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Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank

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Abstract

We start from two empirical facts: (1) Central banks target CPI inflation. (2) Independent central banks are concerned about their balance sheet and the level of their capital. The first fact makes it difficult for a central bank to implement the optimal escape from a liquidity trap, because it makes a commitment to overshoot the inflation target not credible. We show that the second fact helps to solve this credibility problem. The optimal policy can take the form of a currency depreciation combined with a crawling peg, a policy advocated by Svensson as the Foolproof Way to escape from a liquidity trap. (JEL E52, F31, F41)

This paper starts from two empirical facts: (1) Central banks target inflation measured by the consumer price index (CPI). (2) Independent central banks are concerned about their balance sheet and the level of their capital. We provide evidence for these facts below. The first fact adds to the well-known credibility problem of the optimal escape from a liquidity trap. The second fact provides a mechanism through which the credibility problem can be solved and the optimal escape from a liquidity trap can be implemented.

In a liquidity trap, the nominal interest rate is zero, but the real interest rate is higher than optimal, due to private-sector expectations of low inflation or even deflation. It is well known since Krugman (1998) that the optimal way to escape from a liquidity trap is to generate expectations of a higher future price level and thereby expectations of higher than normal future inflation. In particular, a central bank with an inflation target should generate expectations of a future overshooting of the inflation target. This will lower the real interest rate and stimulate the economy out of the liquidity trap, even though the nominal interest rate is zero. The problem, also emphasized by Krugman (1998), is how to make the higher future price level credible. A promise of a higher future price level may not be credible, since the central bank may renege on its promise in the future and achieve a lower price level than promised, so as to maintain low and stable inflation in line with its inflation target. CPI inflation targeting makes this credibility problem worse, since the exchange rate provides relatively fast channel to affect the CPI via the impact on domestic-currency prices of imported final goods. Thus, the central bank may renege ex post by a currency appreciation, which will reduce the CPI and achieve the inflation target ex post.

The main new result in this paper is that the second fact mentioned above, that independent central banks are concerned with the level of their capital, creates a commitment mechanism that allows an independent central bank to commit to a higher future price level through a current currency depreciation and, in particular, creates an incentive *not to appreciate* the currency in the future. This commitment mechanism provides support for the Foolproof Way to escape from a liquidity trap that has been suggested by Svensson (2001) and (2003a).[1]

As we show in section I.D, there is considerable evidence that central bankers care about the capital (the net worth) of the bank. The bank wishes to maintain its independence from the government. A negative capital would, under existing accounting rules, require a capital injection from the government and put the bank at the government's mercy. In order to avoid this, the bank never voluntarily allows its capital to fall below a certain minimum level. Because undoing the current currency depreciation by a future currency appreciation would imply a future capital loss on the bank's foreign-exchange reserves, a minimum capital level provides a lower bound on the future exchange rate (an upper bound on the future currency appreciation). By managing its capital such that the minimum capital level is reached for the exchange-rate level consistent with the desired higher future price level, the bank can commit itself to that higher future price level.[2]

Although several recent papers on liquidity traps and the experience of Japan have emphasized the credibility problem of committing to future inflation for a central bank with an established low-inflation reputation (for instance, Krugman, 1998; Svensson, 2001-2003b; and Eggertsson, 2003), this literature has not explicitly incorporated the specific concerns of independent central banks for the level of their capital.

For both Krugman (1998) and Eggertsson (2003), central-bank independence and a lack of coordination of monetary and fiscal policy imply a problem and a barrier to the escape from a liquidity trap. In our paper, central-bank independence and related balance-sheet concerns provide the solution to this problem.

Eggertsson (2003) models the role of the nominal liabilities of a combined government and central bank in providing incentives to future inflation that will reduce the real value of the public debt and, thereby, distortionary taxation. Our setup differs from Eggertsson's in several respects.

More realistically (at least for advanced industrial countries), the central bank is not subordinated to the fiscal authority, and monetary policy is not used to reduce the public debt and lower future taxation. Instead the central bank is independent and concerned about its capital solely in order to maintain its independence. It is not concerned about the net worth of the consolidated public sector but about capital losses on its foreign-exchange reserves.

The balance-sheet concerns of central bankers is an area of research that has been left largely unexplored in the literature on monetary policy. However, there is a great deal of evidence that central bankers pay attention to the capital of the central bank because its level matters for the financial autonomy and the independence of the central bank. As more and more central banks become independent, it would seem important to understand better the channels by which the balance-sheet concerns of central bankers and monetary policy influence each other. As this paper illustrates, the balance-sheet concerns of central banks may have implications for the conduct of monetary policy that are non-trivial from an analytical point of view and relevant for real-world policy problems.

In the context of a liquidity trap, the concerns of central banks for their level of capital have been the subject of some informal discussion. Many commentators, for instance, Bernanke (2003), have suggested that such concerns have been a barrier to more aggressive policies in Japan and that monetary and fiscal cooperation, including compensation for central-bank losses from risky open-market purchases, would contribute to Japan's escape from the liquidity trap.

In other contexts than a liquidity trap, more formal models of the balance-sheet concerns of central banks have been presented. Isard (1994) presents a model of currency crises in which the central bank cares about the value of its foreign-exchange reserves. Sims (2004) shows that a low level of capital may prevent a central bank from avoiding self-fulfilling hyperinflationary equilibria. In Sims's model, balance-sheet concerns are an impediment: they prevent the central banker from taking the right policy actions in a situation of economic distress. In contrast, in this paper, we show that, in the context of a liquidity trap, balance-sheet concerns and central-bank independence may provide a solution, a commitment mechanism for optimal escape from a liquidity trap.

Section I lays out the model of the paper. Section II shows how a liquidity trap can arise, and specifies the suboptimal policy under discretion and the optimal policy under commitment. Section III shows how an independent central bank can commit itself to the optimal policy. Section IV provides some conclusions.

I Model

We consider a small open economy, a simplified version of many current open-economy models, for instance, that in Svensson (2003b). Households consume a traded good and a nontraded good. The nontraded good is produced with nontraded intermediate inputs whose nominal prices are set one period ahead by monopolistic producers. The central bank conducts flexible inflation targeting by minimizing a standard quadratic loss function of inflation and the output gap, where inflation is CPI inflation.[3] We show that the economy may fall into a liquidity trap with excessively low output and inflation as a result of an unanticipated fall in expected productivity growth and a related fall in the natural interest rate.[4]

I.A Structure of the economy

Time is separated into discrete periods, $t = ..., -1, 0, 1, ...$ There is one traded good, the foreign good. The foreign-currency price of the good is unity in all periods. Let S_t denote the exchange rate in period t, measured in home-currency units per foreign currency. The Law of One Price holds, so the home-currency price of the foreign good is simply equal to the exchange rate. A foreign-currency bond, with a constant continuously compounded interest rate, $r^* > 0$, is the only traded asset.

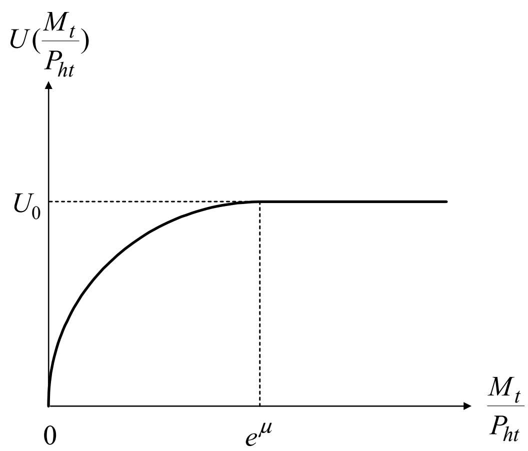
The home country has a private sector, consisting of a household and firms, and a public sector, consisting of a central bank and a government. The household consumes the traded foreign good and the nontraded home good and supplies labor. The utility function of the household in period t is

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau [(1-\alpha)\ln C_{h,t+\tau} + \alpha \ln C_{f,t+\tau} + U(\frac{M_{t+\tau}}{P_{h,t+\tau}}) - N_{t+\tau}],$$

where E_t denotes expectations conditional on information available in period t, $\delta \equiv e^{-r^*}$ is the discount factor, C_{ht} denotes consumption of the home good in period t, C_{ft} denotes consumption of the foreign good, $\alpha \in (0,1)$ denotes the consumption share of the foreign good, M_t denotes the household's holding of home currency, P_{ht} denotes the home-currency price of home goods, and N_t denotes the supply of labor. The function $U(M_t/P_{ht})$ represents the liquidity services of real money, which consist of time saved in the transactions of the home good. Real money is consequently measured in terms of the home good.

The liquidity-services function is continuous and continuously differentiable for $M_t/P_{ht} > 0$ and has the additional properties $U(\frac{M_t}{P_{ht}}) < U_0$, $U'(\frac{M_t}{P_{ht}}) > 0$, $U''(\frac{M_t}{P_{ht}}) < 0$ for $\frac{M_t}{P_{ht}} < e^{\mu}$; $U(\frac{M_t}{P_{ht}}) = U_0$

Figure 1: The liquidity-services function



for $\frac{M_t}{P_{ht}} \geq e^{\mu}$; and $U'(\frac{M_t}{P_{ht}}) \to \infty$ for $\frac{M_t}{P_{ht}} \to 0$; where $e^{\mu} > 0$ is the satiation level of real money. That is, $U(M_t/P_{ht})$ is increasing at a decreasing rate for $M_t/P_{ht} < e^{\mu}$ and has a maximum equal to U_0 for $M_t/P_{ht} \geq e^{\mu}$. There is a positive demand for real balances regardless of how high the home-currency interest rate is. The liquidity-services function is illustrated in Figure 1.

[Insert Figure 1 around here]

The consumer price index (CPI), P_t, will be given by

(1) $$P_t = P_{ht}^{1-\alpha} S_t^{\alpha},$$

where we use the fact that the home-currency price of the foreign good is equal to the nominal exchange rate.

The nontraded home good is produced in two stages. In the final stage, final-good firms produce the output Y_t of the home good with inputs of a continuum of nontraded intermediate goods, $Y_t(\iota)$, $\iota \in [0, 1]$, according to the production function,

(2) $$Y_t \equiv [\int_0^1 Y_t(\iota)^{1-1/\xi} d\iota]^{\frac{1}{1-1/\xi}},$$

where $\xi > 1$ denotes the elasticity of substitution between the intermediate inputs. The final-good firms operate under perfect competition and take the prices of the home good and intermediate inputs as given. The corresponding price index satisfies

(3) $$P_{ht} = [\int_0^1 P_{ht}(\iota)^{1-\xi} d\iota]^{\frac{1}{1-\xi}},$$

where $P_{ht}(\iota)$ denotes the home-currency price of intermediate good ι. It follows that demand for intermediate good ι is given by

$$Y_t(\iota) = Y_t \left(\frac{P_{ht}(\iota)}{P_{ht}} \right)^{-\xi}. \tag{4}$$

In the initial stage, each intermediate good ι is produced by a single firm ι with a technology that is linear in labor input with a country-wide exogenous stochastic productivity, A_t,

$$Y_t(\iota) = A_t N_t(\iota),$$

where $N_t(\iota)$ denotes labor input in the production of intermediate good ι. There is hence a continuum of firms producing intermediate goods. Firm ι maximizes profits, subject to perfect competition in the labor market and monopolistic competition in the intermediate goods market (with the gross markup $\xi/(\xi-1)$ over marginal cost), and distributes the profits to the home household. Firm ι sets its price for period t one period in advance, that is, in period $t-1$, so as to maximize the expected utility value of profits. Aggregate labor supply and demand will be given by

$$N_t \equiv \int_0^1 N_t(\iota) d\iota.$$

The budget constraint in period t for the home household is

$$P_{ht}C_{ht} + S_t C_{ft} + M_t + B_t + S_t B_t^* = P_{ht}Y_t + M_{t-1} + e^{i_{t-1}} B_{t-1} + S_t e^{r^*} B_{t-1}^* + Z_t, \tag{5}$$

where B_t denotes the number of home-currency one-period bonds held between periods t and $t+1$, i_t is the continuously compounded interest rate paid in period $t+1$ on those bonds, $B_t^* \gtrless 0$ denotes the number of foreign-currency one-period bonds held between period t and period $t+1$ (positive if the household is a lender, negative if it is a borrower), and $Z_t \gtrless 0$ denotes the home-currency value of net transfers from the government. We use that the sum of total profits and wages received by the household will equal $P_{ht}Y_t$.

The budget constraint for the central bank is

$$Z_t + S_t R_t = M_t - M_{t-1} + S_t e^{r^*} R_{t-1}, \tag{6}$$

where Z_t is the home-currency value of the central bank's dividend paid to the government, $R_t \geq 0$ denotes the number of foreign-currency bonds held as foreign-exchange reserves between period t and $t+1$, and $M_t - M_{t-1}$ is the change in the central bank's supply of home currency. In this simple model, the supply of home currency, the monetary base, and the stock of money are identical. For simplicity, we also assume that the only asset on the central bank's balance sheet is foreign-exchange reserves. As shown in Jeanne and Svensson (2004), introducing domestic credit on the asset side of the central bank's balance sheet does not change our results.

There is no government consumption. The government collects the dividend from the central bank and passes it on as a lumpsum transfer to the household. We assume that no home-currency bonds are held in the foreign country, and that the net supply of home-currency bonds is zero,

$$B_t = 0. \tag{7}$$

I.B Equilibrium relationships

In equilibrium, consumption and production of the home good are equal,

$$C_{ht} = Y_t. \tag{8}$$

Adding (5) and (6) and using (7) and (8), we can write the consolidated budget constraint for the home country in terms of the foreign good,

$$C_{ft} + F_t = e^{r^*} F_{t-1}, \tag{9}$$

where

$$F_t \equiv B_t^* + R_t$$

denotes the home-country's net foreign assets.

The first-order condition for optimal intertemporal consumption of the foreign good will be

$$e^{-r^*} = \frac{\delta \mathrm{E}_t(1/C_{f,t+1})}{1/C_{ft}} \tag{10}$$

(see Jeanne and Svensson, 2004, for details). Since $\delta = e^{-r^*}$, we realize from (9) and (10) that C_{ft} and F_t will be constant over time and satisfy

$$\begin{aligned} C_{ft} &= (e^{r^*} - 1)F_{t-1} \equiv \bar{C}_f, \\ F_t &= F_{t-1}. \end{aligned} \tag{11}$$

The current account is hence constant and unaffected by monetary policy. This property—which considerably simplifies the analysis—stems from the assumptions that utility is separable in consumption of home and foreign goods.

Firm ι's optimal price is given by the first-order condition

$$P_{h,t}(\iota) = \frac{\xi}{\xi - 1} \mathrm{E}_{t-1} \frac{W_t}{A_t}. \tag{12}$$

where W_t is the nominal wage in period t. The price is simply set equal to the gross markup times the expected home-currency marginal cost (see Jeanne and Svensson, 2004, for details).

The first-order condition for the household's labor supply implies that the wage in home goods, W_t/P_{ht}, is equal to the marginal rate of substitution of home goods for labor,

$$\frac{W_t}{P_{ht}} = \frac{1}{(1-\alpha)/C_{ht}} = \frac{Y_t}{1-\alpha}, \tag{13}$$

where we use (8). It follows from the two previous equations that

$$P_{ht}(\iota) = P_{ht} \mathrm{E}_{t-1} \frac{Y_t}{\bar{Y}_t}, \tag{14}$$

where

$$\bar{Y}_t \equiv \frac{\xi - 1}{\xi} (1-\alpha) A_t \tag{15}$$

is the flexprice level of output, the level that would prevail under flexible prices and constant money supply.[5] Potential output is proportional to productivity.

Equation (14) shows how the decision of an individual price-setter ι depends on the prices set by the competitors as well as the expected ratio of actual to potential output. It says that an intermediate-good firm sets its price above (below) its competitors' if and only if it expects the ratio of actual output to potential output to be above (below) one. Since in equilibrium all intermediate-good producers set their prices at the same level, the expected ratio of actual to potential output must be equal to one

$$\mathrm{E}_{t-1} \frac{Y_t}{\bar{Y}_t} = 1. \tag{16}$$

Actual output can deviate from potential output in the short run because of nominal stickiness. Given that the consumption of the traded good is constant, a change in the production (and consumption) of the nontraded good must be associated with a change in the real exchange rate. The real exchange rate, defined as the price of the traded foreign good in terms of the nontraded home good,

$$Q_t \equiv \frac{S_t}{P_{ht}}, \tag{17}$$

will equal the marginal rate of substitution of the home good for the foreign good,

$$Q_t = \frac{\alpha/C_{ft}}{(1-\alpha)/C_{ht}} = \frac{\alpha}{1-\alpha}\frac{Y_t}{\bar{C}_f}, \tag{18}$$

where we have used (8) and (11). The real exchange rate is proportional to output. We define the natural real exchange rate as the flexprice level, the level that would prevail under flexible prices (and constant money supply),

$$\bar{Q}_t \equiv \frac{\alpha}{1-\alpha}\frac{\bar{Y}_t}{\bar{C}_f}. \tag{19}$$

We conclude this section with the equilibrium relationships for interest rates and money. The first-order condition for optimal intertemporal consumption of the home good is

$$e^{-i_t} = \delta \mathrm{E}_t \frac{P_{ht}Y_t}{P_{h,t+1}Y_{t+1}}, \tag{20}$$

where we use (8) and that the marginal utility of nominal income is proportional to $1/(P_{ht}Y_t)$. The real (CPI) interest rate, r_t, will satisfy

$$e^{-r_t} = \delta \mathrm{E}_t \frac{P_{ht}Y_t/P_t}{P_{h,t+1}Y_{t+1}/P_{t+1}} = \delta \mathrm{E}_t \left(\frac{Y_t}{Y_{t+1}}\right)^{1-\alpha}, \tag{21}$$

where we have used (1), (17), and (18). The natural (CPI) interest rate, $\bar{r}_t$, is defined as the flexprice real (CPI) interest rate. By (21), it will satisfy

$$e^{-\bar{r}_t} = \delta \mathrm{E}_t \left(\frac{\bar{Y}_t}{\bar{Y}_{t+1}}\right)^{1-\alpha}. \tag{22}$$

The home-currency interest rate is subject to a zero lower bound,

$$i_t \geq 0. \tag{23}$$

The first-order condition for money can be written

$$U'(\frac{M_t}{P_{ht}}) = \frac{1-\alpha}{C_{ht}}(1-e^{-i_t}).$$

Given the assumptions about the liquidity-services function and (8), the logs of money, prices, and output (denoted by lowercase variables) will be related by

$$\begin{aligned} m_t - p_{ht} &= g(y_t, i_t) \quad (i_t > 0), \\ m_t - p_{ht} &\geq \mu \qquad\quad\;\; (i_t = 0). \end{aligned} \tag{24}$$

where $g(y_t, i_t) < \mu$, $\partial g/\partial y_t > 0$, $\partial g/\partial i_t < 0$ for $i_t > 0$ and $g(y_t, 0) = \mu$ (see Jeanne and Svensson, 2004, for details).

I.C Productivity

The dynamics of the economy are driven by the exogenous stochastic process for productivity. We specify this process in such a way that the economy may fall in a liquidity trap in period 1 and in that period only. This simplifies the analysis without substantial restriction of generality, since the policies that succeed in extracting the economy from a liquidity trap in period 1 can be implemented in other periods, too.

We assume that productivity is equal to a constant positive level A up to period 1, and falls to a constant lower level $e^{-b}A$ from period 2 onwards, where b is a nonnegative random variable that captures the size of the negative productivity shock. With lowercase variables continuing to denote the log of uppercase variables, this assumption can be written

$$
\begin{aligned}
a_t &= a \qquad (t \leq 1),\\
a_t &= a - b \quad (t \geq 2).
\end{aligned}
$$

The realization of the random variable b is observed in period 1, one period before the productivity fall takes place. It could be equal to zero, in which case productivity remains unchanged. As we shall see, the economy falls in a liquidity trap when the realization of b is sufficiently large. The realization of b is governed by a probability distribution function that is known by all agents, and expectations are rational (except for a simplifying assumption specified below).

It then follows from (15) and (18) that the log of potential output and the log of the natural real exchange rate also fall by b in period 2

$$
\begin{aligned}
&\bar{y}_t = \bar{y} \text{ and } \bar{q}_t = \bar{q} \qquad (t \leq 1),\\
&\bar{y}_t = \bar{y} - b \text{ and } \bar{q}_t = \bar{q} - b \quad (t \geq 2),
\end{aligned} \tag{25}
$$

with $\bar{y}$ and $\bar{q}$ defined by

$$
\begin{aligned}
\bar{y} &\equiv \ln\left(\frac{\xi - 1}{\xi}(1-\alpha)\right) + a,\\
\bar{q} &\equiv \ln\frac{\alpha}{1-\alpha} + \bar{y} - \bar{c}_f.
\end{aligned}
$$

Taking the log of (22)—and using that potential output is known one period in advance—we get

$$
\bar{r}_t = r^* + (1-\alpha)(\bar{y}_{t+1} - \bar{y}_t) \;=\; r^* + (1-\alpha)(a_{t+1} - a_t). \tag{26}
$$

Thus, the natural interest rate is directly related to the rate of time preference, $-\ln\delta = r^*$, and to *expected productivity growth*, $a_{t+1} - a_t$. Expected productivity growth is constant and equal to zero in all periods, except in period 1 when it is falls to $-b$. Consequently, the natural interest rate is equal to r^* in all periods, except in period 1 when it falls to

$$\bar{r}_1 = r^* - (1-\alpha)b. \tag{27}$$

As shown in Jeanne and Svensson (2004), the model can be solved under rational expectations for an arbitrary distribution of b. For expositional simplicity, however, we assume here that economic agents view the lower expected productivity growth in period 1 as very unlikely ex ante. Hence, until period 0 they behave as if b were going to be equal to zero, and the lower expected productivity growth comes to them as an unexpected surprise in period 1. Examining this simple case makes the algebra easier and does not affect the essence of our results on the optimal policy to escape a liquidity trap.

I.D Monetary policy

We assume that the central bank has an objective function corresponding to flexible (CPI) inflation targeting, augmented by concerns about the central bank's balance sheet. The central bank's intertemporal loss function can be written,

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} (1-\delta)\delta^{\tau} L_{t+\tau} \tag{28}$$

with the period loss function

$$L_t = \frac{1}{2}[(\pi_t - \pi)^2 + \lambda(y_t - \bar{y}_t)^2] + \eta_t \gamma. \tag{29}$$

The first term in the loss function captures the concerns of the central banker in a flexible (CPI) inflation targeting regime: $\pi_t = \log(P_t/P_{t-1})$ is inflation, $\pi \geq 0$ is a given inflation target, $y_t - \bar{y}_t$ is the output gap, and $\lambda > 0$ is a given weight on output-gap stabilization (Svensson, 2002; Truman, 2003).

The assumption that the central bank targets the CPI-inflation rate is very realistic.[6] More than twenty central banks have by now announced numerical inflation targets. All these central banks have announced targets for a CPI or a core CPI (Roger and Scott, 2005). This is also the fact for European Central Bank and the Swiss National Bank, who have announced numerical targets but for some reason prefer not to call themselves inflation targeters. Furthermore, the Bank of Japan and the Federal Reserve System, who aim to achieve low and stable inflation but have so far preferred not to announce numerical inflation targets, have nevertheless made clear that they focus on consumer price indices. The Bank of Japan focuses on the CPI excluding fresh food, whereas the Federal Reserve System focuses on the so-called PCE deflator (the Implicit Price Deflator for Personal Consumption Expenditures).

The second term in the loss function captures the central bank's concerns about the capital of the central bank. The parameter $\gamma \geq 0$ is a fixed loss suffered by the central bank if the capital of the central bank falls below a critical level, and η_t is an indicator variable for this event. This term should be justified carefully as it is a new feature of our model and it plays a central role in our results.

Although central banks do not need capital for the same obvious reasons as their commercial counterparts (reducing the risk of a run and a bankruptcy), they seem to be no less concerned by their balance sheets. This is suggested by several facts. First, central-bank capital, although it can be quite low, is generally positive. In 2003, the median level of central-bank capital (including reserves and provisions) represented 8.8 percent of its total assets in emerging economies and 15.3 percent in advanced countries (Hawkins, 2003). In some countries, the central banks have had negative capital for prolonged periods of time, but these are mostly Latin American countries with a history of monetary instability (Stella, 2005).

Second, an increasing number of central banks explicitly define an objective in terms of a capital-adequacy ratio. Since 1998, the Bank of Japan targets a capital-adequacy ratio, consisting of the capital divided by a period average of outstanding bank notes, of around 8–12 percent (Cargill, 2004; Stella, 2005). Recent legislation in Indonesia prevents the transfer of profits to the government unless the central bank has a capital of at least 10 percent of its monetary liabilities. The Reserve Bank of India aims at capital and reserves of 8 percent of assets (Hawkins, 2003).

The central banks' revealed preferences suggest a strong aversion to negative or low capital. But why is it so? The reasons given by central banks suggest that this has to do mainly with the autonomy of their institutions relative to the government. As the governor of the Bank of Japan puts it (Fukui, 2003),

> [C]entral banks' concern with the soundness of their capital base might not be grounded purely in economic theory but may be motivated rather by the political economy instincts of central bankers. In other words, once the restriction that "the central bank should only take risks consistent with the level of its self-imposed capital base" is violated, the boundary between the functions of the central bank and those of the government may become difficult to discern.

A loss-making central bank is likely to become the object of increased oversight from the government and see its independence eroded over time—as any loss-making public entity should be. The government might take advantage of increased oversight to influence monetary policy (Ernhagen et al., 2002; Pringle, 2003; Vaez-Zadeh, 1991). In a long speech on the role of capital for central banks, Ueda (2004)—at the time a member of the Policy Board of the Bank of Japan—mentions several problems for a central bank that becomes insolvent central banks. In particular, he warns:

> [T]he government may take advantage of the opportunity of capital injection to the central bank to influence monetary policy if it wants a different target inflation rate from the central bank.
>
> ...During the [central bank's] insolvency, it is quite likely that the fiscal authorities, which are often responsible for regulating a central bank, would interfere in various aspects of the central bank's policy actions. A more crucial point for an insolvent central bank is that the amount of seigniorage it can generate with a reasonable inflation rate is limited. If, therefore, the central bank intended to overcome insolvency in a short period of time solely by earning seigniorage, it would have to aim for a high inflation rate, sacrificing its goal of price stability.

Turning back to the model, recalling the central bank's budget constraint, (6), we define the central bank's capital at the end of period t, V_t, as

$$V_t \equiv S_t R_t - M_t. \tag{30}$$

We note that, by (6) and (30),

$$V_t = S_t e^{r^*} R_{t-1} - M_{t-1} - Z_t, \tag{31}$$

so the only non-predetermined variables in period t that the capital V_t depends on are the exchange rate, S_t, and the dividend paid to the government during period t, Z_t.

As a simple way of capturing its balance-sheet concerns, as mentioned above, we assume that the central bank suffers an additional loss $\gamma \geq 0$ if its capital falls below a given nonnegative lower bound, $\bar{V} \geq 0$. We represent this additional loss with the term $\eta_t \gamma$ in the loss function (29), where the indicator variable η_t takes values 0 or 1 according to whether the central bank capital violates the lower bound or not:

$$\eta_t = \begin{cases} 0 & (V_t \geq \bar{V}), \\ 1 & (V_t < \bar{V}). \end{cases} \tag{32}$$

The rationale for this, as explained above, is that too low a capital indicates mismanagement, causes embarrassment, might require the resignation of the bank management, and, more importantly, may force the central bank to ask for an injection of capital from the government, a negative dividend, for which it may have to give up some of its independence. In line with this, we also assume that the central bank has control over the dividend it pays and that this dividend is always nonnegative,

$$Z_t \geq 0. \tag{33}$$

We assume, furthermore, that the private sector is perfectly aware of the central bank's concerns about the level of their capital.

Finally, we consider equilibria under discretion, what is often called time-consistent equilibria. The central bank cannot directly commit to future policy actions. Hence, in each period t, it minimizes the period loss function (29) under discretion, for given private-sector expectations of future inflation.[7]

II Liquidity trap

In this section, we temporarily assume that the central bank does not have any balance-sheet concerns, so $\gamma = 0$. We show that, as a result, the economy may fall in a liquidity trap in which the monetary authorities are impotent because their announcements (of future inflation or exchange rate depreciation) are not credible.

II.A Equilibrium without a liquidity trap

We first specify equilibrium inflation. Taking the log of (1) and (17), inflation can be written

$$\pi_t \equiv p_t - p_{t-1} = p_{ht} + \alpha q_t - p_{t-1}. \tag{34}$$

Taking the log difference of (18) and (19), we find that the log deviations of output and of the real exchange rate from their natural levels are equal,

$$q_t - \bar{q}_t = y_t - \bar{y}_t. \tag{35}$$

Using this equation to substitute for q_t in (34) gives the following Phillips curve,

$$\pi_t = (\alpha \bar{q}_t + p_{ht} - p_{t-1}) + \alpha (y_t - \bar{y}_t). \tag{36}$$

The first term on the right side involves variables that are either exogenous ($\bar{q}_t$) or predetermined (p_{ht} and p_{t-1}). The second term implies that the slope of the short-run Phillips curve is α.

Subtracting the log of (22) from (21) gives the following aggregated-demand relation,

$$y_t - \bar{y}_t = \mathrm{E}_t(y_{t+1} - \bar{y}_{t+1}) - \frac{1}{1-\alpha}(r_t - \bar{r}_t) \tag{37}$$

The central bank will be able to control i_t by setting m_t. For given inflation and output-gap expectations, this will determine r_t and y_t. Via (36), π_t is then determined. For simplicity, we can think of the central bank as choosing y_t directly. The central bank can then infer the corresponding level of i_t from (37) and the given inflation and output-gap expectations, and finally the corresponding level of m_t from (24).

We will see below that a liquidity trap can only arise in period 1 and when expected productivity growth and thereby the natural interest rate are low. In all other periods, there is no liquidity trap. Then (23) is not binding, and (36) is the only relevant constraint. In those periods, the central bank's optimization problem under discretion is to minimize the period loss function (29) subject to the Phillips curve (36). This implies the targeting rule (first-order condition)

$$\pi_t - \pi = -\frac{\lambda}{\alpha}(y_t - \bar{y}_t). \tag{38}$$

The only period in which new information is revealed is period 1. In all other periods $t \neq 1$, there is no new information, and y_t is known in the previous period, period $t-1$. Equation (16) then implies that output equals potential output, $y_t = \bar{y}_t$, equation (35) implies that the real exchange rate equals the natural real exchange rate, $q_t = \bar{q}_t$, and the first-order condition (38) implies that inflation equals the inflation target. The real exchange rate is constant for $t \geq 2$, implying that the price of the home good and the nominal exchange rate increase at the same rate π. The same argument applies for $t \leq 0$, and we have the following result (s_t denotes the log exchange rate):

Proposition 1 *Assume that the central bank has no balance-sheet concerns ($\gamma = 0$). Under discretion, for $t \leq 0$ and $t \geq 2$, we have $y_t = \bar{y}_t$, $q_t = \bar{q}_t$, and $p_{ht} - p_{h,t-1} = \pi_t = s_t - s_{t-1} = \pi$.*

II.B Liquidity trap in period 1

Having characterized the equilibrium in periods other than 1, we now focus on period 1. We can summarize the model in period 1 as

$$y_1 - \bar{y} = -\frac{1}{1-\alpha}(r_1 - \bar{r}_1) \leq \frac{1}{1-\alpha}(\bar{r}_1 + \pi_2), \tag{39}$$

$$\pi_1 - \pi = \alpha(y_1 - \bar{y}). \tag{40}$$

Equation (39) follows from (37) for $t = 1$ and observing that perfect foresight applies from period 1 onwards and $\bar{y}_1 = \bar{y}$, $y_2 = \bar{y}_2 = \bar{y} - b$. The inequality follows from (23) and the relation between the nominal and real interest rate in period 1,

$$i_1 = r_1 + \pi_2 \geq 0. \tag{41}$$

Equation (40) follows from (36), $\bar{y}_1 = \bar{y}$, $\alpha\bar{q}_1 + p_{h1} - p_0 = \alpha(\bar{q}_1 - q_0) + \Delta p_{h1}$, $q_0 = \bar{q}_1 = \bar{q}$, and the fact that $\Delta p_{h1} \equiv p_{h1} - p_{h0}$ was set equal to π in period 0 (because the information in period 1 about the fall of productivity in period 2 was unanticipated in period 0).

The economy is in a liquidity trap in period 1, if the constraint (41) prevents the central bank from setting output at its potential level, that is, if $\bar{r}_1 + \pi_2 < 0$. By Proposition 1, in a discretion equilibrium, inflation will be set to $\pi_2 = \pi$ in period 2. By (27), we then have the following result.

Proposition 2 *Assume that the central bank has no balance-sheet concerns ($\gamma = 0$). The economy falls in a liquidity trap in period 1 if and only if the natural interest rate is sufficiently negative,*

$$\bar{r}_1 < -\pi \leq 0, \tag{42}$$

that is, if and only if the fall in expected productivity growth is sufficiently large,

$$b > \frac{r^* + \pi}{1 - \alpha} > 0. \tag{43}$$

We now assume that conditions (42) and (43) holds, so there is a liquidity trap in period 1. In a discretion equilibrium with a liquidity trap in period 1, the best the central bank can do is to set $m_1 \geq p_{h1} + \mu$, so the nominal interest rate will equal zero, $i_1 = 0$. We let $\hat{}$ denote the values of variables in the discretion equilibrium, so $\hat{y}_1$, $\hat{\pi}_1$, and $\hat{r}_1$ denote the corresponding values of output, inflation, and the real interest rate. From the binding constraints (39) and (41), it follows that

$$\hat{y}_1 - \bar{y} = \frac{1}{1-\alpha}(\bar{r}_1 + \pi) < 0, \tag{44}$$

$$\hat{\pi}_1 - \pi = \frac{\alpha}{1-\alpha}(\bar{r}_1 + \pi) < 0, \tag{45}$$

$$\hat{r}_1 - \bar{r}_1 = -(\bar{r}_1 + \pi) > 0.$$

The output gap and the inflation gap are negative, and the real interest-rate gap is positive.

II.C Optimal policy under commitment

The above equilibrium is suboptimal, with a negative output gap that is unnecessary large, because private-sector inflation expectations are equal to the inflation target. If possible, it would be better for the central bank to credibly commit to exceeding the inflation target next period, period 2, and this way create private-sector expectations in period 1 of a higher period-2 inflation. This would lower the real interest rate and reduce the magnitude of the output gap in period 1. In order to specify this optimal policy—the optimal escape from a liquidity trap—we consider the optimal policy in a liquidity trap under commitment (continuing to assume that the central bank does not have any balance-sheet concerns, $\gamma = 0$).

The relevant loss function in period 1 is then

$$L_1 + \frac{1}{2}\delta(\pi_2 - \pi)^2 = \frac{1}{2}[(\pi_1 - \pi)^2 + \lambda(y_1 - \bar{y})^2 + \delta(\pi_2 - \pi)^2]. \tag{46}$$

In period 2, we have $y_2 = \bar{y}_2 = \bar{y} - b$, so there cannot be any surprise and liquidity trap in period 2. Hence, the period-2 output-gap term vanishes from (46). In addition, since there will not be any surprise in period 2, there will not be any uncertainty about period-2 inflation, π_2, so actual rather than expected inflation appears in (46).

The central bank minimizes (46) subject to (39) and (40). We now consider π_2 as a control variable, in addition to y_1. The Lagrangian is

$$\mathcal{L}_1 = \frac{1}{2}[(\pi_1 - \pi)^2 + \lambda(y_1 - \bar{y})^2 + \delta(\pi_2 - \pi)^2] - \varphi_1[\frac{1}{1-\alpha}(\bar{r}_1 + \pi_2) - y_1 + \bar{y}],$$

where $\varphi_1 \geq 0$ is the Lagrange multiplier for the constraint (39) and we will use (40) to substitute for $\pi_1 - \pi$. The first-order conditions with respect to y_1 and π_2 and the complementary-slackness condition are then, respectively,

$$\alpha(\pi_1 - \pi) + \lambda(y_1 - \bar{y}) + \varphi_1 = 0, \tag{47}$$

$$\delta(\pi_2 - \pi) - \frac{1}{1-\alpha}\varphi_1 = 0, \tag{48}$$

$$\varphi_1[\frac{1}{1-\alpha}(\bar{r}_1 + \pi_2) - y_1 + \bar{y}] = 0. \tag{49}$$

In a liquidity trap, $\varphi_1 > 0$. Eliminating φ_1 from (47)–(49) results in the targeting rule under commitment in a liquidity trap:[8] Set $m_1 \geq p_{h1} + \mu$ and thereby $i_1 = 0$, and choose $\pi_2 > \pi$, and thereby

$$y_1 - \bar{y} = \frac{1}{1-\alpha}(\bar{r}_1 + \pi_2), \tag{50}$$

so as to satisfy the target criterion

$$\pi_2 - \pi = -\frac{\alpha}{\delta(1-\alpha)}(\pi_1 - \pi) - \frac{\lambda}{\delta(1-\alpha)}(y_1 - \bar{y}) > 0. \tag{51}$$

The commitment equilibrium in the liquidity trap is then determined by (40), (50), and (51). We let $\tilde{}$ denote the values of variables in the commitment equilibrium. Combining (40), (50), and (51), we get

$$\tilde{y}_1 - \bar{y} = \frac{\delta(1-\alpha)}{\delta(1-\alpha)^2 + \alpha^2 + \lambda}(\bar{r}_1 + \pi) < 0, \tag{52}$$

$$\tilde{\pi}_2 - \pi = -\frac{\alpha^2 + \lambda}{\delta(1-\alpha)^2 + \alpha^2 + \lambda}(\bar{r}_1 + \pi) > 0, \tag{53}$$

$$\tilde{r}_1 - \bar{r}_1 = -\frac{\delta(1-\alpha)^2}{\delta(1-\alpha)^2 + \alpha^2 + \lambda}(\bar{r}_1 + \pi) > 0,$$

where the last equation follows from (39) and (52).

Comparing the output gap in the commitment equilibrium, (52), with that in the discretion equilibrium, (44), we see that

$$\tilde{y}_1 - \bar{y} > \hat{y}_1 - \bar{y}.$$

The magnitude of the negative output gap is less than under discretion. By (40), it follows that the magnitude of the negative inflation gap in period 1, $\tilde{\pi}_1 - \pi$, is also smaller, $\tilde{\pi}_1 - \pi > \hat{\pi}_1 - \pi$. The optimal policy trades off the right amount of period-2 overshoot of the inflation target, $\tilde{\pi}_2 - \pi > 0$, for the right amount of increase in the period-1 output and partial closing of the output gap. The higher period-2 inflation results in a smaller real-interest-rate gap, $\tilde{r}_1 - \bar{r}_1 < \hat{r}_1 - \bar{r}_1$.

II.D Problems in implementing the commitment equilibrium

The commitment equilibrium is not time consistent under discretion. This section discusses the problems that the central bank would encounter in trying to implement the commitment solution (under the assumption that it has no balance-sheet concerns, $\gamma = 0$).

The commitment equilibrium will imply a period-2 money supply, $\tilde{m}_2$, given by

$$\tilde{m}_2 = \tilde{p}_{2h} + g(\bar{y} - b, r^* + \pi) = \tilde{p}_2 - \alpha(\bar{q} - b) + g(\bar{y} - b, r^* + \pi), \tag{54}$$

where $\tilde{p}_{2h}$ and $\tilde{p}_2 \equiv p_0 + \tilde{\pi}_1 + \tilde{\pi}_2$ are the period-2 home-good and CPI price levels, respectively, and we use that $\tilde{p}_{2h} = \tilde{p}_2 - \alpha\bar{q}_2$, $\bar{q}_2 = \bar{q} - b$, $y_2 = \bar{y}_2 = \bar{y} - b$, and $i_2 = r^* + \pi$.

The commitment equilibrium can be implemented, if the central bank can commit in period 1 to a period-2 money supply equal to $\tilde{m}_2$. Indeed, this would be the most direct way to implement the optimal escape from a liquidity trap, and as noted by Krugman (1998) and more recently by Auerbach and Obstfeld (2005), an expansion of the money supply that is perceived to be permanent will be effective. However, as emphasized in Krugman (1998) and Svensson (2003a), in the real world, there is no direct mechanism through which central banks can commit to a particular future money supply.

Indeed, from March 2001 to January 2006 the Bank of Japan expanded the monetary base by about 74 percent. If this expansion had been perceived as permanent, it would have resulted in dramatically increased expectations of future inflation in Japan, which would have shown up in either a large depreciation of the yen or a large rise in long interest rates. Obviously, none of this happened. The obvious explanation is that the Bank of Japan was expected to contract the monetary base in the future. Indeed, the Policy Board at the Bank of Japan started discussing reducing the monetary base in August 2005 (Bank of Japan, 2005), and from January 2006 the monetary base has been rapidly contracted.

As argued by Svensson (2003a, 2003b), the exchange rate may provide a better focal point to influence the expectations of the private sector than money supply. Using (17) and (35), the period-1 log exchange rate in the commitment equilibrium is

$$\tilde{s}_1 = p_{h1} + \tilde{q}_1 = p_{h1} + \bar{q} + \tilde{y}_1 - \bar{y}, \tag{55}$$

with $\tilde{y}_1 - \bar{y}$ given by (52). The period-2 exchange rate can be derived from uncovered interest parity, which follows from the first-order conditions for home- and foreign-currency bonds (see Jeanne and Svensson, 2004, for details) and the fact that, under our assumptions, there will be no surprise in period 2,

$$i_1 = r^* + s_2 - s_1, \tag{56}$$

which, since the nominal interest rate is equal to zero in the liquidity trap, implies

$$\tilde{s}_2 = \tilde{s}_1 - r^*. \tag{57}$$

The central bank can implement the commitment equilibrium, if it can credibly commit itself to pegging the exchange rate at the optimal levels $\tilde{s}_1$ and $\tilde{s}_2$ given by (55) and (57). That is, the central bank would make a credible commitment to buy and sell unlimited amounts of foreign exchange at those exchange rates. In a commitment equilibrium, it would then not have to make any unlimited foreign-exchange interventions, but just the foreign-exchange interventions that result in money supplies $m_1 \geq p_{h1} + \mu$ and $m_2 = \tilde{m}_2$. The problem with such a commitment is, however, that it will not be credible, if the central bank reoptimizes under discretion in period 2.

In order to see this, suppose that the central bank has credibly implemented the optimal policy in period 1, resulting in the output gap $\tilde{y}_1 - \bar{y}$ and the inflation gap $\tilde{\pi}_1 - \pi$ in period 1, as well as expectations in period 1 of inflation in period 2 equal to $\tilde{\pi}_2$. These expectations would then result in period-1 pricing decisions resulting in the corresponding home-good price level $\tilde{p}_{h2}$. By (36), this implies that, in period 2, the central bank faces the Phillips curve

$$\pi_2 - \tilde{\pi}_2 = \alpha(y_2 - \tilde{y}_2), \tag{58}$$

where ex post inflation π_2 and output y_2 could differ from the levels expected in period 1, $\tilde{\pi}_2$ and $\tilde{y}_2$. The period loss in period 2 under the commitment policy is (with $\gamma = 0$)

$$\tilde{L}_2 \equiv \frac{1}{2}(\tilde{\pi}_2 - \pi)^2 > 0. \tag{59}$$

If the central bank reoptimizes under discretion in period 2, it can reduce the loss in period 2. If it minimizes the period loss function (29) for $t = 2$, the corresponding first-order condition is

$$\pi_2 - \pi = -\frac{\lambda}{\alpha}(y_2 - \tilde{y}_2). \tag{60}$$

Combining (58) and (60) results in

$$\pi_2 = \frac{\alpha^2}{\alpha^2 + \lambda}\pi + \frac{\lambda}{\alpha^2 + \lambda}\tilde{\pi}_2 < \tilde{\pi}_2.$$

That is, ex post, the central bank would renege on its commitment to implement the optimal inflation $\tilde{\pi}_2$ in period 2 and instead implement lower inflation. The corresponding loss in period 2 is then

$$L_2 = \frac{1}{2}[(\pi_2 - \pi)^2 + \lambda(y_2 - \tilde{y}_2)^2] = \frac{1}{2}[\frac{\lambda^2}{(\alpha^2 + \lambda)^2}(\tilde{\pi}_2 - \pi)^2 + \lambda\frac{\alpha^2}{(\alpha^2 + \lambda)^2}(\tilde{\pi}_2 - \pi)^2] = \frac{1}{2}\frac{\lambda}{\alpha^2 + \lambda}(\tilde{\pi}_2 - \pi)^2 \tag{61}$$

and hence lower than $\tilde{L}_2$. Anticipation of this reneging by the private sector in period 1 would unravel the commitment equilibrium and prevent private-sector inflation expectations in period 1 from rising to $\tilde{\pi}_2$.[9]

Conditional on the assumption of reoptimization under discretion in period 2 (and $\gamma = 0$), the only equilibrium in the present setup in period 1 is the discretion equilibrium, the equilibrium we have characterized in section II.B. The corresponding log exchange rates in the discretion equilibrium are given by

$$\begin{aligned} \hat{s}_1 &= p_{h1} + \hat{q}_1 = p_{h1} + \bar{q} + \hat{y}_1 - \bar{y} < \tilde{s}_1, \\ \hat{s}_2 &= \hat{s}_1 - r^* < \tilde{s}_2. \end{aligned}$$

The period-1 and -2 exchange rates in the discretion equilibrium are lower than in the commitment equilibrium (the home currency is stronger in period 1 and 2), by the same amount in the two periods.

III How an independent central bank can commit to a higher future price level

This section shows that the optimal commitment policy can be achieved if the balance-sheet concerns of the central banker are sufficiently strong, represented by being sufficient large and exceeding a lower bound $\bar{\gamma} > 0$ to be determined. Consider a central bank that finds itself in a liquidity trap in period 1. The central bank cannot commit to a particular money supply in the future. We will show that it can, nevertheless, move to and implement the commitment equilibrium in period 1 by managing its balance sheet in the right way. Thereby it can commit itself to an exchange-rate in period 2 that corresponds to the desired higher next-period inflation, $\tilde{\pi}_2$.

The central bank enters period 1 with given stocks of monetary base and foreign exchange reserves, M_0 and R_0. Suppose that the economy initially is in the discretion equilibrium in period 1, with the period-1 exchange rate equal to $\hat{S}_1$, the expected period-2 exchange rate equal to $\hat{S}_2$, and with money supply, dividend, reserves and capital, denoted $\hat{M}_1$, $\hat{Z}_1$, $\hat{R}_1$, and $\hat{V}_1$, respectively, satisfying

$$\hat{M}_1 = \hat{Z}_1 + \hat{S}_1 \hat{R}_1 + M_0 - \hat{S}_1 e^{r^*} R_0 \geq P_{h1} e^{\mu}, \tag{62}$$

$$\hat{V}_1 = \hat{S}_1 e^{r^*} R_0 - M_0 - \hat{Z}_1 \geq \bar{V} \geq 0, \tag{63}$$

where we have used (6) and (31) and we recall that $e^{\mu} > 0$ is the satiation level of real money.

We want to show that the central bank can implement the commitment equilibrium by depreciating the currency in period 1 to the exchange rate $\tilde{S}_1 > \hat{S}_1$ and committing to maintaining the exchange rate $\tilde{S}_2 > \hat{S}_2$ in period 2. This will then implement the optimal inflation, $\tilde{\pi}_2 > \pi$, in period 2 and the optimal overshooting of the inflation target. The problem with this commitment, as explained in section II.D, is that the private sector knows that the central bank has an incentive in period 2 to renege on its commitment and appreciate the currency below the exchange rate $\tilde{S}_2$, so as to achieve an ex post inflation closer to the inflation target. How can the central bank commit to not appreciating the currency in period 2? Our main result follows:

Proposition 3 *If γ is sufficiently large and satisfies*

(64)
$$\gamma \geq \bar{\gamma} \equiv \frac{1}{2}\frac{\alpha^2}{\alpha^2+\lambda}(\tilde{\pi}_2-\pi)^2,$$

the central bank can implement the commitment solution, $\tilde{S}_1$ and $\tilde{S}_2$, by pegging its exchange rate to $\tilde{S}_1$ and by setting its capital equal to the minimum level in period 1, $V_1=\bar{V}$.

The central bank will set its policy subject to the constraint $V_1 \geq \bar{V}$, if the additional loss, γ, of allowing the central bank's capital to fall below $\bar{V}$ exceeds the gain from reneging in period 2. This gain is equal to the difference between the period 2 loss under commitment, (59), and the period loss while reneging, which equals the right side of (64). This determines the lower bound $\bar{\gamma}$ in (64).

To show that the proposition is true, we note that, in period 2, for given R_1 and M_1, (31), (32), and (33) imply a lower bound for the period-2 exchange rate,

(65)
$$S_2 \geq \frac{M_1+\bar{V}}{e^{r^*}R_1}.$$

This inequality comes from the currency mismatch in the balance sheet of the central bank. The central bank has foreign-currency assets but home-currency liabilities. The central bank's capital remains above the lower bound $\bar{V}$ only if the value of the home-currency value of its foreign-exchange reserves is sufficiently high, that is, if the home currency is sufficiently depreciated.[10]

We then realize that, subject to (6), the central bank should chose $Z_1=\tilde{Z}_1$, $R_1=\tilde{R}_1$, and $M_1=\tilde{M}_1 > P_{h1}e^{\mu}$ in period 1 so that

(66)
$$\frac{\tilde{M}_1+\bar{V}}{e^{r^*}\tilde{R}_1}=\tilde{S}_2.$$

Then the lower bound for the period-2 exchange rate, the right side of (65), is exactly equal to $\tilde{S}_2$. The central bank will never choose a *higher* exchange rate (weaker currency) in period 2 than this lower bound, because doing so would result in CPI inflation even further away from the target and an even larger period-2 loss L_2. Therefore, it will then implement exactly the desired exchange rate $\tilde{S}_2$.

Let $\tilde{V}_2$ denote the central bank's capital in period 2 for $Z_2=0$ and $S_2=\tilde{S}_2$. By (31) and (66), it satisfies

$$\tilde{V}_2 \equiv \tilde{S}_2 e^{r^*}\tilde{R}_1-\tilde{M}_1=\bar{V}$$

and is hence equal to the lower bound. Furthermore, for $S_1=\tilde{S}_1$, since $\tilde{S}_1=\tilde{S}_2e^{r^*}$ (because $i_1=0$), we have that the bank's capital in period 1 satisfies

$$V_1=\tilde{S}_1\tilde{R}_1-\tilde{M}_1 \equiv \tilde{S}_2e^{r^*}\tilde{R}_1-\tilde{M}_1 \equiv \tilde{V}_2=\bar{V}.$$

Thus, in order to implement the commitment equilibrium, in period 1 the central bank should pay the dividend $\tilde{Z}_1 \geq 0$ to the government so as to make its capital in period 1 equal $\bar{V}$ when evaluated at the exchange rate $\tilde{S}_1$. Furthermore, it should adjust its money supply to exceed the satiation level, $\tilde{M}_1 > P_{h1}e^{\mu}$, and thereby ensure that the period-1 nominal interest rate is zero. Finally, it should make an explicit commitment to the peg of the exchange rate to $\tilde{S}_1$ and $\tilde{S}_2$ in the two periods, and, importantly, publish its balance sheet. This allows the private sector to verify that the central bank's capital in period 1 equals the lower bound when evaluated at the exchange rate $\tilde{S}_1$, which implies that the central bank's capital will equal the lower bound also in period 2 at the exchange rate $\tilde{S}_2$. Then the commitment to the period-2 exchange rate will be credible, and $\tilde{S}_1$ will be the only exchange rate consistent with those period-2 expectations and uncovered interest parity. The commitment equilibrium is then the only possible equilibrium.

It only remains to demonstrate that it is *feasible* for the central bank to choose a nonnegative dividend $\tilde{Z}_1$ that sets its capital equal to the minimum level in period 1. This is the case, if its capital is larger than the minimum level conditional on a zero dividend, that is

$$\tilde{S}_1 e^{r^*} R_0 - M_0 \geq \bar{V}.$$

This is true by (63) and $\tilde{S}_1 > \hat{S}_1$.[11]

If, counter to the above logic and announcements, the private sector, irrationally, would believe that the currency might appreciate back to $\hat{S}_1 < \tilde{S}_1$ in period 1, there will be excess demand for currency and excess supply of foreign-exchange reserves. The central bank can, however, always eliminate that excess demand and excess supply by issuing currency and buying foreign-exchange reserves, that is, by increasing $\tilde{R}_1$ and $\tilde{M}_1$, by the interventions ΔR_1 and ΔM_1 satisfying $\Delta R_1 = \Delta M_1/\tilde{S}_1$. This maintains the central bank's capital at the level $\tilde{V}_1 = \bar{V}$. Thus, we have shown not only that the central bank has the incentive to maintain the exchange rate at $\tilde{S}_1$ and $\tilde{S}_2$ in period 1 and 2 but that, in case there was nevertheless a speculative attack in the direction of appreciating the currency (lowering the exchange rate) from $\tilde{S}_1$ in period 1, the central bank can actually defend currency peg at that rate. The reason is that a speculative attack in the direction of *appreciating* the currency can always be averted, since the resulting excess demand for home currency can always be satisfied.[12]

IV Conclusions

This paper starts from the two empirical facts that central banks target CPI inflation and that independent central banks are concerned about the level of their capital. The first fact implies that a central bank can renege on a commitment to a higher future price level by an ex post currency appreciation. We show, however, that the second fact makes it possible for independent central banks to manage their capital so as to create an incentive not to appreciate the currency ex post. This way an independent central bank can indeed commit to a higher future price level through a currency depreciation and a crawling peg, in line with Svensson's (2001, 2003a) Foolproof Way to escape from a liquidity trap.

We have made these points in a very simple model. The simplicity of the model is an advantage when explaining the logic of the argument, but raises the question of the robustness of our results to realistic changes in the assumptions. The working paper version of our paper, Jeanne and Svensson (2004), shows that our results are robust to several extensions, in particular to liquidity traps that last several periods or to different specifications of the balance-sheet concerns of the central bank (time varying, or specified in real terms). That paper also shows that the commitment to the optimal policy might be achieved by pegging the price of assets other than foreign-exchange reserves. Interestingly not all assets can be used in this way. The important distinction in this regard is between the assets whose returns are predetermined in home currency ("nominal" assets, such as home-currency bonds of all maturities) and assets whose returns are not predetermined in terms of the home currency ("real" assets, such as stocks, real estate or indexed bonds). The latter, and the not the former, provide the appropriate tool for the sort of commitment strategies we have analyzed so far.

More generally, we hope to contribute to more attention of scholars to the balance-sheet concerns of central bankers and their implications for monetary policy. These concerns have not been much analyzed in the academic literature, although real-world central bankers do seem to care about the level of the central banks' capital. This omission may not be a serious one when central bankers' balance-sheet concerns do not significantly influence monetary policy and can be safely ignored to a first approximation. However, there are situations—and a liquidity trap is an important example—where those concerns might matter in a nontrivial way for monetary policy-making. This observation would seem to warrant more theoretical and empirical research on the foundations of central bankers' balance-sheet concerns and on their implication for monetary policy.

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Initial footnote

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Notes

[1]The Foolproof Way consists of announcing and implementing (1) a target path for the domestic price-level, starting above the current price level by the "price gap" that the central bank wishes to undo, (2) a currency depreciation and a crawling peg to achieve the price-level target path, and (3) an exit strategy in the form of abandoning the peg and shifting to flexible inflation or price-level targeting once the price-level target path has been reached.

[2]Although we believe that foreign-exchange reserves provide the most relevant and realistic case, this commitment mechanism could potentially also arise for other assets on central-bank balance sheets, such as equity, property or indexed bonds—but not for fixed-income securities denominated in the home currency, see the working-paper version of this paper, Jeanne and Svensson (2004).

[3]It is also an empirical fact that all inflation-targeting central banks are "flexible" inflation targeters in the sense that they also put some weight on stability of the real economy, here represented by output-gap stability (Svensson, 2002; Truman, 2003).

[4]For simplicity, the liquidity trap is assumed to last one period, the same as the horizon of the nominal stickiness. So although the model is in infinite time, the analysis of the liquidity trap will involve two periods.

[5]One can show that the assumption of constant money supply is sufficient to ensure that no liquidity trap arises under flexible prices. An endogenous future money supply adjusted to maintaining low inflation may prevent an equilibrium and equality between output and potential output also under flexible prices (Jeanne and Svensson, 2004).

[6]This assumption is not derived from the maximization of the representative household's utility. Endogenizing the central bank's loss function is outside the scope of this paper.

[7]Since the central bank's target level for output equals potential output, there will not be any average inflation bias in a discretion equilibrium.

[8]Outside a liquidity trap, under commitment, the central bank would just set $\pi_2 = \pi$ and set m_1 and thereby i_1 such that $y_1 = \bar{y}$.

[9]The above reneging cannot arise in the formulation of the Foolproof Way in Svensson (2003b). There, monetary policy is concerned about inflation in the home-currency price of home goods rather than in the CPI. There cannot be unexpected inflation for home goods since their prices are set one period ahead.

[10]Note the contrast with the currency mismatches that are the focus of the recent literature on international financial crises (Aghion et al., 2001). There, domestic firms are *indebted* in foreign currency so that a depreciation reduces their net value.

[11]The dividend payment, $\tilde{Z}_1$, will hence be positive. Since the government passes on the dividend to the private sector, the private sector receives a positive transfer. This transfer has, in itself, no effect on private-sector consumption and demand in this model, since Ricardian equivalence holds and the private sector fully internalizes the government and central-bank budget constraints (as in the derivation of consolidated home-country budget constraint, (9)).

[12]This argument requires that the central bank stands ready to buy unlimited amounts of reserves. There is no cost for the bank of doing so, since money is neutral in the liquidity trap. If instead the central bank allowed the currency to appreciate when the level of reserves exceeds a certain level, there could be a speculative attack leading to an appreciation of the domestic currency (Grilli, 1986).