Summary

• Model

\[ x_t = E_t^* x_{t+1} - \varphi(u_t - E_t^* \pi_{t+1}) + \gamma_t \]  (1)
\[ \pi_t = \beta E_t^* \pi_{t+1} + \lambda x_t + u_t \]  (2)
\[ \gamma_t = \mu g_t - \bar{g}_t \]  (3a)
\[ u_t = \rho u_{t-1} + \bar{u}_t \]  (3b)

E_t^* private-sector expectations, not necessarily rational
E_t rational expectations
Monetary-policy loss function

\[ E_t^* \sum_{s=0}^{\infty} \beta^s (\pi_{t+s}^2 + \alpha x_{t+s}^2) \]  (4)

• Adaptive learning

Period t, E_t^*, PLM: (a_t, b_t, c_t), \quad y_t \equiv (x_t, \pi_t)', \quad u_t \equiv (g_t, u_t)'

\[ y_s = a_t + \bar{b}_t y_{s-1} + c_t v_s \]  (18)
\[ E_t^* y_{t+1} = a_t + \bar{b}_t E_t^* y_t + c_t E_t^* v_{t+1} \]
\[ = a_t + \bar{b}_t (a_t + \bar{b}_t y_{t-1} + c_t v_t) + c_t F v_t, \quad F = \begin{bmatrix} \mu & 0 \\ 0 & \rho \end{bmatrix} \]  (19)

Combine (19) with (1), (2) and, for instance, reaction function (13), solve for \( y_t \Rightarrow \text{new observation, ALM:} \)
\[ y_t = a_t + \bar{b}_t y_{t-1} + c_t v_t \]

Period t+1, E_{t+1}^*, PLM: (a_{t+1}, b_{t+1}, c_{t+1}), update by recursive least squares,

\[ \xi_t = (a_t', b_t', c_t')', \quad z_t = (1, y_t', v_t')' \]
\[ \xi_{t+1} = \xi_t + \frac{1}{t+1} R_{t+1}^{-1} z_t(y_t - \xi_t z_t) \]
\[ R_{t+1} = R_t + \frac{1}{t+1}(z_t z_t' - R_t) \]

Comments

• Microfoundations of model
• Focus on optimal targeting rule rather than instrument rule
• Alternative reaction functions

Question: Is REE learnable (does E_t^* \to E_t when t \to \infty) under alternative assumptions about monetary-policy implementation?

Results

- Depends on the monetary-policy implementation and parameters (and private-sector information: lagged/current variables)
- Learnability if monetary-policy implementation takes E_t^* into account (expectations-based reaction function) to achieve optimal targeting rule (8)
• Microfoundations of model?
  
  - Distinction aggregate/individual?
  
  - Individual information? Includes representative agent, aggregate equilibrium?
  
  - Preston 2002a,b:
    
    * Individual less information
    * Consumption plans rely on PLM w/ individual budget constraint rather than w/ aggregate equilibrium/representative agent
    * Not use law of iterated expectations for aggregate expectations
    
    \[ E_t^* \equiv \int E_{jt}^* d\eta \]
    
    * Different results

• Alternative reaction functions
  
  - McCallum-Nelson
    
    \[ i_t = \pi_t + \theta[\pi_t + \frac{\alpha}{\lambda}(x_t - x_{t-1})] \]
    
    Very large \( \theta \) to achieve optimal targeting rule

  - Svensson-Woodford “Implementing Optimal Policy...”:
    
    Dangerous, observation/estimation errors

  - Svensson-Woodford (w/ current \( x_t, \pi_t \) predetermined)
    
    \[ i_{t+1,t} = \bar{\pi}_{t+1,t} + \theta[\pi_{t+1,t} + \frac{\alpha}{\lambda}(x_{t+1,t} - x_{t+1,t-1})] \]
    \[ i_{t+1,t} \equiv \psi_x x_{t+1,t} + \psi_{y_g} g_{t+1,t} + \psi_u u_{t+1,t} \]
    
    Implements \( \pi_{t+1,t} + \frac{\alpha}{\lambda}(x_{t+1,t} - x_{t+1,t-1}) = 0 \)
    
    Out-of-equilibrium commitment achieves determinacy
    
    \( \theta > \bar{\theta} \) implies determinacy

• Focus on optimal targeting rule rather than instrument rule
  
  - Commitment to optimal targeting rule rather than to particular reaction function
    
    * CB transparency, announcements: Influence private-sector expectations directly
    
    - Learning: Combine (19) with (3) and (8)
    
    * Cross-equation restriction on \( E_t^* \)?
    
    * Same as CB using \( E_t^* \) to implement (8)
    
    * Real-world CBs take actual private-sector expectations into account
      
      - Extract private-sector expectations
      
      - “Credibility”