Comments on Orphanides, "The Quest for Prosperity without Inflation"

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- Fascinating and controversial paper: Consequences for monetary policy of uncertainty about state of economy
- Controversial conclusion

$$R_t = R_t^* + \gamma(\pi_t - \pi^*) + \delta(y_t - \bar{y}_t)$$

Uncertainty about $\bar{y}_t$ suggests "prudence," $\delta = 0$

Comments

- Status of CEA estimates of potential output?
  - Noisy/biased estimates?
  - Optimal estimates?
  - How much used?
- Conceptual clarification
  - Response to optimal estimates (certainty-equivalence)
  - Response to noisy observations (optimal filtering)

Example (Svensson-Woodford, "Indicators for Optimal Policy")

- Aggregate supply and demand

$$\pi_t = \delta\pi_{t+1|t} + \kappa(y_t - \bar{y}_t) + \nu_t \tag{5.1}$$
$$y_t = y_{t+1|t} - \sigma(i_t - \pi_{t+1|t}) \tag{5.2}$$
$$\bar{y}_{t+1} = \gamma\bar{y}_t + \eta_{t+1} \tag{5.3}$$
$$\nu_{t+1} = \rho\nu_t + \varepsilon_{t+1} \tag{5.4}$$

  π_t inflation, y_t output, $\bar{y}_t$ potential output, ν_t "cost-push" shock, i_t instrument rate, ε_t and η_t iid shocks
  $z_{t+1|t} \equiv \mathrm{E}[z_{t+1}|I_t]$

  – Predetermined variables: $\bar{y}_t$, ν_t

  – Forward-looking variables: π_t, y_t

- Objectives: "Flexible inflation targeting"

  – Period loss function

$$L_t = \frac{1}{2}[\pi_t^2 + \lambda(y_t - \bar{y}_t)^2] \tag{5.5}$$

  – Intertemporal loss function

$$\mathrm{E}\left[\sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} \mid I_t\right]$$

  – Target variables: π_t, y_t

- Information
  - Imperfect observation of potential output, θ_t iid measurement error, variance σ_θ^2
$$\tilde{y}_t = \bar{y}_t + \theta_t \tag{5.6}$$
  - Perfect observation of inflation
  - Observable variables (indicators)
$$Z_t = \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} \tag{5.7}$$
  - Information set
$$I_t = \{Z_t, Z_{t-1}, ...;\ \text{model}\}$$
  - i_t, y_t also observable (from (5.2))
  - $\bar{y}_t$, ν_t not observable

- Equilibrium under discretion (see paper for commitment case)
  - "Targeting rule" for y_t
$$y_t = \bar{y}_{t|t} - \frac{\kappa}{\kappa^2 + \lambda(1 - \delta\rho)}\nu_{t|t} \equiv F \begin{bmatrix} \bar{y}_{t|t} \\ \nu_{t|t} \end{bmatrix}$$
  - CE: Independent of uncertainty (σ_η^2, σ_ε^2, σ_θ^2)
  - Inflation
$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda(1 - \delta\rho)}\nu_{t|t} \equiv G \begin{bmatrix} \bar{y}_{t|t} \\ \nu_{t|t} \end{bmatrix}$$

- Commitment to optimal targeting rule

$$\pi_t = -\frac{\lambda}{\kappa}[(y_t - \bar{y}_{t|t}) - (y_{t-1} - \bar{y}_{t-1|t-1})] \qquad (5.15)$$

  - Results in social optimum (optimum under commitment)
  - Depends only on κ and λ
  - CE: Independent of uncertainty

- Optimal estimation

$$\begin{bmatrix} \bar{y}_{t|t} \\ \nu_{t|t} \end{bmatrix} = (I + KM)^{-1} \left((I - KL)H \begin{bmatrix} \bar{y}_{t-1|t-1} \\ \nu_{t-1|t-1} \end{bmatrix} + KZ_t \right) \qquad (5.28)$$

$$H = \begin{bmatrix} \gamma & 0 \\ 0 & \rho \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 0 & 0 \\ \kappa & \frac{\lambda}{\kappa^2 + \lambda(1-\delta\rho)} - 1 \end{bmatrix}. \qquad (5.21)$$

  - Kalman gain matrix

$$K = \begin{bmatrix} k_{11} & k_{12} \\ \kappa k_{11} & \kappa k_{12} + 1 \end{bmatrix} = \begin{bmatrix} + & - \\ + & + \end{bmatrix} \qquad (5.26)$$

- Instrument response, implied "instrument rule"
  From (5.8)

$$i_t = \tilde{F} \begin{bmatrix} \bar{y}_{t|t} \\ \nu_{t|t} \end{bmatrix}, \quad \tilde{F} \equiv GH + \frac{1}{\sigma} F(H - I) \tag{5.29}$$

  – CE: $\tilde{F}$ independent of σ_η^2, σ_ε^2 and σ_θ^2
  – Response to *optimal estimates* independent of uncertainty

- Response to *indicators*

$$\begin{aligned} i_t \; = \; & \tilde{F}(I + KM)^{-1}(I - KL)H \begin{bmatrix} \bar{y}_{t-1|t-1} \\ \nu_{t-1|t-1} \end{bmatrix} \\ & + \tilde{F}(I + KM)^{-1}(K_1 \tilde{y}_t + K_2 \pi_t), \\ & K = [K_1 \;\; K_2] = \begin{bmatrix} k_{11} & k_{12} \\ \kappa k_{11} & \kappa k_{12} + 1 \end{bmatrix} \end{aligned}$$

  – Not CE: Depends on σ_η^2, σ_ε^2, σ_θ^2
  – $\sigma_\theta^2 \to \infty \Rightarrow k_{11} \to 0$, no response to $\tilde{y}_t$, only to π_t

- "Effective" measurement equation

$$\bar{Z}_t = \begin{bmatrix} \bar{y}_t + \theta_t \\ -\kappa \bar{y}_t + \nu_t \end{bmatrix}. \tag{5.24}$$

  – Observe noisy measure of $\bar{y}_t$
  – Observe linear combination of $\bar{y}_t$ and ν_t

  Transition equation

$$\begin{bmatrix} \bar{y}_{t+1} \\ \nu_{t+1} \end{bmatrix} = H \begin{bmatrix} \bar{y}_t \\ \nu_t \end{bmatrix} + \begin{bmatrix} \eta_{t+1} \\ \varepsilon_{t+1} \end{bmatrix}, \tag{5.25}$$

  – Optimal inference of $\bar{y}_{t|t}$ and $\nu_{t|t}$, also when $\sigma_\theta^2 \to \infty$

- Sum up
  - Implied instrument rule

$$
\begin{aligned}
i_t &= \tilde{F} \begin{bmatrix} \bar{y}_{t|t} \\ \nu_{t|t} \end{bmatrix} \\
&= \tilde{F}(I + KM)^{-1}(I - KL)H \begin{bmatrix} \bar{y}_{t-1|t-1} \\ \nu_{t-1|t-1} \end{bmatrix} \\
&\quad + \tilde{F}(I + KM)^{-1}(K_1 \tilde{y}_t + K_2 \pi_t),
\end{aligned}
$$

  - $\tilde{F}$ independent of uncertainty

$$
\frac{\partial \bar{y}_{t|t}}{\partial \tilde{y}_t} \sim k_{11} > 0
$$

$$
\frac{\partial \bar{y}_{t|t}}{\partial \pi_t} \sim k_{12} < 0
$$

  - k_{11} and k_{12} depend on uncertainty
  - If $\sigma_\theta^2 \to 0$, $\tilde{y}_t$ useless indicators, $k_{11} \to 0$

  - Disregard $\tilde{y}_t$, *not* $\bar{y}_t$. Still $\frac{\partial \bar{y}_{t|t}}{\partial \pi_t} < 0$

- Status of CEA estimates?
  - Optimal estimates?
  - Noisy/biased observations?
  - Useless?
  - Current estimates of $\bar{y}_t$?