geq 0. \tag{8.11}$$

Here U_{ct} and U_{ht} denote $\partial U(c_t, h_t)/\partial c_t$ and $\partial U(c_t, h_t)/\partial h_t$, respectively. The complementary-slackness conditions are (8.3), (8.10), and (8.11).

We get

$$\beta^{t-1} U_{ct} = \lambda_t P_t^c, \tag{8.12}$$
$$\beta^{t-1} U_{ht} = \lambda_{t+1}(M_t - P_{t+1}) + \lambda_t P_t - \mu_t \ell_t P_t, \tag{8.13}$$
$$\mu_t = \lambda_t - \lambda_{t+1}(1 + i_t) \geq 0. \tag{8.14}$$

By (8.12), the Lagrange multiplier λ_t equals $\beta^{t-1} U_{ct}/P_t^c$ and thus equals the marginal utility of nominal wealth in period $t \geq 1$ in terms of utility units of period 1. It follows that the ratio λ_{t+1}/λ_t satisfies

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\beta U_{c,t+1}/P_{t+1}^c}{U_{ct}/P_t^c}, \tag{8.15}$$

where the right-hand side equals the marginal rate of substitution of nominal wealth in period $t+1$ for nominal wealth in period t. Given this, the (nominal) shadow interest rate, i_t^s, is defined as

$$\frac{1}{1 + i_t^s} \equiv \frac{\lambda_{t+1}}{\lambda_t}. \tag{8.16}$$

By (8.14) and (8.16) we get

$$\frac{\mu_t}{\lambda_t} = 1 - \frac{\lambda_{t+1}}{\lambda_t}(1 + i_t) = 1 - \frac{1 + i_t}{1 + i_t^s} = \frac{i_t^s - i_t}{1 + i_t^s} \geq 0. \tag{8.17}$$

It follows from (8.17) that $i_t^s \geq i_t$. Furthermore, with $i_t > -1$, $\mu_t/\lambda_t < 1$. Thus,

$$0 \leq \frac{\mu_t}{\lambda_t} < 1. \tag{8.18}$$

Define the (nominal) user cost, UC_t, as[6]

$$\mathrm{UC}_t \equiv P_t^c \frac{U_{ht}}{U_{ct}}. \tag{8.19}$$

From (8.16), (8.17), and (8.19), we then have

$$\begin{align}
\mathrm{UC}_t \equiv P_t^c \frac{U_{ht}}{U_{ct}} &= P_t + \frac{\lambda_{t+1}}{\lambda_t}(M_t - P_{t+1}) - \frac{\mu_t}{\lambda_t}\ell_t P_t \tag{8.20}\\
&= \frac{M_t + i_t^s P_t - (P_{t+1} - P_t)}{1 + i_t^s} - \frac{(i_t^s - i_t)\ell_t P_t}{1 + i_t^s} \tag{8.21}\\
&= \frac{M_t + [\ell_t i_t + (1-\ell_t) i_t^s] P_t - (P_{t+1} - P_t)}{1 + i_t^s} \tag{8.22}\\
&= \frac{M_t + f_t P_t - (P_{t+1} - P_t)}{1 + i_t^s}, \tag{8.23}
\end{align}$$

where f_t, the (nominal) financing cost (of housing capital), is defined as

$$f_t \equiv \ell_t i_t + (1 - \ell_t) i_t^s. \tag{8.24}$$

Here, the first term represents the cost of the mortgage and the second the cost of (housing) equity.

8.1 Unsecured debt

Introduce unsecured debt, $D_t^u \geq 0$, with interest rate $i_t^u \geq i_t$. Let the budget and other constraints be

$$\begin{align}
P_t^c c_t + P_t h_t - D_t - D_t^u &= (P_t - M_{t-1}) h_{t-1} - (1 + i_{t-1}) D_{t-1} - (1 + i_{t-1}^u) D_{t-1}^u + Y_t, \tag{8.25}\\
D_t &\leq \ell_t P_t h_t, \tag{8.26}\\
D_t^u &\geq 0, \;\; t \geq 1. \tag{8.27}
\end{align}$$

The Lagrangian is then

$$\begin{align}
L = &\sum_{t=1}^{\infty} \beta^{t-1} U(c_t, h_t) \tag{8.28}\\
&+ \sum_{t=1}^{\infty} \lambda_t [(P_t - M_{t-1}) h_{t-1} - (1 + i_{t-1}) D_{t-1} - (1 + i_{t-1}) D_{t-1}^u + Y_t \nonumber\\
&\qquad - P_t^c c_t - P_t h_t + D_t + D_t^u] \tag{8.29}\\
&+ \sum_{t=1}^{\infty} \mu_t [\ell_t P_t h_t - D_t]. \tag{8.30}
\end{align}$$

Here the only constraint on D_t^u is the nonnegativity one—the household cannot invest at a high unsecured interest rate. We could consider more complicated constraints, such as $D_t^u \leq$

[6] As in Díaz and Luengo-Prado (2008), the real user cost, uc_t, is defined as $\mathrm{uc}_t \equiv U_{ht}/U_{ct} = \mathrm{UC}_t/P_t^c$. It corresponds to setting $P_t^c \equiv 1$ and using non-housing consumption as the numeraire.

$\ell_t^u(1-\ell_t)P_t h_t, 0 \leq \ell_t^u \leq 1$, corresponding to a situation when the unsecured loan can only be used to finance at most the fraction ℓ_t^u of a minimum deposit $(1-\ell_t)P_t h_t$.

The first-order conditions are (8.7)–(8.11) and the complementary-slackness conditions (8.31)–(8.33),

$$\frac{\partial L}{\partial D_t^u} = -\lambda_{t+1}(1+i_t^u) + \lambda_t \leq 0, \tag{8.31}$$

$$(-\lambda_{t+1}(1+i_t^u) + \lambda_t)D_t^u = 0, \tag{8.32}$$

$$D_t^u \geq 0. \tag{8.33}$$

We get

$$\beta^{t-1}U_{ct} = \lambda_t P_t^c, \tag{8.34}$$

$$\beta^{t-1}U_{ht} = \lambda_{t+1}(M_{t)-P_{t+1}} + \lambda_t P_t - \mu_t \ell_t P_t, \tag{8.35}$$

$$\mu_t = \lambda_t - \lambda_{t+1}(1+i_t) \geq 0, \tag{8.36}$$

$$\frac{1}{1+i_t^u} \leq \frac{\lambda_{t+1}}{\lambda_t}, \tag{8.37}$$

$$\left(\frac{1}{1+i_t^u} - \frac{\lambda_{t+1}}{\lambda_t}\right) D_t^u = 0. \tag{8.38}$$

By (8.16) and (8.37), we have $i_t^u \geq i_t^s$. By (8.38), if $D_t^u > 0$, we then have $i_t^s = i_t^u$.

In summary, we have

$$i_t^u \geq i_t^s \geq i_t, \tag{8.39}$$

with equality in the first inequality if $D_t^u > 0$ and equality in the second inequality if $D_t < \ell_t P_t h_t$.

The user cost is still given by (8.23) and (8.24).

One can also consider amortization and refinancing restrictions, as well as payment restrictions.

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