The Effect on Housing Prices of Changes in Mortgage Rates and Taxes[*]

Lars E.O. Svensson

SIFR – The Institute for Financial Research,
Swedish House of Finance, Stockholm School of Economics

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Abstract

This paper presents some simple calculations of the effect on housing prices of temporary changes in the one-year mortgage rate and permanent changes in short and long mortgage rates, the capital-income tax, the effective property and wealth tax, the capital-gains tax, the expected growth rate of the value of housing services, and the CPI inflation rate. A new element in the calculation is to take the capital-gains tax on housing into account. The semi-elasticity of housing prices with respect to *temporary* changes in the one-year mortgage rates is quite small. This semi-elasticity is less sensitive to the assumptions about the parameters. The semi-elasticities of *permanent* changes in mortgage rates, taxes and the tax-deductibility of mortgage rates are substantial. These semi-elasticities are more sensitive to the assumptions about the parameters.

With the help of the user-cost approach to housing prices used in, for example, Englund (2011), Kuttner (2012), Poterba (1984), and Sørensen (2013), this paper provides some simple calculations of the effect on housing prices of temporary changes in the one-year mortgage rate and permanent changes in short and long mortgage rates, the capital-income tax, the effective property and wealth tax, the capital-gains tax, the expected growth rate of the value of housing services, and the CPI inflation rate. A new element in the calculation is to take the capital-gains tax on housing into account.

The main result is that, for realistic parameters, the semi-elasticity of housing prices with respect to *temporary* changes in the one-year mortgage rates is quite small, and actually somewhat smaller and more temporary than the rule of thumb that I myself have applied in policy discussions

the last few years. According the calculations in the present paper, a temporary increase in the one-year mortgage rate for one year of 1 percentage point (equivalent to a temporary increase in the variable mortgage rate of 1 percentage point for one year) reduces housing prices only temporary and only between 0.6 and 0.8 percent. Here the lower number applies if the capital-gains tax is disregarded and the higher number applies if the capital-gains tax is fully internalized, including that it would be paid each year regardless of whether the housing is sold or not.

A *permanent* increase in the mortgage rate has a substantial effect and a permanent increase by 1 percentage point reduces housing prices permanently by about 7 percent if the capital-gains tax is disregarded and about 6.5 percent if the capital-gains tax is fully internalized. A permanent elimination of the tax deductibility of mortgage-rate payments also has a substantial effect and would permanently reduce housing prices by about 8 percent if the capital-gains tax is disregarded and by about 15 percent if the capital-gains tax is fully internalized. Permanent changes in effective property and wealth taxes have a large effect, and a 1 percentage point increase in the tax rate would reduce housing prices permanently by about 10 percent if the capital-gains tax is disregarded, somewhat more if it is fully internalized. A permanent elimination of a fully internalized capital-gains tax would permanently increase housing prices by about 10 percent. A permanent increase in the expected annual rate of growth of the real value of housing services by 1 percentage point (equivalent to an increase of 1 percentage point of the growth rate of real market rents in a perfect rental market) would permanently increase housing prices by about 8 percent if the capital-gains tax is disregarded, somewhat more if the capital-gains tax is fully internalized. A permanent increase in annual CPI inflation by 1 percentage point would, if the real before-tax interest rate is unchanged, permanently increase housing prices by about 3 percent if the capital-gains tax is disregarded and by 0.8 percent if it is fully internalized.

If the monetary policy rate is assumed to affect variable mortgage rates one-to-one, the conclusion is that the policy-rate has a smaller and more temporary impact on housing prices than previously thought, including what I have thought myself. Svensson (2013) looks at the implications for household debt.[1]

1 The user-cost approach to housing prices

Let h_t denote the (real) value at the end of year t (the beginning of year $t+1$) of the housing services during year t from a unit of housing. Suppose that h_t is known at the beginning of year t, that

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[1] The rule of thumb that I have applied myself is that a 1 percentage point higher monetary policy rate for four quarters and then a return to a baseline path would reduce housing prices by up to 2 percent below the baseline path in a couple of years. If the variable mortgage rate is assumed to vary one-to-one with the policy rate, this change in the policy rate might translate into an increase in the one-year mortgage rate of 1 percentage point above the baseline the first year and then a fall back to perhaps 0.5 percentage point above the baseline the second year. According to this paper, this would reduce housing prices by 0.7 percent (taking the middle of 0.6 and 0.8) below the baseline in the first year, then housing prices would rise back to a level of 0.35 percent below the baseline in the second year, and have returned to the baseline the third year. This is clearly a smaller and more temporary effect than the rule of thumb I have applied.

there is a perfect market at the beginning of year t for renting houses during year t, and that the contracted market rent is paid at the end of year t. Then, that market rent will equal h_t. Suppose that there is also a perfect market for buying and selling housing units. Let $p_t \equiv P_t/P_t^c$ denote the (real) housing price of a housing unit at the beginning of year t, where P_t is the nominal price of the housing unit and P_t^c is the CPI at the beginning of year t. Then, the following equilibrium condition will hold,

$$h_t = [(1 - \tau^i)i_t - \mathrm{E}_t\pi_{t+1}^c + \delta + \tau^h + \sigma + \theta]p_t - (\mathrm{E}_t p_{t+1} - p_t). \tag{1.1}$$

Here τ^i denotes the tax rate on interest income and i_t denotes the nominal one-year mortgage rate before tax. Mortgage rate payments are assumed to be tax-deductible, so then $(1 - \tau^i)i_t$ is the after-tax nominal mortgage rate. Furthermore, $\pi_{t+1}^c \equiv P_{t+1}^c/P_t^c - 1$ denotes the CPI inflation rate during year t (from the beginning of year t to the beginning of year $t + 1$), so $\mathrm{E}_t\pi_{t+1}^c$ denotes expectations at the beginning of year t of CPI inflation during the year. Then, the sum of the first two terms in the bracket on the right-hand side of (1.1) is the real one-year after-tax mortgage rate, denoted r_t,

$$r_t \equiv (1 - \tau^i)i_t - \mathrm{E}_t\pi_{t+1}^c. \tag{1.2}$$

Furthermore, δ denotes the depreciation rate of the housing unit, τ^h denotes the effective property and wealth tax rate, σ denotes a risk premium associated with owning the housing unit, and θ denotes a premium associated with down-payment restrictions and the opportunity cost of the house-owner's equity.[2]

We can write this as

$$h_t = \gamma_t p_t - (\mathrm{E}_t p_{t+1} - p_t), \tag{1.3}$$

where

$$\gamma_t \equiv r_t + \delta + \tau^h + \sigma + \theta. \tag{1.4}$$

Then, $\gamma_t p_t$ in (1.3) represents the expected real (gross) cost in year t associated with owning the housing unit, $\mathrm{E}_t p_t - p_t$ denotes the expected real capital gain on the house, so the right-hand side of (1.3) is the net cost – the user cost – of owning the housing unit during year t. In equilibrium, this must equal the value of the housing services provided by the housing unit, h_t, the left-hand side of (1.3). If the housing owner would sublet the housing unit on a perfect rental market, the real market rent would equal the value of the housing services, h_t.

Above, the rental market is assumed to work so as to determine the real market rent at the beginning of year t to be paid at the end of year t. Alternatively, we could assume a rental market that works so as to determine a *nominal* market rent to paid at the end of the year. That nominal

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[2] The expression $(1 - \tau^i)i_t - \pi_{t+1}^c$ in (1.1) is an approximation of $(1 - \tau^i)i_t/(1 + \pi_{t+1}^c)$. As further discussed by Englund (2011, appendix) and Sørensen (2013, appendix), the premium θ is positive if there are binding down-payment restrictions and the opportunity cost of the house-owner's equity is greater than the real after-tax mortgage rate.

market rent would then be given by $h_t + \mathrm{E}_t \pi_t^c$, the sum of the value of the housing services and the expected CPI inflation rate, so the expected real rent equals the value of the housing services. The results below do not depend on which interpretation of the rental market is chosen.

The asset-pricing equation for the housing price follows from (1.3), although with the term γ_t replacing the real interest rate at the standard asset-pricing equation,

$$p_t = \frac{h_t + \mathrm{E}_t p_{t+1}}{1 + \gamma_t}. \tag{1.5}$$

We can solve this equation forward to express the housing price as the expected present value of future housing services, the expected discounted sum of the value of future housing services,

$$p_t = \mathrm{E}_t \sum_{s=1}^{\infty} d_{t+s,t} h_{t+s-1}, \tag{1.6}$$

where the discount factor $d_{t+s,t}$ satisfies

$$d_{t+s,t} = \prod_{j=1}^{s} \frac{1}{1 + \gamma_{t+j-1}} \quad \text{for} \quad s \geq 1.$$

It is assumed that the infinite sum in (1.6) converges, that is, that the value of housing services is not growing at a rate higher than γ_t.

1.1   Bubbles

The solution for p_t in (1.6) is the so-called fundamental solution and excludes so-called bubbles. A bubble b_t is any stochastic process that satisfies

$$b_t = \frac{\mathrm{E}_t b_{t+1}}{1 + \gamma_t}. \tag{1.7}$$

For such a stochastic process, by adding (1.7) and (1.5), we have

$$(p_t + b_t) = \frac{h_t + \mathrm{E}_t(p_{t+1} + b_{t+1})}{1 + \gamma_t},$$

so the sum $p_t + b_t$ of the fundamental value p_t in (1.6) and any bubble b_t satisfying (1.7) is also a solution to the asset-price equation (1.5).

The bubble b_t has the property that the expected real capital gain from the beginning of year t to the beginning of year $t + 1$ equals $\gamma_t b_t$,

$$\mathrm{E}_t b_{t+1} - b_t = \gamma_t b_t.$$

Thus, the bubble must be expected to grow at the expected rate γ_t. As we shall see, for conventional parameters for Sweden, the rate γ_t will typically be around 11 percent per year, so a bubble requires expectations of long-run real capital gains of about 11 percent per year. In Sweden, there is no indication that house-owners' expectations of permanent capital gains would be so large. Therefore, it seems reasonable to exclude such bubbles for Sweden.[3]

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[3] Case and Shiller (2003) give examples of such high expectations in the US before the crisis.

1.2 A steady state

Consider now a steady state where γ_t is constant, $\gamma_t = \gamma$, and the value of housing services is growing at the constant rate g. Then

$$d_{t+s,t} = \left(\frac{1}{1+\gamma}\right)^s, \quad (s \geq 1),$$

$$h_{t+s} = (1+g)^s h_t = (1+g)^{t+s} h_0,$$

and we can write the steady-state housing price as

$$p_t = \mathrm{E}_t \sum_{s=1}^{\infty} d_{t+s,t} h_{t+s-1} = \sum_{s=1}^{\infty} \left(\frac{1+g}{1+\gamma}\right)^s \frac{1}{1+g} h_t = \frac{1/(1+\gamma)}{1 - (1+g)/(1+\gamma)} h_t$$

$$= \frac{h_t}{\gamma - g}, \tag{1.8}$$

where I have used the expression for the sum of an infinite geometric series and assumed that $\gamma > g$ so the sum converges. As already mentioned, γ is likely to be around 11 percent, and g, the steady-state rate of growth of the value of housing services, might be about the same as the steady-state rate of growth of disposable income, which might be about 2 percent, so the condition $\gamma > g$ is then fulfilled by a substantial margin.

Thus, according to (1.8), the steady-state housing price is the steady-state value of housing services divided by

$$\gamma - g \equiv r + \delta + \tau^h + \sigma + \theta - g$$

which expression is the steady-state user cost – to be precise, the percentage user cost, that is, the user cost as a rate relative to the value of the housing price. Since the housing price by (1.8) is proportional to the value of housing services, it will also grow by the constant rate g,

$$p_{t+s} = p_t (1+g)^s.$$

1.3 A capital-gains tax

The above treatment disregards any capital-gains tax. Assume now that there is a capital-gains tax, $\tau^g (P_{t+1} - P_t)$, paid in *nominal* terms at the beginning of year $t+1$, where τ^g, $0 \leq \tau^g < 1$, is the tax rate on the *nominal* capital gain, $P_{t+1} - P_t$.[4] The *real* capital-gains tax paid at the beginning of year $t+1$ is then

$$\tau^g \frac{P_{t+1} - P_t}{P_{t+1}^c} = \tau^g \frac{P_{t+1}^c p_{t+1} - P_t^c p_t}{P_{t+1}^c} = \tau^g (p_{t+1} - \frac{p_t}{1 + \pi_{t+1}^c}) \approx \tau^g [p_{t+1} - (1 - \pi_{t+1}^c) p_t],$$

where I have used the approximation $1/(1 + \pi_{t+1}^c) \approx 1 - \pi_{t+1}^c$, which is sufficient when the CPI inflation rate is only a few percent. The real after-tax capital gain is then

$$p_{t+1} - p_t - \tau^g [p_{t+1} - (1 - \pi_{t+1}^c) p_t] = (1 - \tau^g)(p_{t+1} - p_t) - \tau^g \pi_{t+1}^c p_t.$$

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[4] A more precise calculation would take into account that the capital-gains tax can be postponed, subject to an imputed tax.

The equilibrium condition (1.1) is then modified to

$$h_t = [(1 - \tau^i)i_t - \mathrm{E}_t\pi^c_{t+1} + \delta + \tau^h + \sigma + \theta]p_t - (1 - \tau^g)(\mathrm{E}_tp_{t+1} - p_t) + \tau^g\mathrm{E}_t\pi^c_{t+1}p_t, \qquad (1.9)$$

where the last two terms on the right-hand side are the negative of the real after-tax capital gain. We can write this as

$$h_t = (\gamma_t + \tau^g\mathrm{E}_t\pi^c_{t+1})p_t - (1 - \tau^g)(\mathrm{E}_tp_{t+1} - p_t), \qquad (1.10)$$

where γ_t is defined as in (1.4).

From (1.10) follows that the asset-pricing equation (1.5) is modified to

$$p_t = \frac{h_t + (1 - \tau^g)\mathrm{E}_tp_{t+1}}{1 + \gamma_t - (1 - \mathrm{E}_t\pi^c_{t+1})\tau^g}. \qquad (1.11)$$

For simplicity, assume that inflation expectations are constant, $\mathrm{E}_t\pi^c_{t+1} = \pi^c$, and solve this forward to get

$$
\begin{aligned}
p_t &= \frac{h_t}{1 + \gamma_t - (1 - \pi^c)\tau^g} + \mathrm{E}_t\frac{(1 - \tau^g)(h_{t+1} + p_{t+1})}{[1 + \gamma_t - (1 - \pi^c)\tau^g][1 + \gamma_{t+1} - (1 - \pi^c)\tau^g]} \\
&= \mathrm{E}_t\sum_{s=1}^{\infty}d_{t+s,t}h_{t+s-1}, \qquad (1.12)
\end{aligned}
$$

where the discount factor, $d_{t+s,t}$, is given by

$$d_{t+s,t} \equiv \frac{1}{1 - \tau^g}\prod_{j=1}^{s}\frac{1 - \tau^g}{1 + \gamma_{t+j-1} - (1 - \pi^c)\tau^g} \text{ for } s \geq 1$$

and it is assumed that the infinite sum in (1.12) converges.

In a steady state, we have

$$
\begin{aligned}
d_{t+s,t} &= \frac{1}{1 - \tau^g}\left(\frac{1 - \tau^g}{1 + \gamma - (1 - \pi^c)\tau^g}\right)^s, \quad (s \geq 1), \\
h_{t+s} &= (1 + g)^sh_t = (1 + g)^{t+s}h_0.
\end{aligned}
$$

Then the housing price is given by

$$
\begin{aligned}
p_t &= \sum_{s=1}^{\infty}\left(\frac{(1 - \tau^g)(1 + g)}{1 + \gamma - (1 - \pi^c)\tau^g}\right)^s\frac{1}{(1 - \tau^g)(1 + g)}h_t = \frac{1/[1 + \gamma - (1 - \pi^c)\tau^g]}{1 - (1 + g)(1 - \tau^g)/[1 + \gamma - (1 - \pi^c)\tau^g]}h_t \\
&= \frac{h_t}{\gamma + \tau^g\pi^c - (1 - \tau^g)g}, \qquad (1.13)
\end{aligned}
$$

where $\gamma + \tau^g\pi^c - (1 - \tau^g)g$ is the steady-state user cost taking the capital-gains tax into account.

1.4 Some reasonable numbers

What are some reasonable numbers? I follow Englund (2011) and Sørensen (2013), who assume that the sum $\delta + \tau^h + \sigma + \theta - g$ equals 7 percent (per year). As for g, the growth rate of the value of housing services, assume that it grows at the same rate as real disposable income, and assume that real disposable income grows at 2 percent (per year), so g is 2 percent. It remains to consider the real after-tax mortgage rate, (1.2). Assume a relatively high nominal mortgage rate before tax of 6 percent, a 30 percent capital-income tax, and CPI inflation equal to 2 percent.[5] Then $r = 0.7 \cdot 6 - 2 = 2.2$ percent, so $\gamma = 2.2 + 7 + 2 = 11.2$ percent and the steady-state user cost without any capital-gains tax, $\gamma - g = 11.2 - 2 = 9.2$ percent.

Without any capital-gains tax, by (1.8), the housing price is $1/0.092 = 10.9$ times the value of housing services. Suppose that the value of housing services is about 30 percent of disposable income. Then, the housing price is about 3.3 times disposable income.

The capital-gains tax, τ^g, is 22 percent in Sweden. With the capital-gains tax, the steady-state user cost is $\gamma + \tau^g \pi^c - (1 - \tau^g)g = 11.2 + 0.22 \cdot 0.02 - (1 - 0.22)0.02 = 0.101$. Then the housing price is $1/0.101 = 9.9$ times the value of housing services, about 3 times disposable income with the above assumption.

The effect of the capital-gains tax is to reduce steady-state housing prices by about 8 percent. Since the Swedish capital-gains tax is not paid each year but only when the housing is sold, and it can be further postponed with the purchase of new housing, the effective – not to speak of the perceived – capital-gains tax is probably less than that, perhaps half .

2 The effect of changes in mortgage rates and taxes

2.1 The effect a temporary change in the mortgage rate

What is the short-run *partial* semi-elasticity of the housing price p_t with respect to the one-year mortgage rate i_t, that is, when future one-year mortgage rates are kept unchanged and there is no impact of the mortgage rate on the value of housing services and no impact on the future housing price. By the asset-pricing equation (1.11), we then have

$$\frac{\partial \ln p_t}{\partial i_t} = -\frac{\partial \ln[1 + \gamma_t - (1 - \pi^c)\tau^g]}{\partial i_t} = -\frac{\partial \gamma_t / \partial i_t}{1 + \gamma_t - (1 - \pi^c)\tau^g} = -\frac{1 - \tau^i}{1 + \gamma_t - (1 - \pi^c)\tau^g}. \quad (2.1)$$

With $\tau^i = 30$ percent, $\gamma_t = 11.2$ percent, $\pi^c = 2$ percent and $\tau^g = 22$ percent, the semi-elasticity is $-0.7/0.896 = -0.78$. An increase of the nominal one-year mortgage rate by 1 percentage point for one year reduces housing prices by 0.78 percent.

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[5] A steady-state nominal mortgage rate of 6 percent is consistent with a steady-state nominal repo rate of 4 percent (consistent with a 2 percent real repo rate and 2 percent inflation) and a spread of 2 percentage points between mortgage rate and the repo rate. If anything, these numbers are probably on the high side.

Suppose the capital-gains tax is zero, $\tau^g = 0$. Then the semi-elasticity is $-0.7/1.112 = -0.63$, smaller in magnitude than with the capital-gains tax. A 1 percentage point increase in the one-year nominal mortgage rate reduces housing prices by 0.63 percent. Since the denominator in (2.1) is decreasing in the capital-gains tax, the magnitude of the semi-elasticity is increasing in the capital-gains tax.

Since the above treatment of the capital-gains tax most likely exaggerates its impact, by assuming that it is not postponed, an intermediate value of the partial semi-elasticity, say -0.7, may be the best estimate.

What about the *total* semi-elasticity of the housing price with respect to the mortgage rate, taking into account possible effects on the current and future value of housing services and hence future housing prices? Suppose that variable mortgage rates vary one-to-one with the Riksbank's policy rate, the repo rate. That is, suppose that mortgage issuers keep a constant margin between variable mortgage rates and the repo rate. Then, we may consider the case when the repo rate is increased by 1 percentage point during year t only and future repo rates are kept unchanged at their original level. This results in a 1 percentage point higher one-year mortgage rate for year t but no change in future one-year mortgage rates.

Looking at the asset-pricing equation (1.5) and (1.6) (for simplicity disregarding the capital-gains tax), we realize that the issue is whether current and future housing services are affected or not by the repo rate. Log-linearizing the asset pricing equation around the steady state, we have the expression for the total semi-elasticity

$$\frac{d \ln p_t}{d i_t} = -\frac{\partial \ln(1 + \gamma_t)}{\partial i_t} + E_t \sum_{s=1}^{\infty} \kappa_s \frac{\partial \ln h_{t+s-1}}{\partial i_t} \tag{2.2}$$

where

$$\kappa_s = \frac{d_{t+s,t} h_{t+s-1}}{\sum_{j=1}^{\infty} d_{t+s,t} h_{t+s-1}} = \frac{\frac{1}{1+g} \left(\frac{1+g}{1+\gamma} \right)^s h_t}{h_t/(\gamma - g)} = \frac{\gamma - g}{1 + g} \left(\frac{1+g}{1+\gamma} \right)^s. \tag{2.3}$$

With the numbers above, $\kappa_1 = 0.083$, $\kappa_2 = 0.076, ...$ Thus, the first coefficient, κ_1, is just 0.083, and then the coefficients decrease exponentially by the factor $(1 + g)/(1 + \gamma) = 0.92$.[6] Even if the magnitude of the semi-elasticity of h_{t+s-1} for the first few years was relatively high, even as high as 1, each year's effect would be multiplied by the small number 0.083 or less. From an economic point of view, the value of the housing services for a particular housing owner would mainly depend on his or her permanent income (the present value of future disposable income plus wealth), which would be very little affected by a year's higher repo rate. Then, the semi-elasticity of housing services with respect to the repo rate would be quite low. Moreover, even if it were to take present values (and sums in (2.2)) not over infinity but over 15 or 20 years, corresponding to a realistic

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[6] Taking into account the capital-gains tax and using the asset pricing equations (1.11) and (1.12) , $\kappa_1 = 0.10$, $\kappa_2 = 0.08$ and κ_s falls faster, by a factor of 0.80.

length of housing ownership, the κ_s would be equally small. Intuitively, monetary policy can affect housing owners' disposable income over a couple of years, but not over 15 or more years.

The conclusion is that the partial semi-elasticity of the housing price with respect to the one-year mortgage rate in year t is a good approximation of the total semi-elasticity, so the effect through the value of future housing services can be disregarded.

2.2 The effect a temporary change in inflation expectations

What is the short-run partial semi-elasticity of the housing price p_t with respect to inflation expectations, $\mathrm{E}_t\pi_{t+1}^c$. By the asset-pricing equation (1.11), we then have

$$
\begin{aligned}
\frac{\partial \ln p_t}{\partial \mathrm{E}_t\pi_{t+1}^c} &= -\frac{\partial \ln[1+\gamma_t-(1-\mathrm{E}_t\pi_{t+1}^c)\tau^g]}{\partial \mathrm{E}_t\pi_{t+1}^c} = -\frac{\partial\gamma_t/\partial\mathrm{E}_t\pi_{t+1}^c+\tau^g}{1+\gamma_t-(1-\mathrm{E}_t\pi_{t+1}^c)\tau^g} \\
&= \frac{1-\tau^g}{1+\gamma_t-(1-\mathrm{E}_t\pi_{t+1}^c)\tau^g},
\end{aligned}
\tag{2.4}
$$

where I have used that γ_t depends on inflation expectations through the effect of inflation expectations on the real after-tax mortgage rate according to (1.2) and (1.4). The semi-elasticity is $0.78/0.896 = 0.87$, when the capital-income tax is included as above. If the capital-gains tax is disregarded ($\tau^g = 0$), the semi-elasticity is $1/1.112 = 0.90$. Thus, the semi-elasticity is not very sensitive to the capital-gains tax.

2.3 The effect of a permanent change in the mortgage rate

To assess the effect of permanent changes in the mortgage rate, we use the steady-state asset-pricing equation (1.13). The steady-state semi-elasticity with respect to i is then (1.13)

$$
\frac{\partial \ln p_t}{\partial i} = -\frac{\partial \ln[\gamma+\tau^g\pi^c-(1-\tau^g)g]}{\partial i} = -\frac{\partial\gamma/\partial i}{\gamma+\tau^g\pi^c-(1-\tau^g)g} = -\frac{1-\tau^i}{\gamma+\tau^g\pi^c-(1-\tau^g)g}. \tag{2.5}
$$

With the above numbers, the semi-elasticity is $-0.7/0.101 = -6.9$. Without the capital-gains tax ($\tau^g = 0$), it is -7.6. Thus, the semi-elasticities with respect to permanent changes in the mortgage rate is an order of magnitude larger.

Furthermore, since the numerator in (2.5) is small, about 0.1 instead of the 0.9 in (2.1), there is some nonlinearity, so one has to distinguish between the *infinitesimal* semi-elasticity, $\partial \ln p_t/\partial i$, computed for an infinitesimal change in the mortgage rate, and the *finite* semi-elasticity, $(\Delta p_t/p_t)/\Delta i_t$, computed for a finite change in the mortgage rate. For an increase of 1 percentage point in the mortgage rate, housing prices fall by 6.5 percent, less in magnitude than the infinitesimal semi-elasticity. For a decrease of 1 percentage point in the mortgage rate, housing prices rise by 7.5 percent, more in magnitude than the infinitesimal semi-elasticity. The infinitesimal semi-elasticity is hence between the finite semi-elasticities for an increase and a decrease in the mortgage rate. Without the capital-gains tax, the fall and the increase are 7.1 and 8.2 percent, respectively.

2.4 The effect of a permanent change in taxes

The steady-state infinitesimal semi-elasticity with respect to the *capital-income tax*, τ^i, is by (1.13)

$$\frac{\partial \ln p_t}{\partial \tau^i} = -\frac{\partial \ln[\gamma + \tau^g \pi^c - (1 - \tau^g)g]}{\partial \tau^i} = -\frac{\partial[\gamma + \tau^g \pi^c - (1 - \tau^g)g]/\partial \tau^i}{\gamma + \tau^g \pi^c - (1 - \tau^g)g} = \frac{i}{\gamma + \tau^g \pi^c - (1 - \tau^g)g}.$$

With the above numbers, the semi-elasticity is $0.06/0.101 = 0.60$. Without the capital-gains tax, it is 0.65. That is, increasing the capital-income tax rate by 1 percentage point, from 30 to 31 percent, increases housing prices by 0.60 percent without and 0.65 percent with the capital-gains tax.

Eliminating the deductibility of mortgage rate payments, here equivalent to setting the capital-income tax equal to zero, $\tau^i = 0$, reduces housing prices by 15.2 percent. Without the capital-gains tax, eliminating deductibility reduces housing prices by 8.4 percent. Clearly, eliminating the deductibility has a large effect.

The steady-state infinitesimal semi-elasticity with respect to effective *property and wealth taxes*, τ^h, is by (1.13)

$$\frac{\partial \ln p_t}{\partial \tau^h} = -\frac{\partial \ln[\gamma + \tau^g \pi^c - (1 - \tau^g)g]}{\partial \tau^h} = -\frac{\partial[\gamma + \tau^g \pi^c - (1 - \tau^g)g]/\partial \tau^h}{\gamma + \tau^g \pi^c - (1 - \tau^g)g} = -\frac{1}{\gamma + \tau^g \pi^c - (1 - \tau^g)g}.$$

The semi-elasticity is $-1/0.101 = -9.9$. Without the capital-gains tax, it is -10.9. Clearly, a permanent change in property and wealth taxes has a large effect on housing prices. With the capital-gains tax, increasing (reducing) the effective property and wealth tax rate by 1 percentage point reduces (increases) housing prices by 10 (11) percent.

The steady-state infinitesimal semi-elasticity with respect to the *capital-gains tax*, τ^g, is by (1.13)

$$\frac{\partial \ln p_t}{\partial \tau^g} = -\frac{\partial \ln[\gamma + \tau^g \pi^c - (1 - \tau^g)g]}{\partial \tau^g} = -\frac{\partial[\gamma + \tau^g \pi^c - (1 - \tau^g)g]/\partial \tau^g}{\gamma + \tau^g \pi^c - (1 - \tau^g)g} = -\frac{\pi^c + g}{\gamma + \tau^g \pi^c - (1 - \tau^g)g}.$$

The semi-elasticity is $-0.04/0.0964 = -0.40$. Without the capital-gains tax, here meaning that the capital gains tax is initially zero, the semi-elasticity is -0.44. The capital-gains tax has a relatively modest effect on housing prices.

Eliminating the capital-gains tax in a situation where it is fully internalized and paid each year would increase housing prices by 9.6 percent.

2.5 The effect of a permanent change in the growth rate of the value of housing services

The steady-state infinitesimal semi-elasticity with respect to the growth rate of the real value of housing services, g (which in steady state is also the growth rate of real housing prices) is by (1.13)

$$
\begin{aligned}
\frac{\partial \ln p_t}{\partial g} &= -\frac{\partial \ln[\gamma + \tau^g \pi^c - (1 - \tau^g)g]}{\partial g} = -\frac{\partial[\gamma + \tau^g \pi^c - (1 - \tau^g)g]/\partial g}{\gamma + \tau^g \pi^c - (1 - \tau^g)g} \\
&= \frac{1 - \tau^g}{\gamma + \tau^g \pi^c - (1 - \tau^g)g}.
\end{aligned}
\tag{2.6}
$$

The semi-elasticity is $0.78/0.101 = 7.4$. Without the capital-gains tax, it is 10.9. Thus, the impact of the steady-state growth rate is large, but the impact is substantially reduced by the capital-gains tax. We can interpret g as the expected long-run real growth of housing services, so we see that those expectations have a large impact.

2.6 The effect of a permanent change in the CPI inflation rate

What is the steady-state semi-elasticity with respect to CPI inflation? Here we must be precise about what is held constant when CPI inflation changes. Let r^* denote the steady-state real before-tax interest rate, and let it satisfy the Fisher equation,

$$
r^* = i - \pi^c.
$$

Furthermore, assume that r^* is constant when steady-states with different CPI inflation rates are considered. That is, the mortgage rate for this experiment considered to vary one-to-one with CPI inflation. This implies that the after-tax real interest rate satisfies

$$
r = (1 - \tau^i)i - \pi^c = (1 - \tau^i)r^* - \tau^i \pi^c.
\tag{2.7}
$$

The after-tax real interest rate is decreasing in CPI inflation for a given capital-income tax rate.

Then, by (2.7) and (1.13), we have

$$
\frac{\partial \ln p_t}{\partial \pi^c} = -\frac{\partial \ln[\gamma + \tau^g \pi^c - (1 - \tau^g)g]}{\partial \pi^c} = -\frac{\partial[\gamma + \tau^g \pi^c - (1 - \tau^g)g]/\partial \pi^c}{\gamma + \tau^g \pi^c - (1 - \tau^g)g} = \frac{\tau^i - \tau^g}{\gamma + \tau^g \pi^c - (1 - \tau^g)g}.
$$

Hence, whether the semi-elasticity with respect to inflation is positive or negative depends on the difference between the capital-income tax and the capital-gains tax. With the above numbers, the semi elasticity is $(0.30 - 0.22)/0.101 = 0.79$. Without the capital-gains tax, it is 3.3.

3 Sensitivity to assumptions

How sensitive to the assumptions made in section 1.4 about the numerical value of γ are the elasticities calculated here? We realize that the semi-elasticities of housing prices with respect to

temporary changes in mortgage rates and inflation expectations are less sensitive to the assumptions, since according to (2.1) and (2.4) they have a denominator equal to $1 + \gamma_t$ and $1 + \gamma_t - (1 - \mathrm{E}_t \pi_{t+1}^c)$, respectively. Since γ may be around 0.11, $1 + \gamma$ is not so sensitive to modest changes in γ. The elasticity of the denominator with respect to γ is $\gamma/(1 + \gamma) = 0.1$.

In contrast, the semi-elasticities of housing prices with respect to *permanent* changes in the mortgage rate, taxes, and the value of housing services have a denominator equal to $\gamma + \tau^g - (1 - \tau^g)g$ according to (2.5)-(2.6) . When the capital-gains tax is disregarded ($\tau^g = 0$), the denominator is $\gamma - g = 0.092$. The elasticity of the denominator is then $\gamma/(\gamma - g) = 1.2$, an order of magnitude larger than for temporary changes.

For monetary-policy issues, the semi-elasticities with respect to temporary changes will often be the most relevant (see Svensson (2013)). For taxation and tax-deductibility issues, the semi-elasticities with respect to permanent change will often be the most relevant.

4   Summary of results

The results about the semi-elasticities are summarized in table 1.

Table 1. The effect on housing prices in percent of a temporary change in the one-year mortgage rate and in CPI inflation expectations, of a permanent change in the mortgage rate, tax rates, the growth rate of real value of housing services, and the CPI inflation rate, and of the elimination of the deductibility of the capital-income tax and the capital-gains tax.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Semi-elasticity				**Elimination**	
	Infinitesimal		Finite		of deductability	
Variable	CGT	No CGT	+1 pp	−1 pp	CGT	No CGT
Mortgage rate, temp.	− 0.78	− 0.63				
Inflation expectations, temp.	0.87	0.90				
Mortgage rate, permanent	− 6.94	− 7.61	− 6.5	− 7.5		
Capital-income tax	0.60	0.65			− 15.2	− 8.4
Property and wealth tax	− 9.92	−10.87	− 9.0	− 11.0		
Capital-gains tax	− 0.40	− 0.44			9.6	
Housing services growth	7.74	10.87				
CPI inflation	0.79	3.26				

Note: CGT refers to the case when the capital-gains tax is fully internalized, including that the tax is paid each year. No CGT refers to the case when the capital-gains tax is disregarded. Columns (2) and (3) report the infinitesimal semi-elasticity. Columns (4) and (5) report the finite semi-elasticity of housing prices with respect to plus and minus 1 percentage point change in the variable only for the case when the capital-gains tax is fully internalized. When the capital-gains tax is disregarded, the magnitudes are somewhat larger. Columns (6) and (7) report the change in percent of housing prices with respect to an elimination of the deductibility of the capital-income tax and to an elimination of the capital-gains tax when it is fully internalized before the elimination.

5   Conclusions

The paper presents some simple calculations of the effect on housing prices of temporary changes in the one-year mortgage rate and permanent changes in short and long mortgage rates, the capital-income tax, the effective property and wealth tax, the capital-gains tax, the expected growth rate of the value of housing services, and the CPI inflation rate. A new element in the calculation is to take the capital-gains tax on housing into account. The semi-elasticity of housing prices with respect to *temporary* changes in the one-year mortgage rates is quite small. This semi-elasticity is less sensitive to the assumptions about the parameters of the user-cost model. The semi-elasticities of permanent changes in mortgage rates, taxes and the tax-deductibility of mortgage rates are substantial. These semi-elasticities are more sensitive to the assumptions about the parameters.

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