Thomas Laubach and John C. Williams,
Measuring the Natural Interest Rate

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- "Our goal is to provide a robust method of identifying lower-frequency movements in r_t^* that can be included in the kind of 'simple' policy rules that have been shown to perform well in a wide variety of models"
  - Intercept in instrument rules like the Taylor rule
  - Not the only reason to estimate r_t^*, though
- Conceptual discussion of r_t^*
- Empirical framework for estimating an r_t^*
- Estimation of an r_t^*, US 61:1–00:4
- Robustness analysis
- Consequences of mismeasuring the natural rate for a Taylor rule

- Conceptual discussion: Background

$$\begin{array}{ccccc} \text{Inflation gap} & \overset{\text{Phillips/AS}}{\longleftrightarrow} & \text{Output gap} & \overset{\text{AD}}{\longleftrightarrow} & \text{Interest rate gap} \\ \pi_t - \pi^* & & y_t - y_t^* & & r_t - r_t^* \end{array}$$

Observed: π_t, π^*, y_t, r_t
Unobserved: y_t^*, r_t^*

- Estimate r_t^*?
- Intuition: Use inflation gap and AS relation to infer output gap. Then use output gap and AD relation to infer interest rate gap
- Note: Relation inflation gap-interest gap "indirect", via "direct" relations inflation gap-output gap and output gap-interest rate gap.

- Authors' definition of natural interest rate
  - Wicksell (1898): "... a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise or lower them."
  - "the real short-term interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation"
  - "represents a medium-run real rate 'anchor' for monetary policy"
  - "varies over time in response to shifts in preferences and technology"
  - Reference to standard optimal-growth model

$$r = \frac{1}{\sigma} g + n + \theta \tag{1}$$

  - Converging? "Trend growth"?

- More precise definition desirable
  - Real interest rate in flex-price equilibrium
  - Actual output = potential output ≡ output in flex-price equilibrium
    - ∗ Given capital stock, or capital stock in LRE/SS (Woodford, Nelson & Neiss)

- Minimum setup

$$\begin{aligned} \frac{1}{1+r_t^n} &= \mathrm{E}_t \frac{\beta U_C(C_{t+1}, \xi_{t+1})}{U_C(C_t, \xi_t)} \\ N_t C_t &= Y_t^n - I_t \\ Y_t^n &= A_t F(N_t Q_t L_t, K_t) \\ K_{t+1} &= K_t + I_t - \delta_t K_t \end{aligned}$$

- Time-varying (time preference/consumption shocks, population growth, technology shocks, capital stock, ...)
- Low-frequency components? Derive!

- Reduced-form model for Kalman-filter estimation
  Measurement equations

$$y_t = y_t^* + A_y(\mathrm{L})(y_{t-1} - y_{t-1}^*) + A_r(\mathrm{L})(r_{t-1} - r_{t-1}^*) + \varepsilon_{1t} \tag{2}$$

$$\pi_t = B_\pi(\mathrm{L})\pi_{t-1} + B_y(\mathrm{L})(y_{t-1} - y_{t-1}^*) + B_x(\mathrm{L})x_t + \varepsilon_{2t} \tag{3}$$

  Transition equations

$$r_t^* = c g_t + z_t \tag{4}$$

$$z_t = D_z(\mathrm{L}) z_{t-1} + \varepsilon_{3t} \tag{5}$$

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{4t} \tag{6}$$

$$g_t = g_{t-1} + \varepsilon_{5t} \tag{7}$$

  - Relation between (2)-(7) and above theory?
    - ∗ Example model where (2)-(7) is true? What assumptions are needed?
    - ∗ Consistency of (4)-(7) with above theory?
    - ∗ Cross-equation restrictions?

- Estimation of an r_t^*, US 61:1–00:4, Stock & Watson median-unbiased estimator
- Robustness analysis

- Consequences of mismeasuring the natural rate for a Taylor rule
  - Academic issue: No central bank would commit to a Taylor rule or a particular estimate of r_t^*
  - Other reasons for interest in r_t^*
    - ∗ Related to output gap
    - ∗ Measures of monetary-policy stance
    - ∗ Constructing forecasts conditional on alternative instrument-rate paths