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Inflation Targeting: Some Extensions

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Abstract

Previous analyses of the implementation of inflation targeting are extended to monetary policy responses to different shocks, consequences of model uncertainty, and effects of interest rate smoothing and stabilization. Model uncertainty, output stabilization, and interest rate stabilization or smoothing all call for a more gradual adjustment of the conditional inflation forecast toward the inflation target. The conditional inflation forecast is the natural intermediate target during inflation targeting. The optimal way of reacting to shocks is hence to check how they affect the inflation forecast and then take the appropriate action.

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I. Introduction

Explicit inflation targeting has been subject to considerable attention during the last few years. New Zealand, Canada, UK, Sweden, Finland, Australia and Spain have monetary policy regimes with explicit inflation targets. Two recent conference volumes, Leiderman and Svensson (1995) and Haldane (1995), and an increasing number of research papers, deal with different aspects of inflation targeting. At a recent symposium (Federal Reserve Bank of Kansas City (1996)), the four major papers (by Stanley Fischer, Charles Freedman, Mervyn King and John Taylor) recommended explicit inflation targeting as the best way of achieving and maintaining low and stable inflation.[1]

In Svensson (1997a), I examined both the implementation and the monitoring of inflation targeting. In a simple closed-economy model, I showed that inflation targeting implies that the central bank's conditional inflation forecast for a horizon corresponding to the control lag becomes an intermediate target (in line with explicit statements in King (1994) and Bowen (1995)). Under what we can call *strict inflation targeting*, with low and stable inflation being the only goal for monetary policy (a zero weight on output stabilization), this implies that the central bank should adjust its instrument such that the conditional inflation forecast for the control lag equals the inflation target. Under what we may call *flexible inflation targeting* (with a positive weight on output stabilization), the conditional inflation forecast should instead be adjusted gradually towards the inflation target. I also argued that inflation targeting allows efficient monitoring of monetary policy by the public, especially if the central bank makes the conditional inflation forecast an explicit intermediate target, and publishes and allows public scrutiny of its inflation forecast, including models, analyses and judgements. The conditional inflation forecast then becomes an ideal intermediate target in that it is the current variable most

[1] See Svensson (1997a), for instance, for further references to the literature on inflation targeting.

correlated with the goal, is easier to control than the goal, is easier to observe than the goal, and by implying extremely transparent principles for monetary policy is most conducive to public understanding of monetary policy. I also showed that inflation targeting is more efficient than money growth or exchange rate targeting, in the sense of bringing lower inflation variability.

In the present paper, I extend the analysis of the implementation of inflation targeting to the monetary policy response to different shocks (section III), to the consequences of model uncertainty (section IV), and to the effects of interest rate smoothing and stabilization (section V). Section II restates the result that inflation targeting implies that the conditional inflation forecast becomes an intermediate target, and shows how that intermediate target is affected by a positive weight on output-gap stabilization. This section goes beyond Svensson (1997a) in incorporating a stochastic "natural rate" level of output and exogenous variables. Section VI concludes. Appendix A contains some technical details.

II. Inflation forecast targeting

This section shows that conditional inflation targeting implies that the conditional inflation forecast for a horizon corresponding to the control lag becomes an intermediate target. Although the result can be demonstrated in a much more elaborate model with a more explicit role for agents' expectations, a much simpler model is sufficient.[2] The model nevertheless has some structural similarity to more elaborate models used by certain central banks. Svensson (1998b) discusses some issues that arise with a more forward-looking model in an open economy.

The important aspects of the model are that the monetary authority has imperfect control over inflation, that inflation and the output gap react with a lag to changes in the monetary policy

[2] For instance, it is not necessary to assume the systematic discretionary inflation bias (due to 'time-consistency' problems) emphasized in the modern 'principal-agent' approach to central banking (for instance in the work by Barro and Gordon (1983), Rogoff (1985), Walsh (1995), Persson and Tabellini (1993) and Svensson (1997b)) and disputed in the 'traditional' approach (for instance in McCallum (1995) and Romer and Romer (1996)); see Tabellini (1995) for discussion of these approaches.

instrument, that inflation reacts with a longer lag than the output gap, and that a stochastic persistent "natural (rate)" level of output and some exogenous variables (like oil prices) also are of importance. Consider the following model with an accelerationist Phillips curve and an aggregate demand equation,

$$\pi_{t+1} = \pi_t + \alpha_y y_t + \varepsilon_{t+1} \tag{1}$$

$$y_{t+1} = \tilde{\beta}_y y_t + \beta_x x_t - \beta_r (i_t - \pi_{t+1|t}) + \eta_{t+1} \tag{2}$$

$$x_{t+1} = \gamma x_t + \theta_{t+1}, \tag{3}$$

where $\pi_t = p_t - p_{t-1}$ is the inflation (rate) in year t, p_t is the (log) price level, y_t is the output gap (the log of the ratio of output to the natural output level), x_t is an exogenous variable, i_t is the monetary policy instrument or operating target (for instance, a short repo rate or the federal funds rate), $\pi_{t+1|t}$ denotes $\mathrm{E}_t \pi_{t+1}$ (the inflation in year $t+1$ expected in year t), and ε_t, η_t and θ_t are iid shocks in year t which are not known in year $t-1$. The coefficients α_y, $\tilde{\beta}_y$ and β_r are assumed to be positive; γ fulfills $0 \leq \gamma \leq 1$.

In this annual discrete-time model, the instrument i_t can be interpreted as a short interest rate held constant by the monetary authority from one year to the next. Then i_t can be interpreted as a one-year interest rate controlled by the monetary authority, and $i_t - \pi_{t+1|t}$ as a real one-year interest rate.

The change in inflation is increasing in the lagged output gap. The output gap is serially correlated and decreasing in the lagged real interest rate, $i_t - \pi_{t+1|t}$. The real interest rate affects the output gap with a one-year lag, and hence inflation with a two-year lag, that is, the control lag for inflation in the model. That the instrument affects inflation with a longer lag than the output gap is consistent with results from a number of VAR-studies. The average output gap, $\mathrm{E}[y_t]$, is zero, and the average real interest rate, $\mathrm{E}[i_t - \pi_{t+1|t}]$, is normalized to zero. As clarified in appendix A, the exogenous variable x_t can be interpreted (when $\gamma > 0$) as a persistent disturbance to the natural level of output (in which case η_{t+1} is the difference between a temporary demand shock and a shock to the natural output level), or a persistent disturbance to aggregate demand.

Inflation expectations $\pi_{t+1|t}$ in year t are by (1) predetermined and fulfill

$$\pi_{t+1|t} = \pi_t + \alpha_y y_t. \tag{4}$$

Using (4) in (2) results in the reduced form aggregate demand equation

$$y_{t+1} = \beta_y y_t + \beta_x x_t - \beta_r(i_t - \pi_t) + \eta_{t+1}, \tag{5}$$

where

$$\beta_y = \tilde{\beta}_y + \alpha_y \beta_r,$$

and $i_t - \pi_t$ may be called a "pseudo-real" repo rate. Thus, the model can be represented by (1), (5) and (3).

Interpret inflation targeting as monetary policy conducted by a monetary authority with a long-run inflation target π^* (say 2 percent per year) but with no long-run output-gap target (other than the long-run average, zero). Furthermore, in the short-run, the monetary authority wants to reduce inflation fluctuations around the long-run inflation target, and output-gap fluctuations around zero.[3] This can be formalized as the monetary authority's intertemporal loss function being

$$\mathrm{E}_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\pi_\tau, y_\tau), \tag{6}$$

where E_t denotes expectations conditional upon information available in year t, the discount factor δ fulfills $0 < \delta < 1$, and the period loss function $L(\pi_\tau, y_\tau)$ is

$$L(\pi_\tau, y_\tau) = \frac{1}{2}\left[(\pi_\tau - \pi^*)^2 + \lambda y_\tau^2\right], \tag{7}$$

[3] Cf. Fischer (1996), King (1996) and Svensson (1997b) on whether inflation targeting also involves implicit goals for output or employment.

where $\lambda \geq 0$ is the weight on output-gap stabilization. That is, the monetary authority wishes to minimize the expected sum of discounted squared future deviations of inflation and output from the inflation target and the natural output level, respectively.[4] [5]

Svensson (1997a) shows that the first-order condition for minimizing (6) over the repo rate can be written

$$\pi_{t+2|t}(i_t) = \pi^* + c(\lambda)\left(\pi_{t+1|t} - \pi^*\right). \tag{8}$$

Here $\pi_{t+2|t}(i_t)$ denotes the "two-year conditional inflation forecast", $\mathrm{E}\left[\pi_{t+2}|i_t; \pi_t, y_t, x_t\right]$, the forecast for annual inflation from year $t+1$ to year $t+2$, conditional upon a given instrument level i_t, and conditional upon the predetermined state variables in year t (π_t, y_t and x_t). It is given by

$$\pi_{t+2|t}\left(i_t\right) \equiv \pi_t + a_y y_t + a_x x_t - a_r\left(i_t - \pi_t\right), \tag{9}$$

where

$$a_y = \alpha_y(1+\beta_y),\ a_x = \alpha_y \beta_x \text{ and } a_r = \alpha_y \beta_r. \tag{10}$$

The one-year inflation forecast, $\pi_{t+1|t}$, is predetermined and given by (4). The coefficient $c(\lambda)$ is a function of the relative weight λ given by

$$c(\lambda) \equiv \frac{\lambda}{\lambda + \delta\alpha_y^2 k(\lambda)} \tag{11}$$

and fulfills $0 \leq c(\lambda) < 1$, and the coefficient $k(\lambda)$ is another function of λ given by

$$k(\lambda) \equiv \frac{1}{2}\left(1 - \frac{\lambda(1-\delta)}{\delta\alpha_y^2} + \sqrt{\left(1 + \frac{\lambda(1-\delta)}{\delta\alpha_y^2}\right)^2 + \frac{4\lambda}{\alpha_y^2}}\right) \geq 1. \tag{12}$$

Under *strict inflation targeting*, when the weight on output-gap stabilization is zero ($\lambda = 0$) and only inflation enters in the loss function, the coefficients fulfill $c(0) = 0$ and $k(0) = 1$. Then

[4] Since the central bank does not have perfect control over inflation, it is not meaningful to minimize the *realized* squared deviations, only the *expected* squared deviations (conditional upon the information available when the repo rate is set).

[5] Since there is an asymmetry between inflation and output in that there is a level target subject to choice only for the former, I find it appropriate to refer to this as "(flexible) inflation targeting" rather than "inflation-and-output-gap-targeting" (at least as long as the relative weight λ is not excessive), especially since the former name is already used for the monetary policy regimes in New Zealand, Canada, U.K., Sweden and Australia.

the first-order condition simplifies to

$$\pi_{t+2|t}(i_t) = \pi^*. \tag{13}$$

The monetary authority should adjust its instrument such that the two-year conditional inflation forecast always equals the inflation target.

Under *flexible inflation targeting*, when there is a positive weight on output-gap stabilization ($\lambda > 0$) and both inflation and the output gap enter the loss function, the interpretation of the first-order condition (8) is still intuitive. The monetary authority should adjust the instrument such that the deviation of the two-year conditional inflation forecast from the long-run inflation target is a fraction $c(\lambda)$ of the deviation of the pre-determined one-year inflation forecast from the inflation target. Instead of always adjusting the two-year conditional inflation forecast all the way to the long-run inflation target, the monetary authority should adjust the two-year conditional inflation forecast gradually towards the long-run inflation target. The intuition is that this reduces output-gap fluctuations, which is apparent from (1). The higher the weight on output-gap stabilization, the slower the adjustment of the conditional inflation forecast towards the long-run inflation target (the larger the coefficient $c(\lambda)$, see Svensson (1997a)). The right-hand side of (8) can hence be interpreted as a variable short-run target for the two-year inflation forecast.

In general, (8) and its variant (13) imply that the two-year conditional inflation forecast, that is, the conditional inflation forecast corresponding to the control lag, can be interpreted as an explicit intermediate target. As in Svensson (1997a), I call (8) and its variant (13) a(n) (intermediate-)*target*(*ing*) *rule*, that is, a rule that specifies the intermediate-target variable and how its target level is determined. The monetary authority then adjusts the repo rate so as to always fulfill the targeting rule. If the two-year conditional inflation forecast exceeds (falls short of) the right-hand sides of (8) or (13), the repo rate should be increased (decreased). This results in an endogenous reaction function, an *instrument rule*, expressing the instrument as a function of current information.

Thus, substitution of the forecasts (4) and (9) into (8) leads to the optimal reaction function

$$i_t = \pi_t + f_\pi(\lambda)(\pi_t - \pi^*) + f_y(\lambda) y_t + f_x x_t, \tag{14}$$

where

$$f_\pi(\lambda) = \frac{1 - c(\lambda)}{\alpha_y \beta_r}, \; f_y(\lambda) = \frac{\beta_y + 1 - c(\lambda)}{\beta_r} \text{ and } f_x = \frac{\beta_x}{\beta_r}. \tag{15}$$

The reaction function (14) is of the same form as the Taylor rule (1993, 1996), except that it also depends on the exogenous variable. The pseudo-real repo rate $i_t - \pi_t$ is increasing in the excess of current inflation over the inflation target and in the current output gap. The instrument depends on current variables, not because current variables are targeted (they are predetermined) but because they predict future variables. Even if the weight on output-gap stabilization is zero, so that only future inflation is targeted, the instrument will depend on all current variables that help predict future inflation.[6]

Actual inflation in year $t + 2$ will unavoidably deviate from the inflation target and the two-year conditional inflation forecast by a forecast error,

$$\pi_{t+2} - \pi_{t+2|t} = \varepsilon_{t+1} + \alpha_y \eta_{t+1} + \varepsilon_{t+2}, \tag{16}$$

due to disturbances that occur within the control lag, after the monetary authority has set the instrument. Here $\pi_{t+2|t}$ denotes the two-year inflation forecast (9) conditional upon the reaction function (14),

$$\pi_{t+2|t} \equiv \pi_{t+2|t}\left[\pi_t + f_\pi(\pi_t - \pi^*) + f_y y_t + f_x x_t\right].$$

From (8) and (4) the two-year inflation forecast will follow

[6] See Broadbent (1996) for an insightful discussion of Taylor rules in relation to inflation targeting. See also the comment by Svensson (1996) on Taylor (1996).

$$\pi_{t+2|t} - \pi^* = c(\lambda)\left(\pi_t - \pi^*\right) + c(\lambda)\alpha_y y_t.$$

From (5), (14) and (15), the output gap will follow

$$\begin{aligned} y_{t+1} &= \beta_y y_t + \beta_x x_t - \beta_r \left[f_\pi \left(\pi_t - \pi^*\right) + f_y y_t + f_x x_t\right] + \eta_{t+1} \\ &= -\frac{1 - c(\lambda)}{\alpha_y}\left(\pi_t - \pi^*\right) - \left[1 - c(\lambda)\right] y_t + \eta_{t+1}. \end{aligned}$$

To generalize from this model, inflation targeting implies that the conditional inflation forecast for a horizon corresponding to the control lag (two years in the model) becomes an intermediate target. Under strict inflation targeting (no weight on output-gap stabilization), the instrument should be set so as to make the conditional inflation forecast equal to the inflation target. Under flexible inflation targeting (some weight on output-gap stabilization), the instrument should be set so as to make the two-year conditional inflation forecast approach the long-run inflation target gradually. This behavior results in the optimal reaction function of the monetary authority. Since the conditional inflation forecast depends on all relevant information, the instrument will be a function of this information.

III. Response to shocks

How should monetary policy react to shocks?[7] The conventional wisdom is that monetary policy should neutralize aggregate demand shocks, since these move inflation and the output gap in the same direction. With regard to supply shocks, the conventional wisdom is that the response depends on the weight on output-gap stabilization. With a positive weight, it is optimal to partially accommodate supply shocks, since they affect inflation and the output gap

[7] See Freedman (1996) for a more detailed discussion of the optimal response to shocks under inflation targeting, including the response to different shifts in inflation expectations. Such shifts can be examined in the forward-looking model in Svensson (1998b).

in opposite directions. With a zero weight, the supply shock effect on inflation is neutralized, even though this enhances the effect on the output gap.

When lags are taken into account, the conventional wisdom must be modified. First, the monetary authority cannot affect the first-round effects on inflation and the output gap of supply and demand shocks, due to the lags. It can only mitigate the second-round effects. Second, the reaction to temporary demand and supply shocks appears more symmetric. Third, the reaction to both shocks differs with the weight on output-gap stabilization. Under strict inflation targeting (with a zero weight on output-gap stabilization), the two-year conditional inflation forecast is brought in line with the long-run inflation target, regardless of how the shocks have affected the one-year inflation forecast. Hence, shocks must not let the two-year conditional inflation forecast deviate from the long-run target. Under flexible inflation targeting (with a positive weight on output-gap stabilization), the two-year conditional inflation forecast is adjusted less in response to the shocks. The effect of these shocks on future inflation is only gradually eliminated.

A general and operational way of determining the appropriate response to the shocks is to "filter the shocks through the conditional inflation forecast, and then take appropriate action." More specifically, the effects of the shocks on the one-year and two-year inflation forecasts are assessed, and then the instrument is adjusted so that the first-order condition (8) still holds.

In order to see this, consider shocks in year t. By (4) these shocks will change the one-year inflation forecast by

$$\begin{aligned}\pi_{t+1|t} - \pi_{t+1|t-1} &= (\pi_t - \pi_{t|t-1}) + \alpha_y(y_t - y_{t|t-1}) \\ &= \varepsilon_t + \alpha_y(\tilde{\eta}_t - \xi_t), \qquad (17)\end{aligned}$$

where I use the more elaborate model in appendix A in which the shock to the output gap,

$$\eta_t = \tilde{\eta}_t - \xi_t,$$

consists of the difference between a temporary demand shock, $\tilde{\eta}_t$, and a shock to the natural output level, ξ_t. By the analogy of (9) in appendix A, (A9), the shocks will change the two-year conditional inflation forecast by

$$\begin{aligned} \pi_{t+2|t} - \pi_{t+2|t-1} &= \left[(1+a_r)(\pi_t - \pi_{t|t-1}) + a_y(y_t - y_{t|t-1}) + a_z(z_t - z_{t|t-1}) + a_n(y_t^n - y_{t|t-1}^n)\right] \\ &\quad - a_r(i_t - i_{t|t-1}) \\ &= [(1+a_r)\varepsilon_t + a_y(\tilde{\eta}_t - \xi_t) + a_z\zeta_t + a_n\xi_t] - a_r(i_t - i_{t|t-1}), \end{aligned} \tag{18}$$

where z_t is a persistent demand disturbance, ζ_t is a shock to this demand disturbance, y_t^n is (the log of) the natural output level, and the coefficients a_r, a_y, a_z and a_n are given by (A10)–(A13). The term within brackets in (18) is the change in the two-year conditional inflation forecast due to the shocks, and the other term is the change due to the change in the instrument, $i_t - i_{t|t-1}$.

The changes in the one-year and two-year inflation forecasts must obey the first-order condition (8), which implies that they must fulfill

$$\pi_{t+2|t} - \pi_{t+2|t-1} = c(\lambda)\left(\pi_{t+1|t} - \pi_{t+1|t-1}\right). \tag{19}$$

Thus, (17)–(19) determine the required change in the instrument.

Solving for the instrument change results in

$$\begin{aligned} i_t - i_{t|t-1} &= \frac{[(1+a_r)\varepsilon_t + a_y(\tilde{\eta}_t - \xi_t) + a_z\zeta + a_n\xi_t] - c(\lambda)\left[\varepsilon_t + \alpha_y(\tilde{\eta}_t - \xi_t)\right]}{a_r} \\ &= \frac{[1+\alpha_y\beta_r - c(\lambda)]\,\varepsilon_t + \alpha_y[1+\beta_y - c(\lambda)]\tilde{\eta}_t - \alpha_y\left[1+\alpha_y\beta_r + \gamma_n - c(\lambda)\right]\xi_t + \alpha_y\beta_z\zeta_t}{\alpha_y\beta_r}, \end{aligned} \tag{20}$$

where I have used (A5)–(A6) and (A10)–(A13), and γ_n ($0 \leq \gamma_n \leq 1$) is the degree of persistence of the natural output level (for $\gamma_n = 1$ the natural output level is a random walk). The numerator in (20) is the change in the two-year conditional inflation forecast caused by the shocks, less the fraction $c(\lambda)$ of the change in the one-year inflation forecast due to the shock. The denominator is the policy multiplier of the instrument for the two-year conditional inflation forecast.

We see that the response to the shocks vary with the relative weight on output-gap stabilization, λ, via the effect on the coefficient $c(\lambda)$. A positive inflation shock, ε_t, and a positive temporary demand shock, $\tilde{\eta}_t$, both motivate an increase in the instrument. Those increases are smaller with a higher weight on output stabilization, since $c(\lambda)$ is increasing in λ. For an unchanged interest rate, a positive inflation shock and a positive temporary demand shock both increase the one-year and the two-year inflation forecast. The two-year inflation forecast increases more since it is also affected by the fall in the real interest rate, $i_t - \pi_{t+1|t}$. Since by (19) the two-year inflation forecast should optimally increase less, namely by the fraction $c(\lambda)$ of the increase of the one-period forecast, the interest rate should be increased. The higher the fraction $c(\lambda)$, the less the interest rate needs to be increased.

A shock to the natural output level, ξ_t, motivates a fall in the instrument, which is larger with more persistence, γ_n, and a lower relative weight on output-gap stabilization, λ. More persistence lowers the output gap further in year one and leads to a larger fall in the two-year inflation forecast which, everything else equal, requires a larger fall in the interest rate. More weight on output-gap stabilization motivates a more modest fall in the interest rate.

A shock to the persistent demand disturbance, ζ_t, leads to an increase in the instrument which is independent of the weight on output-gap stabilization. This shock increases output in year one and the two-year inflation forecast, but not the one-year inflation forecast. Stabilizing inflation and stabilizing the output gap both call for an increase in the instrument to cancel the effect of the shock.

The response coefficients for the shocks in (20) are, of course, the same coefficients as in the reaction function for the more elaborate model in appendix A, (A14).

IV. Model uncertainty

In this section, I consider model uncertainty, in the form of uncertainty about the coefficients in the model (1)–(3). Let me simplify the model somewhat by disregarding the exogenous variable ($\beta_x = 0$). Restate the model as

$$\pi_{t+1} = \pi_t + \alpha_{yt} y_t + \varepsilon_{t+1} \tag{21}$$

$$y_{t+1} = \tilde{\beta}_{yt} y_t - \beta_{rt} \left(i_t - \pi_{t+1|t} \right) + \eta_{t+1}, \tag{22}$$

where coefficients α_y, $\tilde{\beta}_y$ and β_r have been dated according to the year they refer to. For simplicity, consider only the case of strict inflation targeting ($\lambda = 0$) and the simplification of the problem to minimize (6) to the period-by-period problem (see Svensson (1997a) for details for the case when there is no model uncertainty)

$$\min_{i_t} \delta^2 \mathrm{E}_t \left[\frac{1}{2} \left(\pi_{t+2} - \pi^* \right)^2 \right]$$

subject to

$$\pi_{t+2} = \pi_{t+2|t} \left(i_t \right) + \varepsilon_{t+1} + (\alpha_{y,t+1} - \alpha_{y,t+1|t}) y_{t+1|t} + [\alpha_{y,t+1} \eta_{t+1} - (\alpha_{y,t+1} \eta_{t+1})_{|t}] + \varepsilon_{t+2},$$

where

$$\begin{aligned} \pi_{t+2|t} \left(i_t \right) &= \pi_{t+1|t} + \alpha_{y,t+1|t} y_{t+1|t} + (\alpha_{y,t+1} \eta_{t+1})_{|t} \\ &= \pi_{t+1|t} + \tilde{a}_{y,t+1|t} y_t - a_{r,t+1|t} \left(i_t - \pi_{t+1|t} \right) + (\alpha_{y,t+1} \eta_{t+1})_{|t} \\ \pi_{t+1|t} &= \pi_t + \alpha_{yt} y_t, \end{aligned} \tag{23}$$

and where I use the notation

$$\tilde{a}_{y,t+1} = \alpha_{y,t+1} \tilde{\beta}_{yt} \text{ and } a_{r,t+1} = \alpha_{y,t+1} \beta_{rt}$$

and observe that $\pi_{t+1|t}$ is predetermined.

Assume first that coefficients α_{yt}, $\tilde{\beta}_{yt}$ and β_{rt} remain constant. The first-order condition for this problem is then the targeting rule

$$\pi_{t+2|t}(i_t) = \pi^*, \tag{25}$$

as we saw in section II.

Now, following the classic analysis of Brainard (1967) (the relevance of which has recently been emphasized by Blinder (1995), see also Chow (1975, chapter 10)), consider the alternative problem when there is model uncertainty in the form of uncertainty in year t, when the instrument is chosen, about the coefficient $\tilde{a}_{y,t+1}$ and the policy multiplier $a_{r,t+1}$, resulting from uncertainty about the coefficients α_{yt}, $\tilde{\beta}_{yt}$ and β_{rt}. More precisely, let α_{yt} be known at t, and let

$$\begin{aligned} \alpha_{y,t+1} &= \alpha_y + \nu_{\alpha y,t+1} \\ \tilde{\beta}_{yt} &= \tilde{\beta}_y + \nu_{\beta yt} \\ \beta_{rt} &= \beta_r + \nu_{\beta rt} \end{aligned}$$

where $\nu_{\alpha y,t+1}$, $\nu_{\beta yt}$ and $\nu_{\beta rt}$ are iid stochastic disturbances with zero means and given variances/covariances. The realizations of these disturbances become known in year $t+1$. For simplicity, assume that $\nu_{\alpha y,t+1}$ is uncorrelated with $\nu_{\beta yt}$ and $\nu_{\beta rt}$. Then we can write

$$\begin{aligned} \tilde{a}_{y,t+1} &= \tilde{a}_y + \nu_{y,t+1} \\ a_{r,t+1} &= a_r + \nu_{r,t+1}, \end{aligned}$$

where $\nu_{y,t+1}$ and $\nu_{r,t+1}$ are zero mean iid disturbances, and

$$\tilde{a}_y = \alpha_y \tilde{\beta}_y, \ a_r = \alpha_y \beta_r. \tag{26}$$

Thus, in year t, the parameters in the current Phillips curve are known, but neither those of next year's Phillips curve, nor those of the current aggregate demand equation. These are instead known in year $t+1$. That is, we assume that all uncertainty relevant for the policy decision in year t is resolved in year $t+1$. In particular, there is a new realization of the stochastic disturbance terms each year, with unchanged variances and covariances. Therefore, no attained knowledge can reduce the uncertainty, and there is no point in experimenting in order to learn more about the stochastic properties of the model. The fact that there is no role for experimentation and learning simplifies the analysis considerably.[8]

Under these assumptions, the constraint in year t can be written

$$\pi_{t+2} = \pi_{t+1|t} + (\tilde{a}_y + \nu_{y,t+1})\, y_t - (a_r + \nu_{r,t+1}) \left(i_t - \pi_{t+1|t}\right) + \varepsilon_{t+1} + \alpha_{y,t+1}\eta_{t+1} + \varepsilon_{t+2}, \quad (27)$$

where the one-year inflation forecast, $\pi_{t+1|t}$, remains predetermined and is given by (23). Let $\nu_{y,t+1}$ and $\nu_{r,t+1}$ have variances and covariances σ_y^2, σ_r^2 and σ_{yr}, respectively.[9] Furthermore, let the covariance of $\nu_{r,t+1}$ with $\varphi_{t+1} \equiv \varepsilon_{t+1} + \alpha_{y,t+1}\eta_{t+1}$ be $\sigma_{\varphi r}$, but assume that $\nu_{\alpha y,t+1}$ and η_{t+1} are independent so $(\alpha_{y,t+1}\eta_{t+1})_{|t} = \mathrm{E}[\alpha_{y,t+1}\eta_{t+1}] = 0$. It then follows that the two-year conditional inflation forecast is given by

$$\pi_{t+2|t}(i_t) \equiv \pi_{t+1|t} + \tilde{a}_y y_t - a_r \left(i_t - \pi_{t+1|t}\right). \quad (28)$$

With the constraint (27), the first-order condition is

$$\begin{aligned} 0 &= \delta^2 \mathrm{E}_t \left[(\pi_{t+2} - \pi^*) \frac{\partial \pi_{t+2}}{\partial i_t} \right] \\ &= -\delta^2 \mathrm{E}_t \Big\{ \Big[\pi_{t+1|t} + (\tilde{a}_y + \nu_{y,t+1})\, y_t - (a_r + \nu_{r,t+1})(i_t - \pi_{t+1|t}) \\ &\qquad + \varphi_{t+1} + \varepsilon_{t+2} - \pi^* \Big] \Big\} (a_r + \nu_{r,t+1}) \\ &= -\delta^2 \left(\pi_{t+2|t}(i_t) - \pi^* \right) a_r - \delta^2 \sigma_{yr} y_t + \delta^2 \sigma_r^2 \left(i_t - \pi_{t+1|t} \right) - \delta^2 \sigma_{\varphi r}. \end{aligned}$$

[8] On learning and experimenting, see, for instance, Chow (1975, chapt. 11), Bertocchi and Spagat (1993), Balvers and Cosimano (1994) and Wieland (1996, 1998).

[9] If there is uncertainty in $\alpha_{y,t+1}$ (or $\tilde{\beta}_{yt}$) alone with variance $\sigma_{\alpha y}^2$ (or $\sigma_{\beta y}^2$), we have $\sigma_r^2 = \beta_r^2 \sigma_{\alpha y}^2$ (or $\sigma_r^2 = 0$) and $\sigma_{yr} = \tilde{\beta}_y \beta_r \sigma_{\alpha y}^2$ (or $\sigma_{yr} = 0$). If there is uncertainty in β_{rt} alone, with variance $\sigma_{\beta r}^2$, we have $\sigma_r^2 = \alpha_y^2 \sigma_{\beta r}^2$ and $\sigma_{yr} = 0$.

We can rewrite the first-order condition as

$$\pi_{t+2|t}(i_t) - \pi^* = -\frac{\sigma_{yr}}{a_r} y_t + \frac{\sigma_r^2}{a_r}\left(i_t - \pi_{t+1|t}\right) - \frac{\sigma_{\varphi r}}{a_r}. \tag{29}$$

It is clear that with multiplier uncertainty, the variances and covariances of the multiplier will affect the solution and make it deviate from (25). The standard certainty-equivalence in the linear-quadratic model breaks down.

We can discuss the optimal policy either in terms of targeting rules or reaction functions. Let us first look at reaction functions. Using (28) in (29), we can solve for the optimal reaction function,

$$i_t = \pi_{t+1|t} + \frac{1}{(1+v_r)\,a_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{\tilde{a}_y + \sigma_{yr}/a_r}{(1+v_r)\,a_r} y_t + \frac{\sigma_{\varphi r}/a_r}{(1+v_r)\,a_r}, \tag{30}$$

where

$$v_r = \frac{\sigma_r^2}{a_r^2}$$

is the coefficient of variation of the policy multiplier a_r.

In order to interpret the reaction function (30), consider the special case of "independent multiplier uncertainty", when $\sigma_r^2 > 0$, but ν_r is not correlated with ν_y or φ, that is, $\sigma_{yr} = \sigma_{\varphi r} = 0$. This is the case when there is uncertainty in β_{rt} alone, and when β_{rt} is uncorrelated with φ_{t+1}. Then (30) simplifies to

$$i_t = \pi_{t+1|t} + \frac{1}{(1+v_r)\,a_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{\tilde{a}_y}{(1+v_r)\,a_r} y_t. \tag{31}$$

We see that more uncertainty (a higher coefficient of variation v_r) leads to a more "conservative" and less activist policy, in the sense of reducing the magnitude of the response coefficients.

In order to interpret the policy further, consider two extreme cases. First, consider the case with no (policy) multiplier uncertainty ($\sigma_r^2 = 0$), as in section II. Then $v_r = 0$, and the reaction function is

$$i_t = i_t^0 \equiv \pi_{t+1|t} + \frac{1}{a_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{\tilde{a}_y}{a_r} y_t, \tag{32}$$

which I call the "no-multiplier-uncertainty" policy.

Next, consider the other extreme, with infinite uncertainty ($\sigma_r^2 \rightarrow \infty$). The model and its policy are of course meaningless with unbounded uncertainty, so this case only serves as a hypothetical reference point. It follows from (31) that the optimal policy is then to set the interest equal to expected inflation, so as to make the real interest rate equal to its long-run average (here normalized to zero),

$$i_t = i^\infty \equiv \pi_{t+1|t}. \tag{33}$$

I call this the "infinite-multiplier-uncertainty" policy. Intuitively, with large uncertainty in the coefficient β_{rt} in (22), it is best to choose the instrument so that the real interest rate is close to its long-run average, in order to limit the variability of inflation. For infinite uncertainty, when the real interest rate is held constant at zero, inflation becomes non-stationary.[10]

The reaction function (31) can now be written as a convex combination of the no-multiplier-uncertainty reaction function and the infinite-multiplier-uncertainty reaction function,

$$i_t = \frac{1}{1+v_r} i_t^0 + \frac{v_r}{1+v_r} i_t^\infty. \tag{34}$$

Thus, the monetary authority is more conservative with independent multiplier uncertainty than without any multiplier uncertainty, in the sense that its policy is an average of the policy without uncertainty and the policy for infinite uncertainty (which makes the real interest rate equal to its long-run average).

Next, we shall look at this in terms of targeting rules. The two-year conditional inflation forecasts that correspond to the no-multiplier-uncertainty policy and the infinite-multiplier-

[10] The appropriate response when uncertainty becomes very high is of course dependent on the precise model and nature of the uncertainty. From (21) and (22), it is apparent that if the uncertainty is in α_{yt} or $\tilde{\beta}_{yt}$ rather than in β_{rt}, the appropriate response with infinite uncertainty is to set the instrument such that $y_{t+1|t} = 0$, rather than $i_t - \pi_{t+1|t} = 0$.

uncertainty policy are $\pi_{t+2|t}\left(i_t^0\right) = \pi^*$ and

$$\pi_{t+2|t}\left(i_t^\infty\right) = \pi_{t+2|t}^\infty \equiv \pi_{t+1|t} + \tilde{a}_y y_t,$$

respectively. Since the two-year conditional inflation forecast is linear in the instrument, it follows that it will be a convex combination of the long-run inflation target and the infinite-multiplier-uncertainty two-year conditional inflation forecast with the same weight as in (34),

$$\begin{aligned}
\pi_{t+2|t}\left(i_t\right) &= \frac{1}{1+v_r}\pi^* + \frac{v_r}{1+v_r}\pi_{t+2|t}^\infty \\
&= \pi^* + \frac{v_r}{1+v_r}\left(\pi_{t+2|t}^\infty - \pi^*\right) \\
&= \pi^* + \frac{v_r}{1+v_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{v_r \tilde{a}_y}{1+v_r} y_t.
\end{aligned}$$

Thus, the two-year conditional inflation forecast deviates from the inflation target by a fraction $\frac{v_r}{1+v_r}$ of the deviation of the infinite-multiplier-uncertainty two-year inflation forecast from the inflation target. Equivalently, the two-year conditional inflation forecast deviates from the inflation target by the sum of the same fraction of the deviation of the *one*-year inflation forecast from the inflation target and a term proportional to the output gap. In the case of flexible inflation targeting, the two-year conditional inflation forecast is only gradually adjusted towards the inflation target.

In the general case, when multiplier uncertainty is not independent, the policy (30) involves a constant, $\frac{\sigma_{\varphi r}/a_r}{(1+v_r)a_r}$. The coefficient of y_t is also modified, and affected by the covariance σ_{yr}. The constant will make average inflation deviate from the long-run inflation target. The long-run average follows directly from (29) and fulfills

$$\mathrm{E}\left[\pi_t\right] = \pi^* - \frac{\sigma_{\varphi r}}{a_r}, \tag{35}$$

where I have used that $\mathrm{E}\left[y_t\right] = 0$ and $\mathrm{E}\left[i_t\right] = \mathrm{E}\left[\pi_t\right]$. Thus, the average inflation deviates from the inflation target, the bias being positive or negative depending on the sign of the covariance between the policy multiplier and the disturbance to inflation, $\sigma_{\varphi r}$. The two-year conditional inflation forecast will be

$$\begin{aligned}\pi_{t+2|t}(i_t) &= \pi^* + \frac{v_r}{1+v_r}\left(\pi^{\infty}_{t+2|t} - \pi^*\right) - \frac{\sigma_{yr}/a_r}{1+v_r} y_t - \frac{\sigma_{\varphi r}/a_r}{1+v_r} \\ &= \pi^* + \frac{v_r}{1+v_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{v_r \tilde{a}_y - \sigma_{yr}/a_r}{1+v_r} y_t - \frac{\sigma_{\varphi r}/a_r}{1+v_r}. \end{aligned} \tag{36}$$

The two-year conditional inflation forecast is mean-reverting and gradually adjusted towards (35).

In summary, model uncertainty in the form of policy-multiplier uncertainty motivates deviations from the long-run inflation target. Under strict inflation targeting, without any multiplier uncertainty, the two-year conditional inflation forecast should always equal the long-run inflation target. With independent policy-multiplier uncertainty, the optimal policy is a convex combination of the no-multiplier-uncertainty policy and the infinite-multiplier-uncertainty policy, which results in the two-year conditional inflation forecast being gradually adjusted towards the long-run inflation target. When policy-multiplier uncertainty is not independent, there may be a bias in average inflation, and the response of the two-year conditional inflation forecast to the output gap is modified.

These results are derived under the assumption that parameters are stochastic with a known stationary distribution, so there is no scope for learning and experimentation. Even in that case, for particular covariance patterns for the disturbances to the different parameters, the policy caution can be overturned and a more aggressive policy can be optimal (for instance, if the covariance σ_{yr} in (36) is negative). If the uncertainty is instead due to the central bank having an imperfect knowledge of the true parameters, the problem is very different. As examined in Bertocchi and Spagat (1993) and Wieland (1996, 1998), the central bank may then have an incentive to experiment and, in the short run, pursue a policy that generates more informative data, in order to achieve a better policy in the long run. The variance and covariance of parameters are then endogenous; not exogenous as in the Brainard case. The optimal response often seems to be a compromise between the certainty-equivalent policy in the absence of parameter uncertainty and the cautious Brainard policy.

V. Interest rate stabilization and smoothing

How is inflation targeting affected by attempts to stabilize and/or smooth the instrument?[11] Modify the period loss function to

$$L(\pi_t, y_t, i_t, i_t - i_{t-1}) = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda y_t^2 + \mu\,(i_t - \pi_t)^2 + \nu\,(i_t - i_{t-1})^2\right]. \qquad (37)$$

This allows for a weight $\mu \geq 0$ on stabilizing the pseudo-real rate, $i_t - \pi_t$, as well as a weight $\nu \geq 0$ on smoothing the instrument (stabilizing the first-difference of the instrument). Alternatives to stabilizing the pseudo-real rate are, of course, to stabilize the real interest rate, $i_t - \pi_{t+1|t}$, or the nominal interest rate i_t itself. Since other variables than inflation enters the loss function, this is another case of flexible inflation targeting.

Minimizing the intertemporal loss function (6) with the period loss function (7) replaced by (37) generally seems to require a numerical solution of the standard linear-quadratic optimal control problem. In particular, when $\nu > 0$, the lagged instrument enters as a state variable, which together with inflation and the output gap brings the number of state variables up to three (excluding the exogenous variable).

In order to gain some insight into the effects of interest rate stabilization and smoothing, without having to resort to numerical analysis, let me make a few simplifications. First, the weight on output stabilization is set to zero. Second, each period the monetary authority solves the simple problem

$$\min_{i_t} \frac{1}{2}\left[\delta^2\,(\pi_{t+2} - \pi^*)^2 + \mu\,(i_t - \pi_t)^2 + \nu(i_t - i_{t-1})^2\right] \qquad (38)$$

[11] See Goodhart (1996) for a recent discussion of interest rate smoothing.

subject to

$$\pi_{t+2} = \pi_t + a_y y_t - a_r(i_t - \pi_t) + \varepsilon_{t+1} + \alpha_y \eta_{t+1} + \varepsilon_{t+2},$$

where I use (9), (10) and (16), and for simplicity disregard the exogenous variable ($a_x = \beta_x = 0$).

The monetary authority is assumed to minimize the loss function in (38) each period, taking last year's interest rate as given, but disregarding that today's instrument setting will affect next year's loss function. When $\lambda = \mu = \nu = 0$, this problem is equivalent to the general intertemporal problem, as demonstrated in Svensson (1997a). When either μ or ν differs from zero, this is no longer true. Nevertheless, the simple case of (38) helps to understand the general consequences of instrument stabilization and smoothing.

The first-order condition is

$$\delta^2 \left(\pi_{t+2|t}\left(i_t\right) - \pi^*\right)(-a_r) + \mu(i_t - \pi_t) + \nu(i_t - i_{t-1}) = 0. \tag{39}$$

We can write the first-order condition as

$$\pi_{t+2|t}\left(i_t\right) = \pi^* + \frac{\mu}{\delta^2 a_r}\left(i_t - \pi_t\right) + \frac{\nu}{\delta^2 a_r}(i_t - i_{t-1}),$$

and observe that when μ or ν differ from zero, the two-year conditional inflation forecast will generally deviate from the inflation target.

We can solve for the reaction function and get

$$i_t = \frac{\mu + \delta^2 a_r^2}{\mu + \nu + \delta^2 a_r^2}\pi_t + \frac{\nu}{\mu + \nu + \delta^2 a_r^2}i_{t-1} + \frac{\delta^2 a_r}{\mu + \nu + \delta^2 a_r^2}(\pi_t - \pi^*) + \frac{\delta^2 a_r a_y}{\mu + \nu + \delta^2 a_r^2}y_t. \tag{40}$$

With a zero weight on instrument smoothing ($\nu = 0$), the reaction function does not depend on the lagged interest rate and is given by

$$i_t = \pi_t + \frac{\delta^2 a_r}{\mu + \delta^2 a_r^2}(\pi_t - \pi^*) + \frac{\delta^2 a_r a_y}{\mu + \delta^2 a_r^2}y_t.$$

Hence, the effect of a positive weight on (pseudo-real) interest rate stabilization is simply to reduce the coefficients of $\pi_t - \pi^*$ and y_t.

With a zero weight on interest rate stabilization ($\mu = 0$), the reaction function depends on the lagged interest rate and becomes

$$i_t = \frac{\delta^2 a_r^2}{\nu + \delta^2 a_r^2}\pi_t + \frac{\nu}{\nu + \delta^2 a_r^2} i_{t-1} + \frac{\delta^2 a_r}{\nu + \delta^2 a_r^2}(\pi_t - \pi^*) + \frac{\delta^2 a_r a_y}{\nu + \delta^2 a_r^2} y_t.$$

Note that the reaction function is not simply a rule for the first-difference of the instrument.

In this simple case, the reaction function has an interesting interpretation. Let i_t^π denote the reaction function under strict inflation targeting, when $\mu = \nu = 0$. It is given by

$$i_t^\pi = \pi_t + \frac{1}{a_r}(\pi_t - \pi^*) + \frac{a_y}{a_r} y_t. \tag{41}$$

Furthermore, let i_t^i and $i_t^{\Delta i}$ denote the reaction functions under strict pseudo-real interest rate stabilization ($\mu \to \infty$, $\nu = 0$) and strict interest rate smoothing ($\nu \to \infty$, $\mu = 0$), respectively. These are given by $i_t^i = \pi_t$ and $i_t^{\Delta i} = i_{t-1}$. Then, the optimal reaction function can be written as a convex combination of the three rules,

$$i_t = \frac{\delta^2 a_r^2}{\mu + \nu + \delta^2 a_r^2} i_t^\pi + \frac{\mu}{\mu + \nu + \delta^2 a_r^2} i_t^i + \frac{\nu}{\mu + \nu + \delta^2 a_r^2} i_t^{\Delta i}. \tag{42}$$

Let $\pi_{t+2|t}^j$, $j = \pi$, i, Δi, denote the two-year conditional inflation forecast that corresponds to each strict rule. They are given by

$$\begin{aligned}
\pi_{t+2|t}^\pi &= \pi^* \\
\pi_{t+2|t}^i &= \pi_t + a_y y_t = \pi_t + \alpha_y y_t + \tilde{a}_y y_t = \pi_{t+1|t} + \tilde{a}_y y_t. \\
\pi_{t+2|t}^{\Delta i} &= \pi_{t+1|t} - a_r(i_{t-1} - \pi_t),
\end{aligned}$$

where I use that by (10) and (26) $a_y = \alpha_y + \tilde{a}_y$. It follows that the two-year conditional inflation forecast is the same convex combination of these three forecasts,

$$\begin{aligned}
\pi_{t+2|t}(i_t) &= \frac{\delta^2 a_r^2}{\mu + \nu + \delta^2 a_r^2}\pi_{t+2|t}^\pi + \frac{\mu}{\mu + \nu + \delta^2 a_r^2}\pi_{t+2|t}^i + \frac{\nu}{\mu + \nu + \delta^2 a_r^2}\pi_{t+2|t}^{\Delta i} \qquad (43) \\
&= \pi^* + \frac{\mu}{\mu + \nu + \delta^2 a_r^2}\left(\pi_{t+2|t}^i - \pi^*\right) + \frac{\nu}{\mu + \nu + \delta^2 a_r^2}\left(\pi_{t+2|t}^{\Delta i} - \pi^*\right)
\end{aligned}$$

$$= \pi^* + \frac{\mu+\nu}{\mu+\nu+\delta^2 a_r^2}\left(\pi_{t+1|t} - \pi^*\right) + \frac{\mu+\nu}{\mu+\nu+\delta^2 a_r^2}\tilde{a}_y y_t - \frac{\nu}{\mu+\nu+\delta^2 a_r^2} a_r (i_{t-1} - \pi_t). \tag{44}$$

Equations (43)–(44) can be interpreted as equivalent forms of a targeting rule for the two-year conditional inflation forecast, implying that the two-year conditional inflation forecast is gradually adjusted to the inflation target.

Generally, concerns about interest stabilization and smoothening leads to a less active policy. The two-year conditional inflation forecast, as for flexible inflation targeting, is adjusted gradually towards the inflation target. Numerical analysis of the general intertemporal problem confirms this insight.

As far as I can see, the result that a reaction function can be written as a convex combination of strict reaction functions does not necessarily hold in the general intertemporal problem. In some special cases the result holds, but it is more complicated to determine the weights.[12]

VI. Conclusions

Inflation targeting makes the conditional inflation forecast (conditional upon the current state of the economy and the current instrument setting) an intermediate target. Thus, inflation targeting can be described as a targeting rule, that is, a rule specifying the intermediate-target variable and how its target level is determined. Implementation of this targeting rule then leads to an implicit endogenous reaction function. Inflation targeting can be interpreted as a commitment to a targeting rule, where the monetary authority has discretion in selecting the appropriate reaction function that achieves the targeting rule.

The present paper has examined inflation targeting with regard to the appropriate monetary

[12] Broadbent (1996) observes, for simple loss functions, that the instrument rule can be written as a convex combination of pure instrument rules.

policy response to different shocks, the consequences of model uncertainty, and the effects of interest rate smoothing and stabilization. The analysis distinguishes between strict inflation targeting, when nothing but inflation enters the monetary authority's loss function, and flexible inflation targeting, when the monetary authority is also concerned about the stability of the output gap or the instrument.

Under strict inflation targeting, the targeting rule is very simple. The instrument should be adjusted such that the conditional inflation forecast for a horizon corresponding to the control lag always equals the inflation target. Any shock causing a deviation between the conditional inflation forecast and the inflation target should then be met by an instrument adjustment that eliminates the deviation.

Under flexible inflation targeting, the targeting rule is not quite as simple, but very intuitive. The instrument should be adjusted such that the conditional inflation forecast gradually approaches the long-run inflation target. For instance, when there is some weight on output stabilization in the monetary authority's loss function, the two-year conditional inflation forecast's deviation from the long-run inflation target should be a given proportion of the predetermined one-year inflation forecast's deviation from the same target. When there is some weight on instrument stabilization or smoothing, on the other hand, the conditional forecast should also be gradually adjusted towards the long-run inflation target. As a consequence, there is a more gradual response to shocks. The intuition for this result is, of course, that a more gradual adjustment requires less output and instrument variability.

Interestingly, a gradual adjustment of the conditional inflation forecast towards the long-run inflation target is also the appropriate policy under model uncertainty. Here, the intuition is that uncertainty about the policy multiplier requires a more muted instrument response, in order to reduce the part of the variability in inflation caused by the variability of the policy multiplier.

Thus, both flexible inflation targeting and model uncertainty lead to a gradual adjustment of the conditional inflation forecast toward the long-run inflation target. Since they have similar observation-equivalent consequences the precise reasons for these are not directly revealed by observations of gradual adjustment to the long-run inflation target. In this context, it is interesting to note that the 0–2 percent per year range for the Reserve Bank of New Zealand was increased to 0–3 percent per year in the modification of the Policy Target Agreement in December 1996. In the debate in New Zealand, some observers have suggested that the original target range requires an excessive degree of activism on the part of the Reserve Bank, and that a slightly wider band would be desirable (Brash (1997)).

Many inflation-targeting issues remain and seem suitable for future research. The model used here is annual, and it remains to apply these ideas in a quarterly, more empirical framework. Rudebusch and Svensson (1997) compare different inflation targeting rules and instrument rules, for instance the Taylor rule, in an empirical quarterly model for the United States.

The model is simplified by assuming that there is a one-year control lag for output and a two-year control lag for inflation. Furthermore, the impact at the control lag (the shortest horizon at which the instrument has an effect) is substantial. In a more elaborate model with shorter periods and empirically estimated coefficients, the first effect of the instrument on output and inflation may be quite small, with the effect growing to a maximum several periods later and then declining. For strict inflation targeting, the first effect is still the relevant one, and the task of monetary policy is then to fulfill the inflation target at the shortest possible horizon. That first effect being quite small, drastic adjustments of the instrument will be required, thus making strict inflation even less attractive. For flexible inflation targeting, the question arises which horizon should be emphasized by the central bank, both in its internal policy decisions and in its communications with the private sector. Some central banks have chosen to emphasize the horizon at which the effect is largest (this horizon is often assumed to be about two years). Generally, in a more elaborate model the whole time path of the inflation forecast is of relevance, and the criterion whether policy is optimal is whether the forecasts of inflation and the output gap "look right," more specifically, whether they approach the inflation target and zero, respectively, at an appropriate pace. Svensson (1998b) gives examples of optimal time-paths of these forecasts for different disturbances and weights in the loss function for an open economy with forward-looking aggregate demand and supply. Svensson (1998a) provides a more general discussion of the roles of targeting rules and rules of thumb in inflation targeting. More research on these issues would be desirable.

Inflation-targeting with imperfectly observed shocks results in a signal-extraction problem for the monetary authority. Imperfect identification of shocks may be a separate reason for a gradual adjustment of the conditional inflation forecast toward the long-run inflation target, which remains to be examined.

The real world inflation-targeting regimes are all very open economies. In an open economy, there is also a direct exchange rate channel for the transmission of monetary policy, with a, by all accounts, shorter lag than the aggregate demand channel emphasized in the present paper. In an open economy, there is also a choice between targeting inflation in domestic prices only (the GDP deflator) or a consumer price index where imports enter. These and other issues in open-economy inflation targeting are examined in Svensson (1998b).

The model used here has backward-looking aggregate demand and supply with considerable inertia. More forward-looking aggregate demand and supply in principle reduces the inertia and makes it easier to fulfill the inflation target, as long as the inflation target is credible. If the inflation target is not credible, its forward-looking aspects may possibly make the inflation target more difficult to achieve. Several recent papers examine different aspects of inflation targeting in a more forward-looking setup, for instance, Amano, Coletti and Macklem (1998), Bernanke and Woodford (1997), Black, Macklem and Rose (1997), Haldane and Batini (1998) and Svensson (1998a, 1998b).

Appendix A. The natural output level and the output gap

Consider the model

$$\pi_{t+1} = \pi_t + \alpha_y (y_t^d - y_t^n) + \varepsilon_{t+1} \tag{A1}$$

$$y_{t+1}^d = \tilde{\beta}_y y_t^d + \beta_z z_t - \beta_r (i_t - \pi_{t+1|t}) + \tilde{\eta}_{t+1} \tag{A2}$$

$$z_{t+1} = \gamma_z z_t + \zeta_{t+1} \tag{A3}$$

$$y_{t+1}^n = \gamma_n y_t^n + \xi_{t+1}, \tag{A4}$$

where y_t^d is (log) aggregate demand, y_t^n is the natural output level, z_t is a persistent aggregate demand disturbance, $0 \leq \gamma_z < 1$, $0 \leq \gamma_n \leq 1$, and ε_t, $\tilde{\eta}_t$, ζ_t and ξ_t are iid disturbances. Subtract y_{t+1}^n from (A2),

$$\begin{aligned} y_{t+1}^d - y_{t+1}^n &= \tilde{\beta}_y \left(y_t^d - y_t^n \right) + \beta_z z_t - \beta_r (i_t - \pi_{t+1|t}) + \left(\tilde{\beta}_y y_t^n - y_{t+1}^n \right) + \tilde{\eta}_{t+1} \\ &= \tilde{\beta}_y \left(y_t^d - y_t^n \right) + \beta_z z_t - \beta_r (i_t - \pi_{t+1|t}) + \left(\tilde{\beta}_y - \gamma_n \right) y_t^n + \tilde{\eta}_{t+1} - \xi_{t+1}, \end{aligned}$$

and introduce the output gap,

$$y_t = y_t^d - y_t^n.$$

Then the model can be written

$$\begin{aligned} \pi_{t+1} &= \pi_t + \alpha_y y_t + \varepsilon_{t+1} \\ y_{t+1} &= \tilde{\beta}_y y_t + \beta_z z_t + \left(\tilde{\beta}_y - \gamma_n \right) y_t^n - \beta_r (i_t - \pi_{t+1|t}) + \tilde{\eta}_{t+1} - \xi_{t+1} \\ &= \beta_y y_t + \beta_z z_t + \beta_n y_t^n - \beta_r (i_t - \pi_t) + \eta_{t+1}, \end{aligned}$$

where

$$\beta_y = \tilde{\beta}_y + \alpha_y \beta_r \tag{A5}$$

$$\beta_n = \tilde{\beta}_y - \gamma_n \tag{A6}$$

$$\eta_{t+1} = \tilde{\eta}_{t+1} - \xi_{t+1}. \tag{A7}$$

The one-year and two-year inflation forecasts are

$$\pi_{t+1|t} = \pi_t + \alpha_y y_t \tag{A8}$$

$$\pi_{t+2|t}(i_t) = \pi_t + a_y y_t + a_z z_t + a_n y_t^n - a_r(i_t - \pi_t), \tag{A9}$$

where

$$a_y = \alpha_y(1 + \beta_y) \tag{A10}$$

$$a_z = \alpha_y \beta_z \tag{A11}$$

$$a_n = \alpha_y \beta_n \tag{A12}$$

$$a_r = \alpha_y \beta_r. \tag{A13}$$

With the period loss function (7), the optimal policy rule can be written on the forms

$$i_t = \pi_t + f_\pi(\lambda)(\pi_t - \pi^*) + f_y(\lambda) y_t + f_z z_t + f_n y_t^n, \tag{A14}$$

where $f_\pi(\lambda)$ and $f_y(\lambda)$ are given by (15), $f_z = \frac{\beta_z}{\beta_r}$ and $f_n = \frac{\beta_n}{\beta_r}$.

In (3), x_t represents either the persistent demand disturbance z_t or the natural rate y_t^n (or both, if it is interpreted as a vector and γ as a diagonal matrix).

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