Cost-Benefit Analysis of Leaning Against the Wind

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Abstract

A simple and transparent framework for cost-benefit analysis of “leaning against the wind” (LAW), that is, tighter monetary policy for financial-stability purposes, is presented. LAW has an obvious cost in the form of a weaker economy if no crisis occurs and possible benefits in the form of a lower probability and smaller magnitude of (financial) crises. A second cost—less obvious, overlooked by previous literature, but higher—is a weaker economy if a crisis occurs. For representative empirical benchmark estimates and reasonable assumptions, the result is that the costs of LAW exceed the benefits by a substantial margin. The result is robust to alternative assumptions and estimates. A higher probability, larger magnitude, or longer duration of crises—typical consequences of ineffective macroprudential policy—all increase the margin of costs over benefits. To overturn the result, policy-interest-rate effects on the probability and magnitude of crises need to be more than 5–40 standard errors larger than the benchmark estimates.

JEL Codes: E52, E58, G01

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1 Introduction

“Leaning against the wind” (of asset prices and credit booms) (LAW for short) refers to a monetary policy that is somewhat tighter (that is, with a somewhat higher policy interest rate) than what is consistent with flexible inflation targeting without taking any effects on financial stability into account. LAW has obvious costs in terms of a weaker economy with higher unemployment and lower inflation. It has been justified by possible benefits in the form of a lower probability or smaller magnitude of future (financial) crises (BIS (2014, 2016a), Olsen (2015), Sveriges Riksbank (2013)), although, strikingly, without the support of a credible numerical cost-benefit analysis (Allen, Bean, and Gregorio (2016)). This paper provides a framework for such an analysis and benchmark numerical estimates of the costs and benefits of LAW. The result is that the costs exceed the benefits by a substantial margin. Extensive robustness tests indicate that this result is quite robust. For example, to overturn the result, the effects of LAW on the probability or magnitude of a crisis need to be more than 5–40 standard errors larger than typical empirical estimates in the literature.

Regarding the costs of LAW, one obvious cost is a weaker economy if no crisis occurs. But there is a second, less obvious but higher cost, overlooked by the previous literature but taken into account in this paper. If a crisis occurs when the economy is weaker because of LAW, for a given magnitude of a crisis the economy will be weaker also in the crisis. Thus, for a given magnitude of a crisis the crisis loss level and the cost of a crisis will be higher with LAW than without.

Consider a simple example in terms of unemployment gaps: First, suppose that without LAW the non-crisis unemployment gap is zero. Suppose that a crisis increases the unemployment gap by 5 percentage points (pp). Then the crisis unemployment gap is $0 + 5 = 5$ pp. With a quadratic loss function the non-crisis loss is $0^2 = 0$, and the crisis loss is $5^2 = 25$. The cost of a crisis, defined as the crisis loss less the non-crisis loss, is $25 - 0 = 25$.

Next, suppose that with LAW the non-crisis unemployment gap is 0.5 pp instead of 0. Then the crisis unemployment gap is $0.5 + 5 = 5.5$ pp instead of 5. The non-crisis loss is $0.5^2 = 0.25$ instead of 0, and the crisis loss is $5.5^2 = 30.25$ instead of 25. Then there are two costs of LAW. The first is the non-crisis loss increase, $0.25 - 0 = 0.25$. The second is the crisis loss increase, $30.25 - 25 = 5.25$. It is the main cost of LAW. Furthermore, the cost of a crisis (the crisis loss less the non-crisis loss) is $30.25 - 0.25 = 30$ instead of 25. Importantly, the cost of a crisis is higher with LAW than without. In this example, the first cost of LAW is of the second order, whereas
the second cost is of the first order. It follows that, for a zero non-crisis unemployment gap, the marginal cost is zero for the first cost of LAW but positive for the second cost. Overlooking the second cost misses that the marginal cost of LAW is positive, not zero.

Regarding the possible benefits of LAW (lower probability and smaller magnitude of crises), recent work has emphasized the role of credit growth and credit booms in predicting crises and their magnitude (Borio and Drehmann (2009), Gourinchas and Obstfeld (2012), Schularick and Taylor (2012)). A possible channel for the benefits of LAW is then through an effect of the policy (interest) rate on credit combined with an effect of credit on the probability and magnitude of a crisis. I will use representative empirical estimates surveyed in IMF (2015) of these effects to provide numerical estimates of the benefits of LAW, more precisely Sveriges Riksbank (2014a) estimates of the policy-rate effect on debt, Schularick and Taylor (2012) (ST for short) estimates of effect of debt on the probability of a crisis, and Flodén (2014) and Jordà, Schularick, and Taylor (2013) estimates of the effect of debt on the magnitude of a crisis.

There are, however, at least three limitations of this channel for the benefits of LAW. First, ST show that the probability of a crisis is correlated with lagged growth of real debt or of debt to GDP. But if monetary policy is neutral and has no real effects in the long run, it does not affect the level of real debt or debt to GDP in the long run. Then, if LAW results in lower real debt growth and probability of a crisis, this will later on be followed by higher real debt growth and probability of a crisis; the probability is just shifted between periods. The cumulative policy-rate effect on the probability of a crisis and the cumulative marginal benefit from a lower probability of a crisis will then be small or zero. If instead monetary policy is non-neutral and the policy-rate has a permanent negative effect on real debt, the cumulative policy-rate effect on the probability of a crisis will be negative and the cumulative marginal benefit will be positive, stacking the cards in favor of LAW. Including this case is an obvious robustness test, but as shown below, also for monetary non-neutrality the result is that the costs of LAW exceed the benefits by a substantial margin.

Second, as discussed in Svensson (2013a), the policy-rate effect on real debt and debt to GDP is likely to be small and could be of either sign. The stock of nominal debt, especially mortgages with long maturities, has considerable inertia. A higher policy rate may slow down the growth of housing prices and of new mortgages, but only a fraction of the stock of mortgages is turned over or each year. A higher policy rate also slows down the growth of the price level. Thus, both numerator and denominator of real debt are affected in the same direction, making the policy-rate
effect on the ratio smaller and possibly of the opposite sign. This is even more the case for the
debt-to-GDP ratio (a stock divided by a flow) because then not only the price level but also real
GDP enter in the denominator. The policy-rate effect on the flow of nominal GDP may be larger
and quicker than the effect on the stock of nominal debt. Several recent papers have indeed found
empirical support for a higher policy rate increasing rather than decreasing the debt-to-GDP ratio
(Alpanda and Zubairy (2014), Bauer and Granziera (2017), Gelain, Lansing, and Natvik (2015),
and Robstad (2014)). Nevertheless, I will use empirical estimates according to which the policy-rate
has a negative effect on both real debt and debt to GDP, thereby stacking the cards in favor of
LAW.

Third, the correlation between the probability and magnitude of a crisis and previous growth of
real debt or debt to GDP is obviously a reduced-form result. The underlying determinants of the
probability and magnitude of a financial crisis are mainly the nature of the shocks to the financial
system and the resilience of the system. The latter depends in principle on the loss-absorbing
capacity and liquidity of lenders, intermediators, and borrowers and the debt-servicing capacity of
borrowers. The extent to which higher debt growth increases the probability of a crisis depends
on to what extent it is “bad” credit growth due to, for example, lower lending standards, excessive
loan-to-value ratios, and speculation rather than “good” credit growth due to financial deepening,
new technology, new investment opportunities, and other developments that do not weaken but
rather strengthens the financial system. With better data on the underlying determinants of the
nature and magnitude of shocks and the resilience of the system, it should be possible to assess
the probability and magnitude of a crisis with higher precision. However, given the list of possible
underlying determinants, it seems that the policy rate is unlikely to have much systematic impact
on most or any of them, and that micro- and macroprudential policy is much more likely to have
such an impact.\footnote{Bordo and Meissner (2016), in a thorough survey the literature on financial
crises, warns that “it is not at all obvious from the historical record that credit-financed asset price
boom-bust (i.e., what has come to be known as the financial cycle) have always been, or will always
be, the key explanation despite the recent emphasis on that explanation.” IMF (2015) discusses the
transmission channels from the policy rate to the probability of a crisis and documents its complexity,
uncertainty of direction, and variation over time. Dagher, Dell’Ariccia, Laeven, Ratnovski,
and Tong (2016) shows that more but still relatively moderate bank capital relative to risk-weighted
assets could have had a dramatic effect in reducing the frequency of banking crises in the OECD
countries since 1970. Korinek and Simsek (2016) show that macroprudential policies can be quite
effective in dealing with excess household debt and that interest-rate policies are likely to be inferior
in this respect.}

Some of the estimates, in particular of the policy-rate effects on debt growth and debt to GDP,
are imprecise, in the sense of having large standard errors relative to the point estimates and
therefore not being statistically significant. This makes the estimates of costs and benefits of LAW
A two-step strategy is used to handle this. First, I choose benchmark point estimates of the effects that are either representative or tilted in favor of LAW. The result for these benchmark estimates turn out to be that costs exceed benefits by a large margin. Second, I examine how robust this result is to alternative assumptions about the components of the costs and benefits. In particular, I examine how many standard errors larger than the benchmark estimates the effects need to be in order to overturn the result, in the sense of at least reaching break-even, that is, when the costs and benefits of LAW are equal. One might think that the number of standard errors needed would be relatively small, perhaps even less than two, indicating a less than robust result. But the number of standard errors required to reach break-even turns out to be relatively large, ranging from more than 5 to about 40, indicating a quite robust result.

The previous literature trying to quantify the costs and benefits of LAW (including Ajello, Laubach, Lopez-Salido, and Nakata (2016), Svensson (2014, 2015), and IMF (2015)) has mainly considered a two-period setup where a higher policy rate has a cost in the first period in the form of higher unemployment (and lower inflation) and a benefit in the second period in the form of a lower probability of a crisis because of lower debt. By assumption there is no possibility of a crisis in the first period, and by assumption a crisis in the second period would start from a steady state with a zero non-crisis unemployment gap. The crisis unemployment gap and the crisis loss are then assumed to be constant and independent of LAW. This setup disregards that a crisis could come any time while the unemployment is higher due to LAW. It also assumes a fixed rather than endogenous lag between LAW and lower probability of a crisis. Finally, it implicitly assumes that monetary policy is non-neutral and has a permanent effect on real debt.² ³

Importantly, by assuming that the crisis loss is constant and independent of the non-crisis unemployment gap, the previous literature disregards the second cost of LAW, the main cost of LAW. In terms of the example above: Suppose again that with LAW the non-crisis unemployment gap is 0.5 pp. In the present framework, the crisis unemployment gap, the crisis loss, and the crisis loss increase are then, respectively, 5.5 pp, 5.5² = 30.25, and 30.25 − 25 = 5.25. In the previous literature, they are instead always, respectively, 5 pp, 5² = 25, and 25 − 25 = 0, regardless of the non-crisis unemployment gap. This means that in the previous literature the second cost of LAW, the crisis loss increase, will be zero instead of positive. Then, with the first cost being of

² Diaz Kalan, Laséen, Vestin, and Zdzieńicka (2015) instead use a quarterly multi-period model where the probability of a crisis varies over time and the costs and benefits of LAW are cumulated over time, as does this paper. They nevertheless still assume that the crisis unemployment gap and the crisis loss level are constant and independent of LAW.
³ In Svensson (2014, 2015) the loss function used is not quadratic but linear, in the form of expected unemployment.
second order, it follows that, for a zero non-crisis unemployment gap, the marginal cost of LAW will be zero instead of positive. With a positive marginal benefit of LAW, some LAW will then be optimal. But because the marginal costs rises relatively quickly the optimal LAW is quite small, corresponding to a policy-rate increase of only a few basis points (bp), and is thus of no practical relevance, as shown by Ajello et al. (2016) and further examined in Svensson (2017b).

The assumption of a constant crisis loss has the counter-intuitive implication that the cost of a crisis (the crisis loss less the non-crisis loss) is decreasing in the non-crisis loss. In particular, if the non-crisis unemployment gap for some reason would be above 5 pp, the economy would be better off in a crisis, because then the unemployment gap would drop to 5 pp. It trivially follows that the cost of a crisis is lower with LAW than without, because LAW would increase the non-crisis loss without affecting the crisis loss.

More recent literature, responding to the first version of this paper, Svensson (2016a), is discussed in section 5.

In summary, relative to the previous literature this paper takes into account the second cost of LAW, the crisis loss increase. It also clarifies the role of monetary neutrality and non-neutrality. In particular, it provides a simple, transparent, and robust framework for cost-benefit analysis of LAW that only depends on a few assumptions and empirical estimates. It furthermore provides several robustness tests, including some that handle imprecise estimates.

Section 2 lays out the theoretical framework, section 3 specifies benchmark estimates and assumptions, section 4 provides robustness tests, section 5 discusses some recent literature, and section 6 concludes. Appendices A-D contain some technical details. Data and software are available at www.larseosvensson.se.

2 Theoretical framework

This section sets up the theoretical framework used to assess the costs and benefits of LAW. Let $u_t$ denote the unemployment rate in quarter $t \geq 1$, and let $u^*_t$ denote the optimal unemployment rate under flexible inflation targeting when the possibility of a financial crisis is disregarded. By flexible inflation targeting I mean a monetary policy under which the central bank stabilizes both the inflation rate around an inflation target and the unemployment rate around its long-run sustainable rate. Exogenous cost-push shocks to the Phillips curve create a tradeoff between stabilizing the inflation rate and stabilizing the unemployment rate. Then the optimal unemployment rate depends
on the cost-push shocks, is exogenous, and varies over time. It will here be called the benchmark unemployment rate.

Let the unemployment deviation, $\tilde{u}_t$, be defined as the difference between the unemployment rate and the benchmark unemployment rate, $\tilde{u}_t \equiv u_t - u^*_t$. Thus the unemployment deviation is not the deviation from the steady state (or the conventional unemployment gap as in the simple example in section 1) but the deviation from the optimal policy under flexible inflation targeting when the probability of a crisis is set to zero. As explained in appendix A, the loss from the unemployment rate deviating from the benchmark unemployment rate can be represented by the simple quadratic (indirect) loss function,

$$L_t = (\tilde{u}_t)^2,$$

where $L_t$ denotes the quarter-$t$ (simple) loss. The quarter-1 intertemporal loss function for monetary policy is then

$$L_1 = E_1 \sum_{t=1}^{\infty} \delta^{t-1} L_t = \sum_{t=1}^{\infty} \delta^{t-1} E_1 L_t,$$

where $E_1$ denotes expectations conditional on information available in quarter 1, $\delta \in (0, 1)$ denotes a discount factor, and $E_1 L_t$ denotes the expected quarter-$t$ loss for $t \geq 1$.

The expected quarter-$t$ loss will be given by

$$E_1 L_t = E_1 (\tilde{u}_t)^2 = (1 - p_t) E_1 (\tilde{u}_n^t)^2 + p_t E_1 (\tilde{u}_c^t)^2 = (1 - p_t) E_1 (\tilde{u}_n^t)^2 + p_t E_1 (\tilde{u}_n^t + \Delta u_t)^2.$$

Here it is assumed that in quarter $t \geq 2$ there can be either of two states of the world, namely either a non-crisis or a (financial) crisis, denoted $n$ and $c$, respectively. By assumption, there is no crisis in quarter 1. Furthermore, $p_t$ denotes the probability of (having) a crisis in quarter $t$, conditional on information available in quarter 1. The variable $\tilde{u}_n^t$ denotes the quarter-$t$ non-crisis unemployment deviation, that is, the unemployment deviation if there is no crisis in the quarter. Then the first term of the right side of (3) is the probability of no crisis, $1 - p_t$, times the expected non-crisis loss, $E_1 L_n^t = E_1 (\tilde{u}_n^t)^2$, that is, the expected loss if there is no crisis in quarter $t$.

The second term on the right side of (3) is the probability of a crisis times the expected crisis loss, $E_1 L_c^t = E_1 (\tilde{u}_c^t)^2$, that is, the expected loss if there is a crisis in quarter $t$. A crisis is assumed to be associated with a (possibly random) crisis increase in the unemployment rate, $\Delta u_t > 0$, so the crisis unemployment deviation is $\tilde{u}_c^t = \tilde{u}_n^t + \Delta u_t$, and the crisis loss is $L_c^t = (\tilde{u}_c^t)^2 = (\tilde{u}_n^t + \Delta u_t)^2$. This crisis increase in the unemployment rate is net of any policy response during a crisis. Thus,

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4 The “true” indirect loss function is then an affine function of the simple loss function.
Δut can be interpreted as the unemployment-rate increase that is equivalent to the combination of a demand shock and any shock to the transmission mechanism of monetary policy associated with a crisis, net of the conventional and unconventional policy response at a crisis, including any restriction on the policy response such as the lower bound of the policy rate. It represents the magnitude of a crisis.

Equation (3) can be written as

\[ E_1 L_t = E_1 (\tilde{u}_t^n)^2 + p_t [E_1 (\tilde{u}_t^n + \Delta u_t)^2 - E_1 (\tilde{u}_t^n)^2] = E_1 (\tilde{u}_t^n)^2 + p_t [E_1 (\Delta u_t)^2 + 2E_1 \tilde{u}_t^n E_1 \Delta u_t]. \]  

Here, the expression in square brackets in (4) is the (expected) cost of a crisis, defined as the (expected) crisis loss less the (expected) noncrisis loss. We see in the square brackets on the right side of (4) that the (expected) cost of a crisis is increasing in the (expected) non-crisis unemployment deviation, \( E_1 \tilde{u}_t^n \). (For brevity and when no confusion need arise, “expected” will often be left out but understood in the rest of the paper.)

As explained in section 1, a zero non-crisis unemployment deviation corresponds to the optimal policy under flexible inflation targeting when the possibility of a crisis is disregarded, that is, when the probability of a crisis is set to zero. This can be seen as a policy of no leaning (NL for short). A positive non-crisis unemployment deviation corresponds to tighter policy than NL and can thus be seen as representing LAW. A negative non-crisis unemployment deviation corresponds to easier policy than NL and can be seen as representing leaning with the wind (LWW for short).

Consider the effect on the intertemporal loss (2) of a policy tightening in the form of an increase in the policy rate during quarters 1–4, denoted \( \bar{d}_1 > 0 \). The cumulative net marginal cost of LAW, NMC, is defined as the derivative of the intertemporal loss with respect to the policy rate during quarters 1–4, \( \text{NMC} \equiv \frac{dL_t}{d\bar{d}_1} = \frac{L_t}{\bar{d}_1} = \sum_{t=1}^{\infty} \delta^{t-1}L_t = \sum_{t=1}^{\infty} \delta^{t-1}dE_1 L_t/d\bar{d}_1 \).

Furthermore, define the quarter-\( t \) net marginal cost, \( \text{NMC}_t \), as \( dE_1 L_t/d\bar{d}_1 \), the policy-rate effect on the quarter-\( t \) expected loss. Taking the derivative of the right side of (4) with respect to \( \bar{d}_1 \) gives

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5 The benchmark assumption is for simplicity that the benchmark unemployment rate, \( u_t^* \), is independent of whether there is a crisis or not, in which case \( \Delta u_t \) is the crisis increase in the unemployment rate. As explained in appendix B, if the benchmark unemployment rate is correlated with a crisis, the crisis increase in the unemployment rate and in the unemployment deviation are not the same. Then \( \Delta u_t \) is the crisis increase in the unemployment deviation and equals the crisis increase in the unemployment rate less the crisis increase in the benchmark unemployment rate. Because the benchmark unemployment rate is increasing in cost-push shocks to the Phillips curve and such cost-push shocks are likely to be negative in a crisis, any crisis increase in the benchmark unemployment rate is likely to be negative.

6 I assume that \( E_1 (\tilde{u}_t^n \Delta u_t) = E_1 \tilde{u}_t^n E_1 \Delta u_t \), that is, \( \tilde{u}_t^n \) and \( \Delta u_t \) are uncorrelated conditional on information available in quarter 1.
**NMC_t** \(\equiv MC_t - MB_t\), where

\[
MC_t \equiv 2 (E_1 \tilde{u}_t^n + p_t E_1 \Delta u_t) \frac{dE_1 u^n_t}{dt} = 2 E_1 \tilde{u}_t \frac{dE_1 u^n_t}{dt} \bigg|_{p_t, E_1 \Delta u_t, \text{const.}},
\]

(5)

\[
MB^p_t \equiv [E_1(\Delta u_t)^2 + 2E_1 \tilde{u}_t^n E_1 \Delta u_t] (\frac{dp_t}{dt}),
\]

(6)

\[
MB^{\Delta u}_t \equiv 2p_t E_1(\tilde{u}_t^n + \Delta u_t) (\frac{dE_1 \Delta u_t}{dt}) = 2p_t E_1 \tilde{u}_t^n (\frac{dE_1 \Delta u_t}{dt}),
\]

(7)

\[
MB_t \equiv MB^p_t + MB^{\Delta u}_t.
\]

(8)

Here **MC_t** denotes the *marginal cost of LAW*. It consists of the marginal increase in the expected quarter-\(t\) loss from an increase in the unemployment deviation at constant probability and magnitude of a crisis. By (5) it equals two times the unemployment deviation \((E_1 \tilde{u}_t = E_1 \tilde{u}_t^n + p_t E_1 \Delta u_t)\) times the policy-rate effect on the non-crisis unemployment rate, which equals the policy-rate effect on the unemployment deviation for constant probability and magnitude of a crisis \((dE_1[\tilde{u}_t|p_t, E_1 \Delta u_t \text{ const.}]/dt = dE_1 \tilde{u}_t^n/d\tilde{t}_1 = dE_1 d\tilde{u}_t^n/d\tilde{t}_1)\).

Furthermore, **MB^p_t** denotes the *marginal benefit of LAW from a lower probability of a crisis*. It consists of the marginal reduction of the expected quarter-\(t\) loss from a lower probability of a crisis at a constant non-crisis unemployment deviation and a constant magnitude of a crisis. By (6) it equals the loss increase in a crisis (the cost of a crisis) times the negative of the policy-rate effect on the probability of a crisis.

Similarly, **MB^{\Delta u}_t** denotes the *marginal benefit of LAW from a smaller magnitude of a crisis*. It consists of the marginal reduction of the expected quarter-\(t\) loss from a smaller magnitude of a crisis at constant probability of a crisis and constant non-crisis unemployment deviation. By (7) it equals two times the probability of a crisis times the crisis unemployment deviation times the negative of the policy-rate effect on the magnitude of the crisis. **MB_t** denotes the total marginal benefit, the sum of the two components.

For a zero (expected) non-crisis unemployment deviation \((E_1 \tilde{u}_t^n = 0)\), corresponding to **NL**, the marginal cost and the two marginal benefits are given by

\[
MC_t = 2p_t E_1 \Delta u_t \frac{dE_1 u^n_t}{dt},
\]

(9)

\[
MB^p_t = E_1(\Delta u_t)^2 (\frac{dp_t}{dt}),
\]

(10)

\[
MB^{\Delta u}_t = 2p_t E_1 \Delta u_t (\frac{dE_1 \Delta u_t}{dt}),
\]

(11)

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7 I assume that \(E_1(\tilde{u}_t d\tilde{u}_t^n/d\tilde{t}_1) = E_1 \tilde{u}_t dE_1 u^n_t/d\tilde{t}_1\), that is, that \(\tilde{u}_t\) and \(d\tilde{u}_t^n/d\tilde{t}_1\) are uncorrelated conditional on information available in quarter 1. Furthermore, I have used that, because \(\tilde{u}_t^n\) depends on cost-push shocks and is exogenous, \(dE_1 \tilde{u}_t^n/d\tilde{t}_1 = dE_1 (u^n_t - u_t)/d\tilde{t}_1 = dE_1 u_t^n/d\tilde{t}_1\).

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8
In order to assess whether or not the costs of LAW exceed the benefits when all quarters are considered, we then look at the sign of the cumulative discounted net marginal cost,

\[
\text{NMC} = \sum_{t=1}^{\infty} \delta^{t-1} \text{NMC}_t = \sum_{t=1}^{\infty} \delta^{t-1} \text{MC}_t - \sum_{t=1}^{\infty} \delta^{t-1} \text{MB}_t \geq 0,
\]

where \( \text{MC}_t \) is given by (9) and \( \text{MB}_t \) by (8), (10), and (11).

### 2.1 The case of exogenous probability and magnitude of a crisis: LWW!

Assume now temporariliy that the probability and magnitude of a crisis are exogenous, in the sense that they cannot be affected directly or indirectly by the policy rate and LAW. That is, \( dp_t/d\delta_1 = dE_1 \Delta u_t / d\delta_1 = 0 \) for all \( t \geq 1 \). It then follows from (6) and (7) that \( \text{MB}_t^p = \text{MB}_t^{\Delta u_t} = 0 \) for all \( t \geq 1 \); there are no marginal benefits of LAW, only the marginal cost, given by (5).

The optimal policy would then be, if possible, to set each expected future unemployment deviation equal to zero, \( E_1 \tilde{u}_t = E_1 u_t^\pi + p_t E_1 \Delta u_t = 0 \), which by (5) would make each future marginal cost equal to zero and minimize each quarter-expected loss. But, for a positive future probability of a crisis, \( p_t > 0 \), this implies setting the expected future non-crisis unemployment deviation negative,

\[
E_1 u_t^\pi = -p_t E_1 \Delta u_t < 0.
\]

That is, if the probability and the magnitude of a crisis are exogenous, it is optimal to lean with the wind, LWW, in the sense of setting a lower unemployment rate than the rate that is optimal if the probability of a crisis is set to zero. If the probability and magnitude of a crisis are not exogenous but can be affected by LAW, the issue is then whether any resulting marginal benefits of lower probability or smaller magnitude of a crisis can be so large as to dominate over the marginal cost and make LAW optimal instead of LWW.

### 3 Benchmark numbers

In order to assess whether or not the costs of LAW exceed the benefits, we need numerical estimates of or assumptions about the components of the marginal cost and benefits in (9)-(11). I will use estimates and assumptions that I consider either representative or tilted in favor of LAW. Because of the availability of detail, some of the estimates used are for Sweden by the Riksbank, but they should not be considered “Sweden only.” As is explained below, they are either in line with estimates for other countries surveyed in IMF (2015) or tilted in favor of LAW.
3.1 The marginal cost of LAW

For a numerical estimate of the marginal cost of LAW, by (9) we need representative estimates of or realistic assumptions about the probability of a crisis, the (expected) magnitude of a crisis, and the policy-rate effect on the (expected) non-crisis unemployment rate.

As a representative benchmark policy-rate effect on the (expected) non-crisis unemployment rate, \( dE_1u^n_t/d\hat{\delta}_1 \), I will use the impulse response of the Riksbank’s empirical DSGE model Ramses to a 1 pp higher policy rate during quarters 1–4, shown as the dashed red line in the lower right part of figure 1.\(^8\) The unemployment rate increases above the baseline to about 0.5 pp in quarters 6–8 and then slowly falls back towards the baseline. For an initial zero non-crisis unemployment deviation, the dashed red line then also shows the policy-rate effect on the non-crisis unemployment deviation.

This impulse response has a typical and realistic hump-shaped form. Because the economy responds with a lag to policy-rate changes, the initial effect is approximately zero and the maximum effect is reached after 6–8 quarters. It is similar in shape and magnitude to the impulse response reported by IMF (2015, para. 40 and footnote 42) for its widely used GIMF model for an average of a large, mostly closed economy and a small open economy. It is also similar in shape but larger in magnitude than the impulse response of the unemployment rate for the U.S. reported in Stock and Watson (2001, figure 1).\(^9\) The result for a smaller policy-rate effect is examined in section 4.2.

For the benchmark (expected) crisis increase in the unemployment rate, \( E_1\Delta u_t \), representing the magnitude of a crisis, I will for simplicity use the same assumption as in a crisis scenarios discussed in IMF (2015, para. 41) and in Sveriges Riksbank (2013), that the benchmark crisis increase in the unemployment rate is deterministic, constant, and equal to 5 pp. The result for a larger magnitude

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\(^8\) The figure shows the impulse response for 40 quarters of the unemployment rate that was reported by Riksbank Deputy Governor Karolina Ekholm in Ekholm (2013). It is the same response as the one reported to alternative policy-rate paths for quarters 1–12 in Sveriges Riksbank (2014b).

\(^9\) It is natural that the policy-rate effect on the unemployment rate is larger in a small open economy, with a strong exchange-rate channel in the transmission of monetary policy, than in a large relatively closed economy like the U.S. Stock and Watson’s impulse response peaks at 0.2 pp at around 8 quarters, but the cumulative interest-rate impulse is smaller than in figure 1. (In figure 1 there is an interest-rate impulse in all four quarters 1–4, in order to hold the policy rate 1 pp higher than the baseline for the four quarters.) With a larger and similar cumulative impulse response, Stock and Watson’s peak might be about 0.25–0.3 pp rather than 0.2.
of crisis is examined in section 4.4.

For a zero non-crisis unemployment deviation, if a crisis occurs, the unemployment deviation would then increase to a crisis unemployment deviation of 5 pp, illustrated by the solid black horizontal line in the upper right part of figure 1. Furthermore, with LAW and a positive non-crisis unemployment deviation, if a crisis occurs, the unemployment deviation would increase above 5 pp, as shown by the thick solid red line in the figure. Importantly, LAW leads to a higher crisis unemployment deviation, not only to a higher non-crisis unemployment deviation.

The dashed and solid blue lines in the left part of the figure show the loss and marginal loss (with respect to the unemployment deviation), \( L_t = (\tilde{u}_t)^2 \) and \( ML_t \equiv dL_t/d\tilde{u}_t = 2 \tilde{u}_t \), respectively, measured along the horizontal line to the left of the origin. For a zero non-crisis unemployment deviation, the marginal loss of increasing the non-crisis unemployment deviation is zero. At a positive crisis unemployment deviation equal to \( \Delta u = 5 \) pp, the marginal loss of increasing the crisis unemployment deviation is not zero but positive, \( ML^c = 2\Delta u = 10 \), as shown in the figure. It follows that the marginal crisis loss with respect an increase in the policy rate, \( dL^c/d\bar{i}_1 = ML^c dE_1u^c_t/d\bar{i}_1 = 2\Delta u dE_1u^n_t/d\bar{i}_1 \) is positive. When multiplied by the probability of a crisis, it equals the marginal cost of LAW in (9).

It remains to specify the benchmark probability of a crisis, \( p_t \). I will assume that there is a benchmark annual probability of a crisis start equal to 3.2%, corresponding to a crisis on average every 31 years. Let \( q_t \) denote the (quarterly) probability of crisis start in quarter \( t \). The benchmark (quarterly) probability of a crisis start is thus \( q_t = 3.2/4 = 0.8\% \).

Let \( n \) denote the crisis duration measured in quarters. The benchmark crisis duration is assumed to be \( n = 8 \) quarters. Combined with the benchmark crisis increase in the unemployment rate, a benchmark crisis then corresponds to a substantial 10 point-years of excess unemployment. The result for a longer crisis duration is examined in section 4.5.

In figure 2, the thin blue and thick green lines show the benchmark probability of a crisis start and benchmark probability of a crisis in future quarters, conditional on no crisis in quarter 1. The probability of a crisis is equal to the probability of a crisis starting in the last \( n \) quarters.
This is close to a simple linear approximation used in Svensson (2016a) (equal to the sum of the probabilities of a crisis start over the last \( n \) quarters, \( p_t \approx \sum_{\tau=0}^{n-1} q_{t-\tau} \)). As explained in appendix C, the exact probability of a crisis, which is used in this paper, is given by a Markov process. As shown in figure 2, it rises to 6.25% in quarter 9 and then falls back to a steady-state level of 6% from quarter 14.\(^\text{10}\) The result for a higher probability of a crisis start and a crisis is examined in section 4.3.

In section 2.1, the case of exogenous probability and magnitude of a crisis, we noted that the optimal policy is LWW, that is, to set each future expected non-crisis unemployment deviation negative according to (13). For the benchmark steady-state probability of 6% and crisis increase in the unemployment rate of 5 pp, the optimal non-crisis unemployment deviation is \(-30\) bp, arguably too small to have any practical importance. For LAW to be optimal, when the probability and magnitude are exogenous, this tendency towards LWW has to be overcome by sufficiently large marginal benefits of LAW.

All the components of the benchmark marginal cost (9) have now been specified. It is thus given by \( MC_t = 2p_t \Delta u dE_1 u_t^i / d\tilde{d}_1 = 10 p_t dE_1 u_t^i / d\tilde{d}_1 \), where \( p_t \) and \( dE_1 u_t^i / d\tilde{d}_1 \) are shown in figures 2 and 1, respectively. It is shown as the thin solid red line in figure 5.

3.2 The marginal benefit from a lower probability of a crisis

For the benchmark marginal benefit from a lower probability of a crisis (10), we need an estimate of the policy-rate effect on the probability of a crisis, \( dp_t / d\tilde{d}_1 \), which in turn depends on the policy-rate effect on the probability of a crisis start, \( dq_t / d\tilde{d}_1 \), as shown in appendix C. For a benchmark estimate of the latter, I will combine a benchmark estimate of the effect of debt on the probability of a crisis start with a benchmark estimate of the effect of debt on the policy-rate effect on debt.

First, regarding the effect of debt on the probability of a crisis start, in a seminal paper, ST use annual data for 14 developed countries for 1870–2008 and find that financial crises are predicted by real debt growth lagged 1–5 years. During the work on IMF (2015), IMF staff used a quarterly dataset of Laeven and Valencia (2012) for 35 advanced countries and a more recent sample period, 1970–2011, to predict financial crises. As shown in detail in Svensson (2016a, section 7), the IMF estimates lead to very similar results as the ST estimates.\(^\text{11}\)

\(^{10}\) The linear approximation rises to a steady-state level of 6.4% in quarter 9, see figure C.1. For the Markov process, it is assumed that a new crisis does not start during an existing crisis, which in steady state reduces the probability of a crisis somewhat from the linear approximation.

\(^{11}\) I am grateful to Damiano Sandri for help with and several discussions about the IMF estimates.
As a benchmark, I will thus use the estimates of ST (table 3, specification 5), in a quarterly variant of their main logit regression,

\[ q_t = \frac{1}{4} \frac{\exp(X_t)}{1 + \exp(X_t)}, \]  

(14)

\[ X_t = -3.89 - 0.398 g_{t-4} + 7.138^{***} g_{t-8} + 0.888 g_{t-12} + 0.203 g_{t-16} + 1.867 g_{t-20}. \]  

(15)

Here numbers within parenthesis are robust standard errors\(^{12}\) and \(g_t\) denotes the annual growth rate of (average annual) real debt, defined as \(g_t \equiv \log(\sum_{\tau=0}^{3} d_{t-\tau}/4) - \log(\sum_{\tau=0}^{3} d_{t-4-\tau}/4)\), where \(d_t\) denotes the level of real debt in quarter \(t\).\(^{13}\) The coefficients of the five lags are jointly significant at the 1% level. However, annual real debt growth lagged 2 years, \(g_{t-8}\), has the largest and most significant coefficient in (15) and is thus the major determinant of the probability of a crisis start.

Second, regarding the policy-rate effect on debt, IMF (2015, para. 24 and footnote 19) summarizes the estimates of this effect in several papers. It notes that “real debt levels generally decrease following a temporary monetary policy tightening of 100 basis points, by up to 0.3% and 2%, after 4 to 16 quarters, depending on the model” and that “Sveriges Riksbank (2014a) comes to a middle-of-the-road result.”

As a benchmark, I will therefore use the Sveriges Riksbank (2014a) estimate of the policy-rate effect on real (household) debt from a 1 pp higher policy rate during 4 quarters. It is shown as the red line in figure 3.\(^{14}\) Real debt falls by 1% in two years relative to the baseline and then rises back to the baseline again in about 8 years. Because monetary policy is neutral, there is no long-run effect on real debt. The result for monetary non-neutrality and a permanent effect on real debt will be examined in section 4.1.

Third, in order to find the policy-rate effect on the probability of a crisis start and of a crisis, we note that the yellow line in figure 3 shows the resulting policy-rate effect on \(q_t\), the annual growth rate of (average annual) real debt. Because the real debt level first falls and then rises back to the baseline, real debt growth will first also fall below the baseline but then rise above the baseline. Importantly, because there is monetary neutrality and (after about 40 quarters) no policy-rate effect on real debt, there is no policy-rate effect on the cumulative growth rate.

The blue line in the figure shows the resulting policy-rate effect on the probability of a crisis start, \(dq_t/\bar{d}_1\), that follows from (14) and (15). Recall that real debt growth lagged two years is the

\(^{12}\) One, two, and three stars denote significance at the 10, 5, and 1% level, respectively.

\(^{13}\) What is called real debt here is in ST total bank loans, defined as the end-of-year amount of outstanding domestic currency lending by domestic banks to domestic households and nonfinancial corporations (excluding lending within the financial system) deflated by the CPI.

\(^{14}\) The ST estimates refer to loans to both households and nonfinancial corporations, whereas the estimates in Sveriges Riksbank (2014a) refer to loans to households only. I assume that this difference does not affect the result.
main determinant of the probability of a crisis start. The probability of a crisis start falls below the baseline to a minimum of $-0.04$ pp in quarters 13 and 14, about two years after the minimum of real debt growth rate in quarter 6. Furthermore, because real debt growth rises above the baseline after quarter 11, the probability of a crisis start will rise above the baseline after quarter 19 (barely visible in the figure).

The thick green line in the figure shows the policy-rate effect on the probability of a crisis, $dp_t/di_1$. Because it is close to the linear approximation mentioned above, it is approximately an 8-quarter moving sum of the policy-rate effect on the probability of a crisis start. It has a minimum of $-0.20$ pp in quarter 17 and then rises back to zero and turns positive after quarter 23. It will fall to zero at about quarter 60.15

The logistic function is convex but close to linear for the relevant range of real debt growth rates (figure 9). Then the probability of a crisis start and of a crisis are approximately linear in real debt growth. It follows that monetary neutrality and no permanent effect on real debt in this case implies that the cumulative policy-rate effects of the probability of a crisis start and the probability of a crisis are approximately zero, $\sum_{t=1}^{60} dp_t/di_1 \approx \sum_{t=1}^{60} dq_t/di_1 \approx 0$. Thus, a higher policy rate reduces the probability of a crisis somewhat in 4–5 years and increases it after about 6 years, but with an approximately zero cumulative effect over about 60 quarters.

Finally, we now have all the components of the marginal benefit from a lower probability of a crisis (10). It is given by $MB^p_t = (\Delta u)^2 (dp_t/di_t) = 25 (dp_t/di_t)$, and is shown as the thin solid green line in figure 5. In this case, monetary neutrality implies that the cumulative marginal benefit from a lower probability of a crisis is approximately zero. As mentioned, section 4.1 examines the result for monetary non-neutrality and a permanent effect on real debt, for which the cumulative marginal benefit from a lower probability of a crisis will be positive.

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15 Because real debt growth lagged 5 years has a relatively large coefficient coefficient in (15), for $n = 8$ the probability of a crisis is affected by real debt growth lagged up to 7 years.
3.3 The marginal benefit from a smaller magnitude of a crisis

In order to find a numerical estimate of the marginal benefit from a smaller magnitude of a crisis (11), we need to find an estimate of the policy-rate effect on the magnitude of a crisis. A possible channel is the effect of debt on the magnitude of a crisis combined with the policy-rate effect on debt.

First, regarding the effect of debt on the magnitude of a crisis, for the OECD countries Flodén (2014) finds that a lower household debt-to-income ratio in 2007 is associated with a lower increase in the unemployment rate during 2007–2012. More precisely, a 1 pp lower debt-to-income ratio is associated with a 0.02 pp smaller increase in the unemployment rate. Flodén’s estimate is similar to other estimates. Given this, I will use Flodén’s estimate as a representative benchmark.

Jordà et al. (2013, table 8) (JST for short), with a dataset for 14 advanced countries 1870–2008, report the effect on the GDP downturn in a financial recession of a 1 pp higher “excess credit.” Here, excess credit denotes the yearly pp excess rate of change of aggregated bank loans relative to GDP in the preceding expansion phase (previous trough to peak, where excess is determined relative to the previous mean). The average effect on GDP over 5 years is $-0.8$ pp (the average of the coefficients in JST’s table 8). Assuming an Okun coefficient of 2, this means an average increase in the unemployment rate of 0.4 pp. Post-WWII, the average duration of an expansion phase is 9.46 years in the sample.\(^1\) A 1 pp higher excess credit over 9.46 years implies that the cumulative bank-loans-to-GDP ratio is about 10% higher ($1.01^{9.46} - 1 = 0.0987$). This means that a 1 percent higher bank-loans-to-GDP ratio is associated with a $0.4/10 = 0.04$ pp larger unemployment increase. If the bank-loans-to-GDP ratio is about 100%, 1% is about 1 pp.\(^1\) Then, 1 pp higher bank loans is associated with about 0.04 pp larger unemployment increase, about twice as large as Flodén’s estimate. Krishnamurthy and Muir (2016, table 4), with a dataset for 14 countries 1869–2014, show that a 1 pp higher 3-year growth in the credit-to-GDP ratio is associated with an (insignificant) 0.05 pp larger GDP decline from peak to trough in a financial crisis. With an Okun coefficient of 2, a 0.05 pp decline in GDP is associated with a 0.025 pp rise in the unemployment rate, an estimate similar to Flodén’s.

Second, regarding the policy-rate effect on debt to income and the magnitude of a crisis, as discussed in section 1, the policy-rate effect on debt to income may be positive rather than negative.

\(^1\) JST (table 3) reports an expansion-phase duration of 6.9 years for 30 observations of “Low excess credit” and a duration of 11.8 years for 32 observations of “High excess credit.” The average, taking the numbers of observations into account, is then 9.46 years.

\(^1\) According to BIS (2016b, table F2.3), for advanced economies, bank loans to the private non-financial sector were 83% of GDP in 2016Q1.
The Sveriges Riksbank (2014a) estimate of the effect on the Swedish household debt-to-income ratio of a 1 pp higher policy rate during quarters 1–4 is negative and shown in figure 4. It falls by 1.4 pp in quarter 4. This corresponds to a fall in debt to GDP of 0.8%. But this policy-rate effect on the debt-to-income ratio is too large to be consistent with the policy-rate effect on real debt in figure 3 and on the unemployment rate in figure 1. For an Okun coefficient of about 2, the policy-rate effect on GDP would be a fall by about 1% in quarter 6–8, about the same as the fall in real debt in figure 3. This implies that the effect on the debt-to-GDP ratio would be close to zero.

In order to stack the cards in favor of LAW, I will nevertheless use the Riksbank estimate as a benchmark policy-rate effect on the debt-to-income ratio, even though it seems unrealistically large. Then the policy-rate effect on the magnitude of a crisis, \(dE_1 \Delta u_t / d\bar{u}_1\), is simply 0.02 times the policy-rate effect on the debt-to-income ratio. It is shown as the dashed black line in figure 4 (measured along the right vertical axis). The maximum fall in the crisis unemployment increase is about 0.03 pp for quarter 4.

The resulting marginal benefit from a smaller magnitude of a crisis (11), is then given by
\[
MB_t^{\Delta u} = 2p_t \Delta u \left( -dE_1 \Delta u_t / d\bar{u}_1 \right) = 10 p_t \left( -dE_1 \Delta u_t / d\bar{u}_1 \right)
\]
and is shown as the thick dashed green line in figure 4 and the thin dashed green line in figure 5.

### 3.4 The net marginal cost

We now have all the components of the net marginal cost. Figure 5 shows the marginal cost (thin solid red), (9); the marginal benefit from a lower probability of a crisis (thin solid green), (10); the marginal benefit from a smaller magnitude of crisis (thin dashed green), (11); and the net marginal cost (thick solid blue), (12), the difference between the red line and the sum of the solid and dashed green lines.

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18 Sveriges Riksbank (2014a) estimates the policy-rate effect on the debt-to-GDP ratio and converts that into the debt-to-(disposable)income ratio by assuming that income is proportional to GDP. The percentage response of the debt-to-GDP ratio is then converted into percentage points by multiplying by an initial household debt-to-income ratio of 1.73. Thus, a fall of 1.4 percentage points is a fall of 1.4/1.73 = 0.8 percent.

19 This is consistent with the small or even positive policy-rate effect on GDP found in Alpanda and Zubairy (2014), Bauer and Granziera (2017), Gelain et al. (2015), Robstad (2014), and Svensson (2013a,b).
Figure 5: The marginal cost, marginal benefits from a lower probability and smaller magnitude of a crisis, and net marginal cost

Figure 6: The cumulative marginal cost, marginal benefits from a lower probability and smaller magnitude of a crisis, and net marginal cost

Figure 6 shows the corresponding (undiscounted) cumulative marginal cost, marginal benefits, and net marginal cost. (Because the main marginal benefit occurs later than the main marginal cost, not discounting stacks the cards in favor of LAW.)\textsuperscript{20} The result that the cumulative marginal cost exceeds the cumulative marginal benefit by a substantial margin is clear.

Preliminarly, we also see an indication that an assumption of monetary non-neutrality and a permanent effect on real debt would not change the result. Even if we disregard the negative marginal benefit and positive marginal cost after quarter 23 by just looking at the cumulative net marginal cost for quarter 23, the cumulative marginal cost still exceeds the marginal benefit by a substantial margin. The role of monetary neutrality and non-neutrality is further examined in section 4.1.

4 Robustness tests

The benchmark estimates and assumptions in the previous sections have been chosen to be representative and realistic or, for the policy-rate effect on debt to income, biased in favor of LAW. In this section, I examine the robustness of the result to monetary non-neutrality and a permanent policy-rate effects on real debt; a smaller policy-rate effect on unemployment; a credit boom and a higher probability of a crisis start; a larger crisis magnitude; a longer crisis duration; inefficient macroprudential policy; larger policy-rate effects on the probability and magnitude of a crisis, in-

\textsuperscript{20} The undiscounted cumulative discounted net marginal cost is the limit of (12) for $\delta \to 1-$. 

4.1 Non-neutral monetary policy and a permanent effect on real debt

Assume that there is monetary non-neutrality in the form of a permanent effect on real debt. More precisely, assume that real debt permanently stays down at its maximum deviation from the baseline in figure 3 (−1.03%) from quarter 8 onwards, as shown in figure 7. Then there is a persistent, but not permanent, fall in the probability of a crisis. As shown in figure 8, the resulting marginal benefit is larger, more persistent, and never negative. Nevertheless, it is clear that this marginal benefit is not sufficient to prevent the cumulative marginal cost from exceeding the cumulative marginal benefit by a substantial margin. The result is thus robust to monetary non-neutrality and a permanent policy-rate effect on real debt.

4.2 A smaller policy-rate effect on the non-crisis unemployment rate

A smaller policy-rate effect on the non-crisis unemployment rate will reduce the cumulative marginal cost and make it exceed the cumulative marginal benefit by less, or even fall short of the cumulative marginal benefit. Suppose that the policy-rate effect is only half the benchmark effect. That is, the policy-rate effect is assumed to be substantially smaller than the one reported in IMF (2015,
para. 40 and footnote 42) for its GIMF model and somewhat smaller than the impulse response of the U.S. unemployment rate estimated in Stock and Watson (2001, figure 1) (when adjusted to a similar cumulative policy-rate impulse as discussed in footnote 9). This would result in half the marginal cost in figure 5 and half the cumulative marginal cost in figure 6. Clearly, the cumulative marginal cost would still exceed the cumulative marginal benefit by a substantial margin.

Furthermore, if monetary non-neutrality and a permanent policy-rate effect on real debt is assumed as in figure 7, it is clear from figure 8 that half the benchmark policy-rate effect on the non-crisis unemployment rate would also in that case make the cumulative marginal cost exceed the cumulative marginal benefit by a substantial margin.

4.3 A credit boom and a higher probability of a crisis start

How does a credit boom with an associated higher probability of a crisis start affect the net marginal cost of LAW? The conventional wisdom is certainly that this would be favorable to LAW, in the sense of reducing the net marginal cost of LAW, perhaps even make it negative. But this conventional wisdom is wrong in the current framework.

Figure 9 shows how the annual probability of a crisis start depends on annual steady real debt growth rate for the logit model (14) and (15) and the ST estimates. Point A corresponds to the benchmark annual real debt growth and annual probability of a crisis start of, respectively, 5% and 3.2%. Assume instead a credit boom corresponding to point B, with annual real debt growth and probability of a crisis of, respectively, 7.9% and 4.2% (see appendix D).

In order to stack the cards in favor of LAW, assume again monetary non-neutrality and a permanent policy-rate effect on real debt. In figure 10, the solid lines show the benchmark marginal cost, marginal benefit, and net marginal cost under this assumption, the same as in figure 8, except that only the sum (8) of the two marginal benefits is shown. The dashed lines show the effect of the higher probability of a crisis start. The cumulative net marginal cost will be higher, not lower, so a credit boom with a higher probability of a crisis increases the marginal cost more than the marginal benefit. How can that be?

The higher probability of a crisis start, \( q_t \), leads to a higher probability of a crisis, \( p_t \). In figure 2, the quarterly probability of a crisis start will be 4.2/4 = 1.05%, and the probability of a crisis start will rise to 8.1% in quarter 9 and then fall back to 7.8%.

Both the marginal cost (9) and the marginal benefit (11) are linear in the probability of a crisis, so the higher probability of a crisis then shifts up both in proportion. But because the marginal
cost is higher than the marginal benefit from a smaller magnitude of a crisis, the net effect is an increase the net marginal cost. Furthermore, we see in figure 9 that the probability of a crisis start is slightly convex as a function of real debt growth. This means that, at point B, the probability of a crisis start and the probability of a crisis are somewhat more sensitive to real debt growth. Then the marginal benefit from a lower probability of a crisis (10) will be somewhat higher. This shifts up the marginal benefit a bit. But we see that the net effect is still to shift up the net marginal cost and to make the cumulative net marginal cost higher.

Thus, a credit boom and a corresponding higher probability of a crisis increases the cumulative net marginal cost of LAW, counter to the conventional wisdom.\textsuperscript{21}

\subsection*{4.4 A larger crisis magnitude}

The conventional wisdom is certainly that a larger magnitude of a crisis would be favorable to LAW. Again the conventional wisdom is wrong in the present framework, except for extremely large crisis magnitudes.

A larger magnitude of a crisis is here represented by a larger crisis increase in the unemployment rate, $\Delta u$. The dashed lines in figure 11 show the marginal cost, marginal benefit, and net marginal cost for a $\Delta u$ equal to 6 pp instead of the benchmark 5 pp. Again, I assume monetary non-neutrality and a permanent policy-rate effect on real debt, in order to stack the cards in favor of LAW. But

\textsuperscript{21} Discussions with Helge Berger were helpful for the interpretation of this result.
as we see, the net marginal cost instead increases up to around quarter 17, and the cumulative net marginal cost increases, counter to the conventional wisdom.

To understand this, note that the marginal cost (9) and the marginal benefit from a smaller magnitude of a crisis (11) are both linear in $\Delta u$ and shift up in proportion. But because the marginal cost is larger, the net effect is to increase the net marginal cost. Furthermore, we see from (10) that the marginal benefit from a lower probability of a crisis is quadratic in $\Delta u$. This shifts up the marginal benefit more than in proportion and reduces the net marginal cost. But because the marginal benefit from a lower probability of a crisis is relatively small, the net effect of the shifts in the marginal cost and benefits is an increase in cumulative marginal cost.

However, because the marginal cost is linear but the marginal benefit from lower probability of a crisis is quadratic in $\Delta u$, we realize that a sufficiently large $\Delta u$ can increase the marginal benefit from a lower probability of a crisis so much as to achieve break-even, with the cumulative net marginal cost of LAW equal to zero. But this requires an extremely large $\Delta u$ of 32 pp.\footnote{Because the policy-rate effect on the non-crisis unemployment rate and the marginal cost are still positive at quarter 40, for a precise result the horizon needs to be extended further: 60 quarters is sufficient. Beyond quarter 40, because there is no data on the Riksbank policy-rate effect on the unemployment rate, it is assumed to fall exponentially at a rate 3 times faster than the rate from quarter 32 to 40 (so as to not exaggerate the marginal cost beyond quarter 40). Then $\Delta u = 32$ pp makes the cumulative net marginal cost at quarter 60 close to zero.}
4.5 A longer crisis duration

The conventional wisdom is certainly that a longer duration of a crisis would favor LAW. But instead the cumulative net marginal cost increases, counter to the conventional wisdom.

In figure 12, the dashed lines show the marginal cost, marginal benefit, and net marginal cost for a longer crisis duration of \( n = 12 \) quarters instead of the benchmark 8 quarters. In figure 2, a longer duration of 12 quarters means that the probability of crisis rises further to 9.2\% in quarter 13 and then falls back to 8.8\% instead of 6\%. As a result of the longer duration and higher probability of a crisis, both the marginal cost and the marginal benefit are extended to the right and shifted up. Because the marginal cost is initially higher than the marginal benefit, this will increase the net marginal cost, except around quarter 24, where the marginal benefit increases slightly more than the marginal cost. The result is that the cumulative net marginal cost increases.

4.6 Inefficient macroprudential policy

The conventional wisdom is certainly that LAW is more appropriate in a situation with less effective macroprudential policy. Such a situation would likely imply a more vulnerable economy, with higher probability, larger magnitude, or longer duration of crises. But above we have seen that for each of these changes, the costs exceed the benefits by an even larger margin. Thus, this conventional wisdom is wrong here.

4.7 A larger policy-rate effect on the probability of crisis

As mentioned, the benchmark policy-rate effect on the probability of a crisis is constructed from the Sveriges Riksbank (2014a) estimate of the policy-rate effect on household debt combined with the ST estimate of the effect of debt on the probability of a crisis start. For monetary neutrality and no permanent policy-rate effect on real debt, the cumulative marginal benefit from a lower probability of a crisis is approximately zero. Then a larger policy-rate effect on debt or a larger effect of debt on the probability of a crisis would still have an approximately zero effect on the cumulative net marginal cost.

In order to stack the cards in favor of LAW, let me then again assume monetary non-neutrality and a permanent policy-rate effect on real debt. Then the cumulative marginal benefit from a lower probability of a crisis is positive, as shown in figure 8, and a larger policy-rate effect on the probability of a crisis would increase the cumulative marginal benefit and reduce the cumulative
net marginal cost.

To achieve break-even, that is, a cumulative net marginal cost equal to zero, the policy-rate effect on the probability of a crisis start, $dq_t/d\bar{\alpha}_1$, then needs to be about 6.4 times as large as the benchmark.

A larger policy-rate effect on the probability of a crisis start can be achieved through a larger policy-rate effect on debt or a larger effect of debt on the probability of a crisis start, or both. Assume that it is achieved through a larger policy-rate effect on debt. For break-even, the policy-rate effect on real debt then needs to be 6.4 times as large as the Riksbank estimate (the red line in figure 3). That is, real debt needs to fall by about 6.4% in quarter 8 instead of 1%. With a standard error of about 1 pp, this is an effect about 5.4 standard errors larger than the Riksbank estimate. Clearly, the margin to break-even is large.\textsuperscript{23}

If break-even is achieved through a larger effect of debt on the probability of a crisis start, the effect needs to be about 6.4 times as large as the ST estimate. This implies an effect that is about 15 standard errors larger than the ST estimate. Clearly, the margin to break-even is quite large.\textsuperscript{24}

For some LAW to be optimal, the policy-rate effect on the probability of a crisis needs to be even larger. For a permanent effect on real debt, for $\Delta\bar{\alpha}_1 = 0.5$ pp to be (approximately) optimal an effect 10 times as large as the benchmark is required.

4.8 A larger policy-rate effect on the magnitude of a crisis

The benchmark policy-rate effect on the magnitude of a crisis uses the Sveriges Riksbank (2014a) estimate of the policy-rate effect on household debt to income and combines it with the Flodén (2014) estimate of the effect on debt to income on the magnitude of a crisis. A larger (negative) policy rate effect on the magnitude of a crisis would increase the marginal benefit from a smaller magnitude and reduce the cumulative net marginal cost. But just to achieve break-even requires a policy-rate effect on the magnitude of a crisis that is about 20 times as large as the benchmark effect.

The larger policy-rate effect on the magnitude of a crisis start can be achieved through a larger policy-rate effect on debt to income or a larger effect of debt to income on the magnitude of a crisis, or both. If break-even is achieved through a larger policy-rate effect on debt to income, the effect

\textsuperscript{23} Sveriges Riksbank (2014a, figure A20) reports uncertainty intervals around the point estimate of the policy-rate effect on real debt. The 12.5 + 75 = 87.5th percentile exceeds the point estimate at quarter 8 by 1 pp, which can then be taken to be an approximate (upper bound of the) standard error.

\textsuperscript{24} The largest and most significant coefficient is 7.138, with a standard error of 2.631 (ST (table 3, specification 6)). The required effect needs to be 6.43 times as large. Then the coefficient needs to increase by $(6.43 - 1) \cdot 7.138/2.631 = 14.7$ standard errors.
needs to be about 20 times as large as the Riksbank estimate (the solid red line in figure 4). The average benchmark effect for quarters 4–10 is a fall of 1.13 pp. For break-even, debt to income then needs to fall by about 21 pp for quarters 4–10. This is an effect that is about 13 standard errors larger than the benchmark effect. Clearly, the margin to break-even is quite large. If break-even is achieved through a larger effect of debt to income on the magnitude of a crisis, it needs to be about 18 times as large as the Flodén (2014, table 4, column 1) estimate. This implies an effect that is about 43 standard errors larger than the Flodén estimate and about 12 standard errors larger than the estimate that follows from JST. Clearly, the margin to break-even is quite large.

For some LAW to be optimal, the policy-rate effect on the magnitude of a crisis needs to be even larger. For a permanent effect on real debt, for $\Delta \bar{y}_1 = 0.5$ pp to be (approximately) optimal, an effect 30 times as large as the benchmark is required.

### 4.9 Debt to GDP instead of real debt and 5-year moving averages

One might think that using debt-to-GDP instead of real debt as a predictor of crisis might lead to a different result. But this is not so.

ST (table 4, specification 9) report a specification where the annual growth of real debt in the logistic function is replaced by the annual growth of debt to income. Jordà et al. (2016) use a 5-year moving average of the annual growth of debt to income. Using the debt-to-GDP ratio instead of real debt or using 5-year moving averages of it instead of 5 lags lead to a policy-rate effect on the probability of a crisis and a marginal benefit of LAW similar to the ones in, respectively, figures 3 and 5. Thus, the cumulative marginal cost still exceeds the cumulative marginal benefit by a substantial margin.

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25 Sveriges Riksbank (2014a, figure A22) reports uncertainty intervals around the point estimate of the policy-rate effect on debt to income. The $12.5 + 75 = 87.5$th percentile exceeds the point estimates on average about 1.6 pp around quarters 4–10. This can be taken as an approximate (upper bound of the) standard error. The effect then needs to increase by $(20 - 1) \cdot 1.13/1.6 = 13.4$ standard errors.

26 The benchmark estimate used is 0.02 and the required effect is 0.406. Flodén’s estimate of the effect is actually 0.0230, with a standard error of 0.00897. (Flodén (2014, table 4, column 1) does not report the standard error and reports the point estimate with only two decimals, but the data and programs to replicate his estimate and extract the standard error are available at http://martinfloden.net/files/hhdebt_replicate.zip.) This means that the required effect is $0.41/0.023 = 17.8$ as large as Flodén’s estimate, $(0.41-0.023)/0.00897 = 43.1$ standard errors larger.

The estimated effect of credit to GDP on the unemployment increase that follows from JST (table 8) is 0.04, as explained in section 3.3. The average standard error over years 1–5 for the effect of excess credit on GDP is 0.625 (in the row “Excess credit × financial recession” in table 8). With an Okun coefficient of 2, the average standard error for the effect of excess credit on the crisis increase in the unemployment rate can be considered to be $0.625/2 = 0.313$. Because 1 pp excess credit per year corresponds to about a 10 pp higher credit-to-GDP ratio, the standard error for the effect of credit to GDP on the crisis increase in unemployment is $0.313/10 = 0.0313$. This means that the required effect for break-even is approximately $(0.41 - 0.04)/0.0313 = 11.8$ standard errors larger than the estimate that follows from JST.
ST (table 7, specification 22) contains a logit regression of the annual probability of a crisis start, where the log of the debt-to-GDP ratio is added as an explanatory variable in the logistic function (14) and (15). Svensson (2016a, appendix D) shows that adding this explanatory variable has a negligible effect on the probability of a crisis start and the marginal benefit in, respectively, figures 3 and 5, and thus does not affect the result.

5 Recent literature

After the first version, Svensson (2016a), of this paper was distributed, BIS (2016a, Box IV.B) has disputed its result and argued that it (1) relies on debt growth rather than a “financial cycle” as a predictor of crises, (2) assumes that the magnitude of a crisis is exogenous and independent of the policy rate, and (3) just discusses a one-off policy-rate increase instead of a systematic and optimal policy of LAW.

Regarding (1), it is an empirical issue what the best predictor of future crises is and what the policy-rate effect on that predictor is. I use real debt growth because of the empirical support of ST and IMF (2015); as noted in section 4.9 the result is also robust to instead using debt-to-GDP growth. Nothing prevents the use of some other measure of a financial cycle instead, if it has better empirical support. Regarding (2), an endogenous magnitude of a crisis was actually included in Svensson (2016a, appendix D). As in this paper, it does not change the result. Regarding (3), Svensson (2016a, section 3) actually examined optimal policy, not only a one-off policy tightening. Interestingly, in their review of BIS research, Allen et al. (2016) criticize the BIS for being “excessively focussed on building the case for LAW” and for too much research being “motivated primarily by a desire to overturn [the Svensson (2016a)] conclusion on the inadvisability of LAW.”

As noted in section 1, the previous literature has used the assumption that the crisis loss is independent of the non-crisis unemployment deviation. This is consistent with assuming that \( \tilde{u}_t^c = \Delta u_t \) instead of \( \tilde{u}_t^c = \tilde{u}_t^h + \Delta u_t \), so the expected quarter-t loss instead of (3) satisfies

\[
E_1 L_t = (1 - p_t)E_1(\tilde{u}_t^h)^2 + p_t E_1(\Delta u_t)^2.
\]

It follows that the marginal cost of LAW and the marginal benefits from lower probability and deficit

\[27\] However, with the financial cycle as in Drehmann et al. (2012) including real debt and debt to GDP, by the discussion in section 1, the policy-rate effect on it is likely to be small and possibly even of the opposite sign. Consistent with this, the policy-rate impact on the “leverage gap” in Juselius, Borio, Disyatat, and Drehmann (2016, table 3) is small, positive, and not statistically significant.
magnitude of a crisis satisfy
\[ MC_t \equiv 2 (1 - p_t) E_1 \tilde{u}_t^n \frac{dE_1 u_t^n}{dt_1}, \]  
(17)
\[ MB_t^p \equiv E_1 (\Delta u_t)^2 (\frac{-dE_1}{dt_1}), \]  
(18)
\[ MB_t^{\Delta u} \equiv 2 p_t E_1 \Delta u_t (\frac{-dE_1}{dt_1}), \]  
(19)
instead of (5)-(7). In particular, for a zero (expected) non-crisis unemployment gap, corresponding to NL, the marginal cost is zero for (17) instead of positive for (9), whereas the marginal benefits remain positive. It follows that \textit{some} LAW is optimal. However, as shown in Ajello et al. (2016) and Svensson (2017b), because the marginal cost (17) rises relatively quickly and the marginal benefit is small the optimal LAW is quite small, only a few basis points, and thus hardly economically significant.

More recently, Filardo and Rungcharoenkitkul (2016) and Gourio, Kashyap, and Sim (2016) have responded to Svensson (2016a), arguing that LAW would be optimal and maintaining that this contradicts my result. However, as discussed in Svensson (2016b, 2017b), their results can be explained by their assumption that the cost of a crisis (the crisis loss less the non-crisis loss) is constant and independent of LAW.

This assumption can be represented by the cost of a crisis equaling \((\Delta u_t)^2\), where \(\Delta u_t\) now is just the square root of the cost of a crisis instead of the crisis increase in the unemployment rate. Then the expected quarter-\(t\) loss instead of (4) satisfies
\[ E_1 L_t = E_1 (\tilde{u}_t^n)^2 + p_t E_1 (\Delta u_t)^2, \]  
(20)
where the second term on the right side is the probability of a crisis times the cost of a crisis. The marginal cost is thus
\[ MC_t \equiv 2 E_1 \tilde{u}_t^n \frac{dE_1 u_t^n}{dt_1}, \]  
(21)
whereas the marginal benefits from a lower probability and a smaller magnitude of a crisis are equal to (18) and (19). The only difference between (17) and (21) is that the latter is slightly larger because it is multiplied by unity instead of \(1 - p_t\), the probability of a non-crisis in quarter \(t\) (0.94 in steady state for the benchmark assumptions, see figure 2).

Thus, for a zero non-crisis unemployment gap, the marginal cost is zero, and \textit{some} LAW is then optimal. But again, as shown in Svensson (2017b), the optimal LAW is quite small, only a few basis points. Consistent with this, Gourio et al. (2016) find that the optimal LAW leads to a
reduction of the annual probability of only 6 bp, from 2.08% to 2.02%, implying on average one crisis in 49.5 years instead of one in 48.1 years. This is hardly economically significant.

Finally, Adrian and Liang (2018) (AL for short) have challenged the robustness of my result and argued that alternative reasonable assumptions about the effect of the policy rate on the probability or magnitude of a crisis would overturn it. However, as is shown in Svensson (2017a) and in sections 4.7 and 4.8, AL’s alternative assumptions required to overturn my result imply an effect of debt on the magnitude of a crisis that is more than 43 standard errors larger than the estimate of Flodén (2014) and more than 12 standard errors larger than an estimate that follows from JST, or an effect of credit on the probability of a crisis that is more than 15 standard errors larger than the estimate of ST.28

6 Conclusions

A simple, transparent, and robust framework for a cost-benefit analysis of LAW has been presented. The costs of LAW are not only a weaker economy if no crisis occurs. There is also a second cost, namely, for a given magnitude of a crisis a weaker economy if a crisis occurs. The crisis loss and the cost of a crisis (the crisis loss less the non-crisis loss) will for a given magnitude of a crisis be higher with LAW, something that has been overlooked by the previous literature. The possible benefits of LAW include a lower probability and smaller magnitude of a crisis. For existing representative empirical estimates and reasonable assumptions, the result is that the costs of LAW exceed the benefits by a substantial margin. The reason is that the empirical policy-rate effects on the probability and magnitude of a crisis are far too small to make the benefits of LAW match the cost.

The result is robust to monetary non-neutrality and a permanent policy-rate effect on real debt; a smaller policy-rate effect on unemployment; a credit boom with a higher probability of a crisis; a larger crisis magnitude; a longer crisis duration; less effective macroprudential policy; larger

28 Sutherland, Hoeller, Merola, and Zieman (2012) and Mian, Sufi, and Verner (2016) (cited by AL) examine and estimate the possible effect of credit on the magnitude of downturns in recessions. But, importantly, the estimates that are relevant here are of the possible effect of credit on the magnitude of a financial crisis, that is, the effect on the downturn conditional on a financial crisis, not the possible effect of credit on the unconditional downturn in a general recession, regardless of whether there is a financial crisis or not. In a similar line, Adrian and Duarte (2016) examine possible effects of the policy rate on the volatility of GDP, regardless of whether there is a financial crisis or not. This is about a possible aspect of the transmission mechanism that violates certainty equivalence, but this is a different issue from what is discussed in this paper. Thus, in contrast to what they state, their findings do not contradict but are rather, as far as I can see, orthogonal to the result in this paper.

Brunnermeier, Palia, Sastry, and Sims (2017) question the result in single- and few-equation models that credit has a negative effect on future output and find in a 10-equation structural VAR that the negative effect of credit growth on future output is entirely accounted for by the contractionary monetary policy response credit growth elicits.
policy-rate effects on the probability and duration of a crisis; and using debt to GDP, including 5-year moving averages, instead of real debt as a predictor of crises. In particular, to overturn the result the policy-rate effects on the probability and magnitude of a crisis need to be more than 5–40 standard errors larger than the point estimates, in spite of some of the estimates having large standard errors relative to the point estimates. This indicates that the result is quite robust.

Given the simplicity and transparency of the framework and its dependence on only a few assumptions and empirical estimates of the policy-rate effects on unemployment and the probability and magnitude of a crisis, it is easy to redo the analysis with alternative assumptions or new alternative or better empirical estimates. This way the robustness of the result can be further examined.

Appendix

A The optimal unemployment rate and the indirect loss function

Assume a quadratic loss function of inflation and unemployment,

\[ L^*(\pi_t, u_t) = \pi_t^2 + \lambda(u_t - \bar{u})^2, \]  

(A.1)

where \( \pi_t \) denotes the gap between the inflation rate in quarter \( t \) and and a fixed inflation target, \( u_t - \bar{u} \) denotes the gap between the unemployment rate \( u_t \) in quarter \( t \) and the long-run sustainable unemployment rate \( \bar{u} \), and \( \lambda > 0 \) denotes the constant weight on unemployment-gap stabilization relative to inflation-gap stabilization. With a positive weight on both inflation and unemployment stabilization, this loss function represents flexible inflation targeting.

Assume a simple Phillips curve,

\[ \pi_t = -\kappa(u_t - \bar{u}) + z_t, \]  

(A.2)

where \( z_t \) is an exogenous stochastic process with a zero unconditional mean. It represents cost-push shocks that cause a tradeoff between stabilizing inflation at the inflation target and unemployment rate at its long-run sustainable rate.

Using (A.2) to substitute for \( \pi_t \) in (A.1) results in

\[ L^*[-\kappa(u_t - \bar{u}) + z_t, u_t] = [-\kappa(u_t - \bar{u}) + z_t]^2 + \lambda(u_t - \bar{u})^2. \]  

(A.3)
Choosing $u_t$ to minimize (A.3) gives the optimal unemployment rate under flexible inflation targeting, $u_t^*$, which satisfies

$$u_t^* \equiv \bar{u} + \frac{\kappa z_t}{\lambda + \kappa^2}. \quad (A.4)$$

Using (A.4) to replace $z_t$ in (A.3) results, after some algebra, in the indirect loss function $L^0(u_t, u_t^*)$, which satisfies

$$L^0(u_t, u_t^*) \equiv L_t^* \left( -\kappa (u_t - \bar{u}) + \frac{\lambda + \kappa^2}{\kappa} u_t^*, u_t \right) = (\lambda + \kappa^2) L(u_t - u_t^*) + \frac{\lambda(\lambda + \kappa^2)}{\kappa^2} (u_t^* - \bar{u})^2, \quad (A.5)$$

where the simple loss function $L(u_t - u_t^*)$ is given by

$$L(u_t - u_t^*) \equiv (u_t - u_t^*)^2. \quad (A.6)$$

Clearly, choosing $u_t$ to minimize the simple loss function (A.6) is equivalent to choosing $u_t$ to minimize (A.5) and thus to choosing $u_t$ to minimize (A.1) subject to (A.2).

The optimal unemployment rate under flexible inflation targeting, $u_t^*$, is called the benchmark unemployment rate. The deviation between the actual unemployment rate and the benchmark unemployment rate, $u_t - u_t^*$, is called the unemployment deviation.

### B A cost-push shock correlated with the crisis

A crisis is considered to be a negative demand shock that, net of possible conventional and unconventional policy actions during the crisis to reduce its costs, increases the unemployment deviation, $\tilde{u}_t \equiv u_t - u_t^*$, by the amount $\Delta u_t > 0$. The demand shock and the cost-push shock, $z_t$, are assumed to be independent. Then, by (A.4), $u_t^*$, the benchmark unemployment rate, is independent of a crisis and $\Delta u_t$ is the crisis increase in the unemployment rate, $u_t$.

By (A.5) and (A.6), we can write the quarter-1 expectation of the quarter-$t$ loss increase in a crisis, the cost of a crisis, as

$$E_1[L^0(u_t, u_t^*)|c] - E_1[L^0(u_t, u_t^*)|n] = (\lambda + \kappa^2) \{E_1[(u_t - u_t^*)^2|c] - E_1[(u_t - u_t^*)^2|n]\}$$

$$+ \frac{\lambda(\lambda + \kappa^2)}{\kappa^2} \{E_1[(u_t^* - \bar{u})^2|c] - E_1[(u_t^* - \bar{u})^2|n]\}, \quad (B.1)$$

where $c$ and $n$ denote, respectively, crisis and non-crisis, and where the last term on the right side is zero and can be disregarded if the cost-push shock is independent of the crisis. This is the maintained assumption.
If instead the cost-push shock is assumed to be correlated with the crisis, that last term in (B.1) is exogenous but not zero and has to be included in the cost of a crisis. Furthermore, then $u_t^*$ is not independent of whether there is a crisis or not. With the non-crisis and crisis unemployment gaps satisfying $\tilde{u}_t^n \equiv u_t^n - u_t^{*n}$ and $\tilde{u}_t^c \equiv u_t^c - u_t^{*c}$, for them to satisfy $\tilde{u}_t^c = \tilde{u}_t^n + \Delta u_t$, $\Delta u_t$ has to satisfy $\Delta u_t \equiv (u_t^c - u_t^n) - (u_t^{*c} - u_t^{*n})$. That is, $\Delta u$ is then defined as the crisis increase in the unemployment deviation and equals the crisis increase in the unemployment rate less the crisis increase in the benchmark unemployment rate. With these modifications, the analysis in the paper can be done also for a cost-push shock correlated with a crisis.\textsuperscript{29}

C A Markov process for crisis and non-crisis states

Consider the situation in which the (quarterly) probability of a crisis start is $q$ and the crisis duration $n$ quarters. Assume for simplicity that a new crisis cannot start during an existing crisis, so the probability of crisis start is conditional on no crisis in the previous quarter. This situation can be modeled as a Markov process with $n + 1$ states, where state 1 corresponds to a non-crisis and state $j$ for $2 \leq j \leq n + 1$ corresponds to a crisis in its $(j - 1)$th quarter.\textsuperscript{30}

Let the $(n + 1) \times (n + 1)$ transition matrix be $P = [P_{ij}]$, where $P_{ij} = Pr(j \text{ in } (t + 1) | i \text{ in } t)$ is the probability of a transition from state $i$ in quarter $t$ to state $j$ in quarter $t + 1$. The transition probabilities will be zero except for $P_{11} = 1 - q$, $P_{12} = q$, $P_{i,i+1} = 1$ for $2 \leq i \leq n$, and $P_{n+1,1} = 1$. For example, for $n = 3$ the $4 \times 4$ transition matrix is

$$P = \begin{bmatrix}
1 - q & q & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{bmatrix}.$$  

Let the row vector $\pi_t = (\pi_{ti})_{i=1}^{n+1}$ denote the probability distribution of the states in quarter $t$, and let $\pi_1 = (1, 0, ..., 0)$, representing a non-crisis in quarter 1. Then the probability distribution in quarter $t \geq 1$, conditional on a non-crisis in quarter 1, is given by

$$\pi_t = \pi_1 P^t,$$

and the probability of crisis in quarter $t$, $p_t$, is given by

$$p_t = 1 - \pi_{t1} \quad \text{for } t \geq 1. \quad (C.1)$$

\textsuperscript{29} I am grateful to Simone Manganelli for alerting me to this issue.

\textsuperscript{30} I am grateful for helpful discussions with Stefan Lasèèn and David Vestin on the Markov process of crisis and non-crisis states.
The solid green line in figure C.1 shows the resulting probability of a crisis in quarter $t$, when $q = 0.8\%$ and $n = 8$ quarters. The probability of a crisis rises to 6.25\% and then falls back to 6\%. The dashed green line shows the probability of a crisis for the linear approximation when the probability of crisis is just the sum of the probabilities of a crisis start over the previous $n$ quarters,

$$p_t = \sum_{\tau=0}^{n-1} q_{t-\tau}. \quad (C.2)$$

The linear approximation increases to 6.4\% and exaggerates the probability of a crisis somewhat. It was for simplicity used in Svensson (2016a).

The main advantage with the linear approximation (C.2), is that the policy-rate effect on the probability of a crisis is easy to calculate. Given the effect on the probability of a crisis start, $dq_t/d\bar{\alpha}_1$ for $t \geq 1$, from figure 3, it simply satisfies

$$\frac{dp_t}{d\bar{\alpha}_1} = \sum_{\tau=0}^{n-1} \frac{dq_t}{d\bar{\alpha}_1}. \quad (C.3)$$

For the Markov process, the calculation is a bit more complicated. Let $P_t = [P_{t,ij}]$ for $t \geq 1$, denote the transition matrix from states in quarter $t$ to states in quarter $t+1$, where $P_{t-1,11} = 1-q_t$, $P_{t-1,12} = q_t$ and $P_{t-1,ij} = P_{ij}$ for $(i,j) \neq (1,1), (1,2)$. We have

$$\pi_t = \pi_{t-1} P_{t-1} \text{ for } t \geq 2. \quad (C.4)$$

Then the policy-rate effect on the probability distribution satisfies

$$\frac{d\pi_t}{d\bar{\alpha}_1} = \frac{d\pi_{t-1}}{d\bar{\alpha}_1} P_{t-1} + \pi_{t-1} \frac{dP_{t-1}}{d\bar{\alpha}_1} \text{ for } t \geq 2, \quad (C.5)$$

where $d\pi_1/d\bar{\alpha}_1 = 0$, $dP_{t-1,11}/d\bar{\alpha}_1 = -dq_t/d\bar{\alpha}_1$, $dP_{t-1,12}/d\bar{\alpha}_1 = dq_t/d\bar{\alpha}_1$, and $dP_{t-1,ij}/d\bar{\alpha}_1 = 0$ for $(i,j) \neq (1,1), (1,2)$. Finally,

$$\frac{dp_t}{d\bar{\alpha}_1} = -\frac{d\pi_t}{d\bar{\alpha}_1} \text{ for } t \geq 2. \quad (C.6)$$

Svensson (2016a) shows that the linear approximation (C.3) of the derivative is relatively good for the range of parameters used here. Nevertheless, the exact derivative (C.4)-(C.6) is used in this paper.
D The logistic function

Consider the logistic function

\[ q = \frac{\exp(a + bg)}{1 + \exp(a + bg)} = \frac{1}{1 + \exp[-(a + bg)]}, \]  

(D.1)

where \( q \) here is the annual probability of a crisis start, \( g \) is a the steady annual growth rate of real debt and \( a \) and \( b \) are constants. In a logit regression of crises starts on current and lagged annual growth rates of real debt, \( b \) corresponds to the sum of the coefficients on the lagged annual growth rates.

The derivative of \( q \) with respect to \( g \), the marginal effect of steady real debt growth on the probability \( q \), satisfies

\[ \frac{dq}{dg} = bq(1 - q). \]  

(D.2)

The sum of the coefficients in (15) is \( b = 9.698 \). Given the \( dq/dg = 0.30 \) reported by Schularick and Taylor (2012), it follows from (D.2) that \( q = 0.032 \). (To be precise, the values used are \( dq/dg = 0.3016 \) and \( q = 0.0321 \).) Given \( b \) and \( q \), if \( g = 0.05 \) it follows from (D.1) that \( a = -3.890 \).

Section 4.3 examines the case when \( g = 0.079 \). Then \( q = 0.0421 \), and \( dq/dg = 0.3914 \).

References


