Comment on Isard, Laxton and Eliasson, "Simple Monetary Policy Rules under Model Uncertainty"*

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Isard, Laxton and Eliasson [1] (ILE) have written a fine and impressive paper, with much content. It presents an estimated empirical model of the U.S. economy, with a nonlinear Phillips curve (with both forward- and backward-looking elements) and an unobservable time-varying natural unemployment rate. Stochastic simulations and stability analysis are undertaken with alternative simple "instrument rules," that is, rules specifying the instrument as a given reaction function of directly observable or constructed, synthetic variables. Furthermore, since the natural unemployment rate is unobservable, instrument rules that involve responses to the unemployment gap (the deviation between unemployment and the natural rate) must rely on the policy-makers' estimate of the time-varying natural rate.

A number of interesting detailed results are presented. The main result of the paper, as I interpret it, can be expressed as: "Beware of simple instrument rules (especially conventional linear and backward-looking ones) as an automatic pilot for the economy." Such rules may even result in instability. So-called "forecast-based" rules, where the instrument responds to the deviation of model-consistent inflation forecasts from the inflation target, perform better than the backward-looking instrument rules when the instrument responds to the deviation between current and lagged inflation and the inflation target (in addition to responding to the estimated unemployment gap).

My discussion will focus on an intuitive explanation of why linear backward-looking reaction functions will be inferior in a non-linear model, and why a reaction function, where the

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instrument responds to an inflation forecast, is likely to perform better, but is not optimal.

1. Instrument rules and targeting rules

First, however, I would like to raise the issue about how to handle practical monetary policy, given the paper's warning about the possible instability of conventional instrument rules. The paper states that it is very important for policymakers to calibrate their nominal interest rate adjustments on the basis of forward-looking measures of real interest rates. This can be interpreted as the paper advocating modified instrument rules rather than the conventional backward-looking instrument rules. However, in the real world, instrument rules are never applied mechanically. For several reasons, they are, at best, used as guidelines and benchmarks, which may illuminate the monetary policy decision but never be a substitute for a forward-looking decision framework for monetary policy. One of these reasons is the lack of a commitment mechanism, by which the central bank could commit itself to a given instrument rule. Another is the manifest inefficiency of any simple rule, and the strong incentives to deviate from it, since it relies on much less information than is efficient, including extra-model information that motivate judgemental adjustments.

Instead, as argued in Svensson [5] and [6], I believe that the so-called "targeting rules," which involve a commitment to minimize a given loss function or to fulfill some (approximate first-order) condition for (forecasts of) the target variables, but allow the optimization to be done under discretion, is a more fruitful and realistic formalization of real-world monetary policy. In particular, as discussed in [6], I believe that "forecast targeting" (meaning selecting an instrument path such that resulting forecasts of inflation and the output gap minimizes an intertemporal loss function) rather than a commitment to a simple instrument rule is the best way of maintaining price stability. Furthermore, I believe that a generalization from "mean" forecast targeting to "distribution" forecast targeting is a practical way of handling both nonlinearity and model uncertainty in monetary policy.

2. A simple model with a nonlinear Phillips curve

Let me now illustrate what inflation targeting, interpreted as a commitment to minimize a particular loss function, implies in a model with a nonlinear Phillips curve, and how the resulting equilibrium can be used to illuminate the inferiority of linear backward-looking instrument rules and the somewhat better performance of a forecast-based instrument rule. I choose a simple model which can be seen as a simple variant of the more elaborate model of ILE.

Assume that the aggregate supply is given by the simple accelerationist Phillips curve

$$\pi_{t+1} = \pi_t + f(\tilde{u}_t) + \varepsilon^{\pi}_{t+1}, \tag{2.1}$$

where π_t is inflation in quarter t,

$$\tilde{u}_t \equiv u_t - u^*_t$$

is the unemployment gap (where u_t is the unemployment rate and u^*_t is the natural unemployment rate), and ε^{π}_t is N(0,σ^2_{π}), a normally distributed exogenous shock with zero mean and variance σ^2_{π}. The natural unemployment rate is a random walk,

$$u^*_{t+1} = u^*_t + \varepsilon^{u*}_{t+1},$$

where ε^{u*}_t is N(0,σ^2_{u*}).

Nonlinearity enters in the Phillips curve via the nonlinear function f, which fulfills

$$f' < 0, \;\; f(0) = 0, \;\; f'' \geq 0.$$

Several functional forms can be used. ILE use the form

$$f(\tilde{u}_t) = -\gamma \frac{\tilde{u}_t}{\tilde{u}_t - 4}.$$

Schaling [4] instead uses

$$f(\tilde{u}_t) = -\frac{\gamma \tilde{u}_t}{1 - \varphi \gamma \tilde{u}_t},$$

where $\varphi \geq 0$ is used as an index of convexity. For convenience, I choose a simple quadratic function that allows an (approximate) analytical solution, namely

$$f(\tilde{u}_t) = \begin{cases} -\gamma \tilde{u}_t + \varphi \tilde{u}^2_t & \tilde{u}_t \leq \gamma/2\varphi \\ -\gamma^2/4\varphi & \tilde{u}_t > \gamma/2\varphi, \end{cases} \tag{2.2}$$

for the parameters $\varphi \geq 0$ and $\gamma > 0$. This function is continuous and differentiable. For $\varphi > 0$, it is decreasing and convex for $\tilde{u}_t \leq \gamma/2\varphi$ and constant for $\tilde{u}_t > \gamma/2\varphi$. For $\varphi = 0$, it is linear, $f(\tilde{u}_t) = -\gamma \tilde{u}_t$. Thus, φ can be interpreted as an index of convexity and nonlinearity.

Aggregate demand is taken to a linear function in terms of the unemployment gap,

$$\tilde{u}_{t+1} = \eta_u \tilde{u}_t + \eta_r (i_t - \pi_{t+1|t} - \bar{r}) + \varepsilon^{\tilde{u}}_{t+1}, \tag{2.3}$$

where i_t is a short nominal interest rate (denoted rs_t in ILE) and the central bank's instrument, $x_{t+\tau|t} \equiv \mathrm{E}_t x_{t+\tau}$ for any variable x denotes the expectation of $x_{t+\tau}$ conditional on information available in quarter t, $\bar{r} > 0$ is the "natural" real interest rate, ε^u_{t+1} is N(0,$\sigma^2_{\tilde{u}}$), and parameters η_u and η_r are positive. The natural real interest rate is the constant real interest rate that, in the absence of shocks, would result in a constant zero unemployment gap.

Assume an intertemporal loss function for the central bank,

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau},$$

where $0 < \delta < 1$ is a discount factor and the period loss, L_t, is given by the period loss function

$$L_t = \frac{1}{2}[(\pi_t - \pi^*)^2 + \theta \tilde{u}_t^2],$$

where π^* is the inflation target (denoted π^{TAR} in ILE). (I have simplified the period loss function relative to ILE by setting $\beta = \nu = 0$.)

Let me simplify further by setting $\theta = 0$ and consider "strict" inflation targeting,

$$L_t = \frac{1}{2}(\pi_t - \pi^*)^2,$$

leaving the case of "flexible" inflation targeting, $\theta > 0$, as an extension. We note from (2.1) that π_t and π_{t+1} are predetermined with respect to quarter t. By (2.3), the instrument i_t affects $\tilde{u}_{t+1}$, which, in turn, affects π_{t+2} (and later inflation rates). Since there is no cost to instrument adjustment (since $\nu = 0$), it is clear that i_t should be set so as to minimize

$$\mathrm{E}_t \delta^2 L_{t+2} \tag{2.4}$$

(since $i_{t+\tau}$ for $\tau \geq 1$ can be freely used to minimize $\mathrm{E}_{t+\tau}\delta^{\tau+2}L_{t+\tau+2}$).

Since $\tilde{u}_{t+1}$ by (2.3) is linear in i_t, we realize from (2.1) and (2.4) that the first-order condition for an optimum can be written

$$\begin{aligned} 0 &= \mathrm{E}_t[(\pi_{t+2} - \pi^*)f'(\tilde{u}_{t+1})] = \mathrm{E}_t\{[\pi_{t+1} + f(\tilde{u}_{t+1}) + \varepsilon^\pi_{t+2} - \pi^*]f'(\tilde{u}_{t+1})\} \\ &= (\pi_{t+2|t} - \pi^*)\mathrm{E}_t f'(\tilde{u}_{t+1}) + \mathrm{Cov}_t[f(\tilde{u}_{t+1}), f'(\tilde{u}_{t+1})], \end{aligned} \tag{2.5}$$

(where I have used that $\mathrm{E}_t[x, y] = \mathrm{E}_t[x]\mathrm{E}_t[y] + \mathrm{Cov}_t[x, y]$). Furthermore, assuming that negligible probability mass falls in the interval $\tilde{u}_{t+1} > \gamma/2\varphi$, we have

$$\mathrm{Cov}_t[f(\tilde{u}_{t+1}), f'(\tilde{u}_{t+1})] = 2\varphi \mathrm{Cov}_t[f(\tilde{u}_{t+1}), \tilde{u}_{t+1}].$$

Exploiting a theorem of Rubinstein [2],[1] we have

$$\mathrm{Cov}_t[f(\tilde{u}_{t+1}), \tilde{u}_{t+1}] = \mathrm{E}_t f'(\tilde{u}_{t+1})\sigma^2_{\tilde{u}}.$$

Using this in (2.5), I get the first-order condition

$$\pi_{t+2|t} = \pi^* - 2\varphi\sigma^2_{\tilde{u}}. \tag{2.6}$$

Thus, under strict inflation targeting with a convex Phillips curve, it is optimal to undershoot the inflation target, on average. Average inflation, the unconditional mean of inflation, will fulfill $\mathrm{E}[\pi_t] = \pi^* - 2\varphi\sigma^2_{\tilde{u}}$.

In order to determine the optimal setting of i_t, I need to solve (2.6) for $\tilde{u}_{t+1|t}$. We have

$$\pi_{t+2|t} \equiv \pi_{t+1|t} + \mathrm{E}_t f(\tilde{u}_{t+1}). \tag{2.7}$$

We note, in passing, that by taking the unconditional mean of (2.7), we have $\mathrm{E}[f(\tilde{u}_t)] = 0$. Since $f(\tilde{u}_t)$ is convex and $f(0) = 0$, it follows (as in ILE) that the average unemployment gap will be positive, $\mathrm{E}[\tilde{u}_t] > 0$. We can directly infer from (2.3) that the average real interest rate, $r_t \equiv i_t - \pi_{t+1|t}$, must exceed the natural real interest rate, $\mathrm{E}[r_t] > \bar{r}$.

In order to solve for $\tilde{u}_{t+1|t}$, assume that the variance $\sigma^2_{\tilde{u}}$ is sufficiently small to warrant the second-order Taylor approximation

$$\mathrm{E}_t f(\tilde{u}_{t+1}) = \mathrm{E}_t f(\tilde{u}_{t+1|t} + \varepsilon^{\tilde{u}}_{t+1}) = f(\tilde{u}_{t+1|t}) + \frac{1}{2} f''(\tilde{u}_{t+1|t})\sigma^2_{\tilde{u}} = f(\tilde{u}_{t+1|t}) + \varphi\sigma^2_{\tilde{u}}. \tag{2.8}$$

Combining (2.2) and (2.7)–(2.8) leads to a second-order equation for $\tilde{u}_{t+1|t}$. The solution for the relevant root can be written

$$\tilde{u}_{t+1|t} = g(\pi_{t+1|t} - \pi^*), \tag{2.9}$$

where

$$g(\pi_{t+1|t} - \pi^*) \equiv \begin{cases} \frac{\gamma}{2\varphi}\left(1 - \sqrt{1 - \frac{4\varphi}{\gamma^2}(\pi_{t+1|t} - \pi^* + 3\varphi\sigma^2_{\tilde{u}})}\right), & \varphi > 0 \\ \frac{1}{\gamma}\left(\pi_{t+1|t} - \pi^*\right), & \varphi = 0 \end{cases}$$

Combining the expectation in quarter t of (2.3) with (2.9) results in the optimal reaction function,

$$i_t = \bar{r} + \pi_{t+1|t} + \frac{1}{\eta_r} g(\pi_{t+1|t} - \pi^*) - \frac{\eta_u}{\eta_r}\tilde{u}_t \tag{2.10}$$

$$\equiv \bar{r} + \pi_t + f(\tilde{u}_t) + \frac{1}{\eta_r} g\left(\pi_t - \pi^* + f(\tilde{u}_t)\right) - \frac{\eta_u}{\eta_r}\tilde{u}_t. \tag{2.11}$$

[1] The theorem says that, if x and y are bivariate normal, under some mild regularity conditions,

$$\mathrm{Cov}[f(x), y] = \mathrm{E}[f'(x)]\mathrm{Cov}[x, y].$$

The reaction function can be expressed in terms of $\pi_{t+1|t}$ and $\tilde{u}_t$ as in (2.10). Alternatively, since the predetermined $\pi_{t+1|t}$ fulfills

$$\pi_{t+1|t} \equiv \pi_t + f(\tilde{u}_t) \tag{2.12}$$

(recall that $\tilde{u}_t$ is observable, since I simplify by assuming that u_t^* is observable), it can be expressed in terms of π_t and $\tilde{u}_t$ as in (2.11).

3. Comparing reaction functions

Thus, under strict inflation targeting, the endogenous reaction function on the equivalent forms (2.10) or (2.11) will result. The reaction function (2.10) is nonlinear in $\pi_{t+1|t}$ and linear in $\tilde{u}_t$. The reaction function (2.11) is nonlinear in both π_t and $\tilde{u}_t$. We note that it is optimal to respond to $\tilde{u}_t$, even under strict inflation targeting with no weight on $\tilde{u}_t$ in the loss function, since, as emphasized in Svensson [5], it is generally optimal to respond to the *determinants* of the target variable(s) rather than just the target variable(s) themselves.

We can now compare these optimal reaction functions to a Taylor-type rule,

$$i_t = \bar{r} + \pi_t + w_\pi(\pi_t - \pi^*) - w_u \tilde{u}_t, \tag{3.1}$$

and the two forecast-based instrument rules examined by ILE,

$$i_t = \bar{r} + \pi^4_{t+4|t} + w_\pi(\pi^4_t - \pi^*) - w_u \tilde{u}_t$$

which is denoted IFB1 (inflation-forecast-based rule 1), and

$$i_t = \bar{r} + \pi^4_{t+4|t} + w_\pi(\pi_{t+3|t} - \pi^*) - w_u \tilde{u}_t$$

which is denoted IFB2. Here $\pi^4_t = \frac{1}{4}\sum_{\tau=0}^{3} \pi_{t-\tau}$ denotes 4-quarter inflation. Both IFB rules are somewhat simplified by the assumption that u_t^* and hence $\tilde{u}_t$ are observable.

We see that the Taylor-type rule, as a function of π_t and $\tilde{u}_t$, is quite different from the optimal reaction function (2.11), since the former is linear in both arguments whereas the latter is nonlinear. Thus, it is quite intuitive that, with a nonlinear Phillips curve, the linear Taylor rule is inferior.

Furthermore, for IFB1, the term $\pi^4_{t+4|t} = \frac{1}{4}\sum_{\tau=0}^{3} \pi_{t+4-\tau|t}$ enters. For IFB2, the term $\pi_{t+3|t}$ also enters. Since the terms $\pi_{t+4-\tau|t} = \pi_{t+3-\tau|t} + \mathrm{E}_t f(\tilde{u}_{t+3-\tau|t})$ for $\tau = 0, ..., 3$, are nonlinear functions of $\tilde{u}_t$, this means that the instrument becomes a nonlinear function of $\tilde{u}_t$. The resulting nonlinear function is, of course, not equal to the optimal reaction function (2.10) or (2.11), so it will not be optimal. Still, it may be closer to the optimal reaction function than the backward-looking Taylor-type rule (3.1). This seems to be the reason why IFB1 and IFB2 perform better than the backward-looking linear rule.

Furthermore, note that IFB1 and IFB2 are not reaction functions that are functions of predetermined variables only. Instead, they make the instrument a function of an endogenous model-consistent inflation forecast, which depends on the instrument rule itself and requires the solution of the whole model to be determined. Therefore, IFB1 and IFB2 are actually examples of quite complex equilibrium conditions.[2] For this reason and others discussed in Svensson [5], I remain sceptical about their usefulness in practical monetary policy.

4. An instrument rule involving an optimal response to an inflation forecast

Suppose, however, that we would insist on applying an instrument rule involving a response to an inflation forecast. What could we do within the present model? First, consider the two-period inflation forecast $\pi_{t+2|t}$ as a function $\Pi_{t+2|t}(i_t)$ of the instrument, i_t, and the state of the economy in periods t, π_t and $\tilde{u}_t$. This function is by (2.7) defined by

$$\begin{aligned}\pi_{t+2|t} &= \Pi_{t+2|t}(i_t) \\ &\equiv \pi_{t+1|t} + f[\eta_u \tilde{u}_t + \eta_r (i_t - \pi_{t+1|t})] + \varphi\sigma_{\tilde{u}}^2,\end{aligned}$$

where we recall that $\pi_{t+1|t}$ is given by (2.12) and where I have used the approximation (2.8). Now, we can, of course, consider a first-order Taylor approximation to this forecast around the interest rate i_{t-1} in the previous quarter,

$$\pi_{t+2|t} = \Pi_{t+2|t}(i_{t-1}) + \frac{\partial \Pi_{t+2|t}(i_{t-1})}{\partial i}\Delta i_t,$$

where $\Delta i_t \equiv i_t - i_{t-1}$. Combining this with the first-order condition (2.6) and solving for Δi_t leads to

$$\Delta i_t = \frac{1}{-\frac{\partial \Pi_{t+2|t}(i_{t-1})}{\partial i}}[\Pi_{t+2|t}(i_{t-1}) - (\pi^* - 2\varphi\sigma_{\tilde{u}}^2)]. \tag{4.1}$$

Here, we get an optimal instrument rule, which says that the optimal adjustment of the interest rate, Δi_t, should be proportional to the deviation between the *unchanged-interest-rate* inflation forecasts $\Pi_{t+2|t}(i_{t-1})$ and the adjusted inflation target, $\pi^* - 2\varphi\sigma_{\tilde{u}}^2$. Furthermore, the

[2] Rudebusch and Svensson [3, Appendix] demonstrate the complexity of this instrument rule.

response coefficient is given by

$$\frac{1}{-\frac{\partial \Pi_{t+2|t}(i_{t-1})}{\partial i}} = \frac{1}{-\eta_r f'[\eta_u \tilde{u}_t + \eta_r(i_{t-1} - \pi_{t+1|t} - \bar{r})]} = \frac{1}{\eta_r\{\gamma - 2\varphi[\eta_u \tilde{u}_t + \eta_r(i_{t-1} - \pi_{t+1|t} - \bar{r})]\}}.$$

Several comments are in order. First, the inflation forecast is *not* the model-consistent inflation forecast for the endogenous interest rate but the *unchanged-interest-rate* forecast (that is, for $i_t = i_{t-1}$). Second, the instrument rule involves the *change* in the interest rate, not the level. Third, an *adjusted* inflation target should be applied (when $\varphi > 0$ and the Phillips curve is nonlinear). Fourth, the response coefficient is *not constant* but state-dependent (when $\varphi > 0$). The response coefficient is the reciprocal of the slope of the Phillips curve for an unchanged-interest-rate unemployment-gap forecast for period $t+1$, given by $\eta_u \tilde{u}_t + \eta_r(i_{t-1} - \pi_{t+1|t})$. Finally, even if the response coefficient is state-dependent, the instrument rule is only an approximation, since it follows from a first-order approximation of a nonlinear function.

Clearly, the optimal instrument rule (4.1) is, in several respects, quite different from the IFB rules discussed by ILE. In a linear model (when $\varphi = 0$), the problem of the adjusted inflation target and the state-dependent response coefficient would disappear. Then, the optimal instrument adjustment is proportional to the deviation between an unchanged-interest-rate inflation forecast and the inflation target. Thus, it seems more intuitive that any response should be to the unchanged-interest-rate forecast than to a model-consistent forecast.

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