Our goal is to provide a robust method of identifying lower-frequency movements in \( r_t^* \) that can be included in the kind of ‘simple’ policy rules that have been shown to perform well in a wide variety of models.

- Intercept in instrument rules like the Taylor rule
- Not the only reason to estimate \( r_t^* \), though
- Conceptual discussion of \( r_t^* \)
- Empirical framework for estimating an \( r_t^* \)
- Estimation of an \( r_t^* \), US 61:1–00:4
- Robustness analysis
- Consequences of mismeasuring the natural rate for a Taylor rule

Authors’ definition of natural interest rate
- Wicksell (1898): “... a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise or lower them.”
- “the real short-term interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation”
- “represents a medium-run real rate ‘anchor’ for monetary policy”
- “varies over time in response to shifts in preferences and technology”
- Reference to standard optimal-growth model
  \[
  r = \frac{1}{\sigma} q + n + \theta \tag{1}
  \]
  - Converging? “Trend growth”?

More precise definition desirable
- Real interest rate in flex-price equilibrium
- Actual output = potential output \( \equiv \) output in flex-price equilibrium
  * Given capital stock, or capital stock in LRE/SS (Woodford, Nelson & Neiss)

Minimum setup
\[
\frac{1}{1 + r_t^2} = \frac{\beta U_C(C_t+1, \xi_{t+1})}{U_C(C_t, \xi_t)}
\]
\[
N_t C_t = Y_t^n - I_t
\]
\[
Y_t^n = A_t P(N_t Q_t L_t, K_t)
\]
\[
K_{t+1} = K_t + I_t - b_t K_t
\]
- Time-varying (time preference/consumption shocks, population growth, technology shocks, capital stock, ...)
- Low-frequency components? Derive!

Reduced-form model for Kalman-filter estimation
Measurement equations
\[
y_t = \gamma_t + A_t(L)(y_{t-1} - y_{t-1}^* + A_t(L)(r_{t-1} - r_{t-1}^*) + \varepsilon_{tz} \tag{2}
\]
\[
\pi_t = B_t(L)\pi_{t-1} + B_y(L)(y_{t-1} - y_{t-1}^*) + B_x(L)G_t + \varepsilon_{tx} \tag{3}
\]
Transition equations
\[
r_t^* = c r_t + z_t \tag{4}
\]
\[
z_t = D_2(L)z_{t-1} + \varepsilon_{tz} \tag{5}
\]
\[
y_t^* = y_t - \gamma_t + \varepsilon_{ty} \tag{6}
\]
\[
g_t = y_t - 1 + \varepsilon_{ty} \tag{7}
\]
- Relation between (2)-(7) and above theory?
  * Example model where (2)-(7) is true? What assumptions are needed?
  * Consistency of (4)-(7) with above theory?
  * Cross-equation restrictions?

Estimation of an \( r_t^* \), US 61:1–00:4, Stock & Watson median-unbiased estimator
- Robustness analysis

Consequences of mismeasuring the natural rate for a Taylor rule
- Academic issue: No central bank would commit to a Taylor rule or a particular estimate of \( r_t^* \)
- Other reasons for interest in \( r_t^* \)
  * Related to output gap
  * Measures of monetary-policy stance
  * Constructing forecasts conditional on alternative instrument-rate paths