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Requiem for Forecast-Based Instrument Rules

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Abstract

Frequently used so-called forecast-based instrument rules are shown to correspond to a problematic intertemporal loss function. This loss function is problematic because it is time-inconsistent, arbitrary, and (for the most common form of forecast-based instrument rules) does not incorporate any explicit concern for output-gap stability.

The time-inconsistency of the loss function arises because it does not have exponential discounting. This implies that inflation paths that are optimal ex ante will not be realized ex post. This is a source of inefficiency separate from the one discussed in the literature initiated by Kydland and Prescott and by Barro and Gordon.

It follows that forecast-based instrument rules are not consistent with a conventional intertemporal loss function corresponding to flexible inflation targeting.

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1 Introduction

In the monetary-policy literature, a frequently used class of reaction functions lets the monetary-policy instrument in period t, a short interest rate i_t, respond to an equilibrium inflation forecast and a lagged value of itself,

$$i_t = \bar{f} + f_\pi(\pi_{t+T,t} - \pi^*) + f_i i_{t-1}, \tag{1.1}$$

where $\pi_{t+T,t}$ denotes a T-period-ahead inflation forecast conditional on information available in period t, π^* is a constant inflation target, and $\bar{f}$ is a constant and f_π and f_i are constant response coefficients. To be consistent with a steady state, the constant $\bar{f}$ has to fulfill $\bar{f} = \bar{\imath}/(1 - f_i)$, where $\bar{\imath}$ is the steady-state interest rate. This reaction function is promoted by, for instance, Batini and Haldane [2], and similar reaction functions are used in the Quarterly Projection Model (QPM) of the Bank of Canada [6] and the Forecasting and Policy System (FPS) of the Reserve Bank of New Zealand [4].[1] A somewhat broader class of reaction functions would also involve responses to an output-gap forecast, for instance, with the same horizon T,

$$i_t = \bar{f} + f_\pi(\pi_{t+T,t} - \pi^*) + f_x x_{t+T,t} + f_i i_{t-1}, \tag{1.2}$$

where f_x is a response coefficient and $x_{t+T,t}$ is a T-period-ahead output-gap forecast.

This class of reaction functions has been referred to as *forecast-based* (instrument) rules (Batini and Haldane [2]). For a forecast-horizon T sufficiently long, the forecasts depend on the instrument and are endogenous. For such a horizon, the forecasts in most applications have been taken to be *equilibrium*, or "rule-consistent" forecasts, meaning that they are endogenous rational-expectations forecasts conditional on an intertemporal equilibrium of the model.[2] Thus, this reaction function is really an equilibrium condition that has to be satisfied by simultaneously determined variables. This is called an *implicit* instrument rule in Rudebusch and Svensson [13], Svensson [16] and Svensson and Woodford [19] (reaction functions where the instrument responds to predetermined variables are called *explicit* instrument rules).

This paper shows that forecast-based instrument rules of the form (1.1) or (1.2) correspond to a problematic and arbitrary intertemporal loss function. That loss function is time-inconsistent, in the sense that plans that are optimal relative to the loss function ex ante are not carried out ex post. This is because the loss function does not have exponential discounting, which, as

[1] A similar reaction function is also used by Black, Macklem and Rose [5].

[2] Another possibility is to let the forecasts depend on an exogenous interest rate path, for instance a constant unchanged interest rate, see Jansson and Vredin [9] and Rudebusch and Svensson [13].

shown in the classic paper of Strotz [15], is a necessary condition for time-consistent preferences. The time-inconsistency of the loss function is a source of inefficiency separate from the one in the usual time-consistency problem.

In the traditional literature on the time-consistency problem following Kydland and Prescott [10] and Barro and Gordon [1], the source of the time-consistency problem is the time-inconsistency of the *constraints* of the optimization problem, the fact that the marginal rates of *transformation* between the target variables are different ex ante and ex post (typically, increased inflation increases employment ex post but not ex ante). For a loss function without exponential discounting, there is a separate time-consistency problem, due to the time-inconsistency of the *objective function* in the optimization problem, the fact that the marginal rates of *substitution* between the target variables are different ex ante and ex post. Clearly, time-consistent preferences is a minimum requirement for a monetary-policy loss function.

Furthermore, the commonly used form (1.1) of forecast-based instrument rules implies a loss function without any explicit weight on output-gap stability, and is hence more remote from the idea of "flexible" inflation targeting in the sense of some concern also for output-gap stability. (The form (1.2) involves some concern for output-gap stability, though.)

Section 2 specifies the conventional intertemporal loss function corresponding to flexible inflation targeting and discusses some of its properties. Section 3 derives the loss function corresponding to forecast-based instrument rules and discusses its properties. Section 4 in addition discusses the practical performance of forecast-based instrument rules. Appendix A contains some technical details.

2 A loss function over conditional forecasts corresponding to flexible inflation targeting

Inflation targeting involves stabilizing inflation around an inflation target. In practice, as discussed in a number of recent contributions (see, for instance, Federal Reserve Bank of Kansas City [7]), inflation targeting is "flexible" inflation targeting, in the sense that it also involves some concern about the stability of the real economy.[3] These objectives are conventionally and conveniently expressed as an intertemporal loss function, to be minimized in period t, consisting

[3] I abstract from independent objective to smooth or stabilize interest rates, which objective is difficult to rationalize. See Sack and Wieland [14] for a recent discussion of interest-rate smoothing.

of the expected sum of discounted current and future losses,

$$L_t = (1-\delta)\mathrm{E}_t \frac{1}{2} \sum_{\tau=0}^{\infty} \delta^\tau \left[(\pi_{t+\tau} - \pi^*)^2 + \lambda x_{t+\tau}^2 \right]. \tag{2.1}$$

Here E_t denotes rational expectations conditional on the central bank's information in period t about the state of the economy and the transmission mechanism of monetary policy, δ ($0 < \delta < 1$) is a discount factor, $\pi_t - \pi^*$ is the inflation "gap", where π_t is (the rate of) inflation in period t and π^* is a given inflation target, $x_t \equiv y_t - y_t^*$ is the output gap, where y_t is (log) output and y_t^* is (log) potential output (the (log) of the natural output level), and $\lambda > 0$ is a given weight on output-gap stabilization. "Strict" inflation targeting would be the (unrealistic) special case of $\lambda = 0$. Thus, for $\lambda > 0$, we have flexible inflation targeting and both inflation and the output gap are target variables (target variables in the sense of entering the loss function).[4]

Note that in the limit when the discount factor approaches unity, $\delta \to 1$, (2.1) can be shown to equal

$$L_t = \frac{1}{2}\{(\mathrm{E}[\pi_t] - \pi^*)^2 + \mathrm{Var}[\pi_t] + \lambda \mathrm{E}[x_t]^2 + \lambda \mathrm{Var}[x_t]\}.$$

Thus, if $\mathrm{E}[\pi_t] = \pi^*$ and $\mathrm{E}[x_t] = 0$, the intertemporal loss function is

$$L_t = \frac{1}{2}(\mathrm{Var}[\pi_t] + \lambda \mathrm{Var}[x_t]), \tag{2.2}$$

the weighted unconditional variances, which is frequently used as a loss function for monetary policy.[5]

Suppose the central bank has a linear model of the transmission mechanism. Let $\pi^t \equiv \{\pi_{t+\tau,t}\}_{\tau=0}^{\infty}$ and $x^t \equiv \{x_{t+\tau,t}\}_{\tau=0}^{\infty}$ be conditional mean forecasts of inflation and the output gap, conditional on information available in period t, the bank's model of the transmission mechanism, the bank's mean forecast of exogenous variables, and a given instrument-rate path $i^t \equiv \{i_{t+\tau,t}\}_{\tau=0}^{\infty}$. The conditional forecasts of inflation and the output gap and the corresponding instrument-rate path are given by the bank's forecasting model, which results from the bank's model of the transmission mechanism of monetary policy, when endogenous and exogenous

[4] Note that, since the intertemporal loss function is the *expected* discounted future losses, this formulation includes the realistic case when potential output, y_t^*, is unobservable and has to be estimated.

[5] A common way of evaluating the outcome of alternative instrument rules is to plot the result in a graph with unconditional inflation variance on the horisontal axis and unconditional output-gap variance on the vertical axis and then examine the result in relation to the "Taylor curve" (see Taylor [20]) of efficient combinations of the two variances. This is of course equivalent to using a loss function of the form (2.2) with different relative weights $\lambda \geq 0$. Indeed, a common way to find the Taylor curve is to optimize over a class of reaction functions for values of λ from zero to infinity. See, for instance, Rudebusch and Svensson [13] and several other papers in Taylor [21]. (Taylor [20] plotted the standard deviations along the axes; plotting the variances has the advantage that the (negative) slope at a preferred point on the Taylor curve can be interpreted as revealing $1/\lambda$ in the loss function above.)

variables are replaced by their conditional means, conditional on the information available in period t.[6]

Because of the certainty-equivalence that holds under a linear model and a quadratic loss function, we can substitute the conditional forecasts into the intertemporal loss function (2.1) and consider the intertemporal forecast over forecasts,

$$L_{t,t} \equiv (1-\delta)\frac{1}{2}\sum_{\tau=0}^{\infty}\delta^{\tau}\left[(\pi_{t+\tau,t}-\pi^*)^2+\lambda x^2_{t+\tau,t}\right]. \tag{2.3}$$

The decision problem has now been modified from the stochastic optimization problem of minimizing (2.1) subject to the bank's stochastic model of the transmission mechanism to the deterministic optimization problem of minimizing (2.3) subject to the bank's forecasting model (which, given the forecast of exogenous variables, is deterministic). Because of the certainty-equivalence, the intertemporal loss functions (2.1) and (2.3) will only differ by a constant (consisting of the discounted expected future unanticipated squared shocks to inflation and the output gap), so optimization over (2.1) and (2.3) will lead to the same policy.

The intertemporal loss functions (2.1) and (2.3) have some desirable properties:

1. There is substitution between inflation variability and output-gap variability, corresponding to flexible inflation targeting. This is easy to see in the limit case, when $\delta \to 1$ and the loss function (2.2) can be written

$$L_t = \frac{1}{2}(\mathrm{Var}[\pi_{t+\tau}-\pi^*]+\lambda \mathrm{Var}[x_{t+\tau}]),$$

where $\mathrm{Var}[\pi_{t+\tau}-\pi^*]$ and $\mathrm{Var}[x_{t+\tau}]$ denote the unconditional variance of the future inflation and output gaps. Then the relative weight λ can be seen as the marginal rate of substitution of inflation-gap variability for output-gap variability, the (negative of the) derivative $d\mathrm{Var}[\pi_{t+\tau}-\pi^*]/d\mathrm{Var}[x_{t+\tau}]$ when the intertemporal loss is held constant, that is,

$$-\left.\frac{d\mathrm{Var}[\pi_{t+\tau}-\pi^*]}{d\mathrm{Var}[x_{t+\tau}]}\right|_{dL_t=0} = \lambda.$$

2. There is intertemporal substitution between inflation and output gaps in different periods. Differentiating (2.3) with respect to a change in the inflation-gap forecast for period $t+T_1$,

[6] Constructing conditional forecasts in a backward-looking model (that is, a model without forward-looking variables) is straightforward. Constructing such forecasts in a forward-looking model raises some specific difficulties, which are explained and resolved in the appendix of the working-paper version of Svensson [16]. The conditional forecasts for an arbitrary interest-rate path derived there assume that the interest-rate paths are "credible", that is, anticipated and allowed to influence the forward-looking variables.

$d(\pi_{t+T_1,t} - \pi^*)$, and the output-gap forecast for period $t + T_2$, $dx_{t+T_2,t}$, and holding the resulting change in the intertemporal loss constant, $dL_{t,t} = 0$, we get the marginal rate of substitution of the inflation-gap forecast in period $t + T_1$ for the output-gap forecast in period $t + T_2$,

$$-\left.\frac{d(\pi_{t+T_1,t} - \pi^*)}{dx_{t+T_2,t}}\right|_{dL_{t,t}=0} = \frac{\delta^{T_2-T_1} x_{t+T_2,t}}{\pi_{t+T_1,t} - \pi^*}.$$

3. The loss function has exponential discounting, which is a necessary condition for time-consistent preferences, as noted by Strotz [15]: The marginal rate of substitution of target variables in period $t + T_1$ for target variables in period $t + T_2$ is independent of the period t. Consider the inflation-gap forecasts for period $t + T_1$ and $t + T_2$. The marginal rate of subsitution of the inflation-gap forecast for period $t + T_1$ for the inflation-gap forecast for period $t + T_2$ is

$$-\left.\frac{d(\pi_{t+1,t} - \pi^*)}{d(\pi_{t+2,t} - \pi^*)}\right|_{dL_{t,t}=0} = \frac{\delta^{T_2-T_1}(\pi_{t+T_2,t} - \pi^*)}{\pi_{t+T_1,t} - \pi^*},$$

which is independent of period t. As Strotz observed, this requires exponential discounting with a constant discount factor, which is the case for the loss functions (2.1) and (2.3).

Time-consistency of the loss function implies that future paths of the target variables that are optimal when considered in a given period remain optimal when considered in later periods (assuming that the *constraints*, the marginal rates of *transformation*, for the target variables are time-consistent). Suppose that, in period t, the particular future inflation and output-gap paths $\hat{\pi}^t = \{\hat{\pi}_{t+\tau,t}\}_{\tau=0}^{\infty}$ and $\hat{x}^t = \{\hat{x}_{t+\tau,t}\}_{\tau=0}^{\infty}$ are optimal with respect to (2.3) in period t, $L_{t,t}$. Consider the continuation of these inflation and output-gap paths in period $t + s$, s periods later, denoted $\hat{\pi}^{t+s} = \{\hat{\pi}_{t+\tau,t+s}\}_{\tau=s}^{\infty}$ and $\hat{x}^{t+s} = \{\hat{x}_{t+\tau,t+s}\}_{\tau=s}^{\infty}$, where $\hat{\pi}_{t+\tau,t+s} = \hat{\pi}_{t+\tau,t}$ and $\hat{x}_{t+\tau,t+s} = \hat{x}_{t+\tau,t}$. Time-consistency of the loss function then implies the intuitive property that the paths $\hat{\pi}^{t+s}$ and $\hat{x}^{t+s}$ remain optimal when evaluated with the loss function (2.3) in period $t+s$, $L_{t+s,t+s}$. Thus, the mere passage of time does not affect the optimality of the continuation of given paths of inflation and output.

In contrast, a lack of time-consistency of the loss function implies that the mere passage of time results in the continuation of the same inflation and output-gap paths *not* being optimal (even if the constraints, the marginal rates of transformation between the target variables are time-consistent). This is very counterintuitive, and time-consistency of the loss function seems to be a minimum requirement for sensible intertemporal preferences for a central bank.[7]

[7] Utility functions with so-called hyperbolic discounting (see, for instance, Laibson [11]) are time-inconsistent

3 The intertemporal loss function corresponding to a forecast-based instrument rule

The forecast-based instrument rule implies that the instrument path, i^t, and the inflation and output gap forecasts, π^t and x^t, will be related by

$$i_{t+\tau,t} = \bar{f} + f_\pi(\pi_{t+T+\tau,t} - \pi^*) + f_x x_{t+T+\tau,t} + f_i i_{t+\tau-1,t} \tag{3.1}$$

for $\tau \geq 0$. Suppose that (3.1) together with the bank's forecasting model result in unique inflation and output gap forecasts and instrument-rate path. Let the corresponding impulse responses of inflation and the output gap for an impulse to i_t in period t be denoted $\{\partial\pi_{t+\tau,t}/\partial i_t\}_{\tau=0}^{\infty}$ and $\{\partial x_{t+\tau,t}/\partial i_t\}_{\tau=0}^{\infty}$, respectively, so $\partial\pi_{t+\tau,t}/\partial i_t$ denotes the impulse response of $\pi_{t+\tau}$ to an impulse to i_t in period t, etc.[8]

For what implied loss function would (3.1) be optimal? That is, for what implied loss function would (3.1) be a first-order condition for an optimum? The intertemporal loss function in period t corresponding to (3.1) for $\tau = 0$ turns out to be

$$\tilde{L}_{t,t} \equiv \frac{1}{2}\delta^T[(\pi_{t+T,t} - \pi^*)^2 + \lambda x_{t+T,t}^2] + \lambda_i(i_{t,t} - \bar{\imath})^2 + \lambda_{\Delta i}(i_{t,t} - i_{t-1,t})^2, \tag{3.2}$$

where $\bar{\imath}$ is the steady-state instrument rate and the weights λ, λ_i and $\lambda_{\Delta i}$ are given by

$$\bar{f} \equiv \frac{\lambda_i}{\lambda_i + \lambda_{\Delta i}}\bar{\imath}, \quad f_\pi \equiv \frac{-\delta^T \partial\pi_{t+T,t}/\partial i_t}{\lambda_i + \lambda_{\Delta i}}, \quad f_x \equiv \frac{-\delta^T \lambda \partial x_{t+T,t}/\partial i_t}{\lambda_i + \lambda_{\Delta i}}, \text{ and } f_i \equiv \frac{\lambda_{\Delta i}}{\lambda_i + \lambda_{\Delta i}}. \tag{3.3}$$

In order to show this, we can use that the derivative of (3.2) with respect to an impulse to $i_{t,t}$ must be zero. This results in

$$\delta^T \frac{\partial\pi_{t+T,t}}{\partial i_t}(\pi_{t+T,t} - \pi^*) + \delta^T \lambda \frac{\partial x_{t+T,t}}{\partial i_t} x_{t+T,t} + \lambda_i(i_{t,t} - \bar{\imath}) + \lambda_{\Delta i}(i_{t,t} - i_{t-1,t}) = 0. \tag{3.4}$$

Solving (3.4) for $i_{t,t}$ results in an equilibrium condition exactly like (3.1) for $\tau = 0$, where the parameters $\bar{f}$, f_π, f_x and f_i are given by (3.3).

Thus, take as given the parameters $\bar{f}$, f_π, f_x, f_i (and hence $\bar{\imath}$) and T of the forecast-based instrument rule (3.1), as well as the resulting equilibrium impulse responses $\partial\pi_{t+T,t}/\partial i_t$ and $\partial x_{t+T,t}/\partial i_t$. Furthermore, normalize the parameters λ, λ_i, $\lambda_{\Delta i}$ and δ of the loss function in some way; that is, add a condition that they have to fulfill. For instance, suppose we fix δ, $0 < \delta < 1$. Given δ, T and $\partial\pi_{t+T,t}/\partial i_t$, the identity for f_π in (3.3) allows us to determine the

and give rise to time-inconsistent plans, since they do not have exponential discounting.

[8] That is, $\partial\pi_{t+\tau,t}/\partial i_t$ denotes the derivative of $\pi_{t+\tau,t}$ with respect to a one-time shock to i_t.

sum $\lambda_i + \lambda_{\Delta i}$. Given this sum, the identity for f_i allows us to determine λ_i and $\lambda_{\Delta i}$. Finally, given $\partial x_{t+T,t}/\partial i$, the identity for f_x allows us to determine λ.

Thus, according to the loss function (3.2), in period t the central bank is concerned about stabilizing the inflation and output gaps precisely T periods ahead (with the relative weight λ on output-gap stabilization), in addition to putting the weight λ_i on current interest-rate stabilization and the weight $\lambda_{\Delta i}$ on current interest-rate smoothing.

The loss function (3.2) is problematic for several reasons:

1. The loss function does not have exponential discounting, since it does not consist of an exponentially discounted sum across periods. Indeed, there is *no* trade-off between the same variable in different periods. In period t, the central bank is only concerned with stabilizing inflation and output gaps in period $t+T$, that is, T periods ahead, in addition to instrument stabilization and smoothing in the current period t. The bank is indifferent to inflation- and output-gap stabilization in period $t+ T+1$, for instance, as well as to instrument stabilization and smoothing in period $t+1$. However, when making decisions in the next period, period $t+1$, the bank will be concerned about inflation- and output-gap stabilization precisely in period $T+1$, and instrument stabilization and smoothing precisely in period $t+1$ (and completely indifferent to inflation- and output-gap stabilization in period $t+T$). This highlights the time-inconsistency of the preferences corresponding to the loss function (3.2).

   It follows that the equilibrium resulting from combining the forecast-based instrument rule (3.1) with the model has to be interpreted as an equilibrium under discretionary optimization in each period t, when the central bank anticipates that it will reoptimize with the loss function (3.2) in period $t+1$ and hence choose the instrument rate according to (3.1) in period $t+1$. Thus, the equilibrium can be seen as the result of choosing i_t so as to minimize (3.2) in period t subject to (3.1) being followed in future periods.

2. Instrument stabilization and smoothing has a crucial role, in spite of the difficulties in finding a rationale for such behavior. Thus, as is apparent in (3.4), the sum of the weights λ_i and $\lambda_{\Delta i}$ must be nonzero for the parameters $\bar{f}$, f_π, f_x and f_i to be finite and not unbounded (and the sum must be positive to make intuitive sense). Furthermore, a value of f_i fulfilling $0 < f_i < 1$ requires both λ_i and $\lambda_{\Delta i}$ positive (the case $\lambda_i = 0$, no instrument stabilization objective, would lead to $f_i = 1$).

3. The forecast-based instrument rules used by Batini and Haldane [2] and the QPM [6] and FPS [4] have $f_x = 0$ and hence do not respond to any output-gap forecast. According to (3.3), this implies $\lambda = 0$, that is, no preference for output-gap stabilization, counter to the idea of flexible inflation targeting. Thus, any concern for output-gap stabilization then enters only very indirectly, through a longer horizon T (which is usually taken to be 6–8 quarters in the previous references).

4. Finally, for given "reasonable" values of the parameters f_π, f_x and f_i, there is no guarantee that the resulting weights λ, λ_i and $\lambda_{\Delta i}$ and discount factor δ are reasonable. It is easy to calculate the weights and the discount factor and judge how reasonable they are, though, once the impulse responses $\partial\pi_{t+T,t}/\partial i_t$ and $\partial x_{t+T,t}/\partial i_t$ have been computed.

If this is not at least a theoretical requiem for forecast-based instrument rules, what is?

4 Practical performance

So, if there are severe, perhaps fatal, theoretical problems with forecast-based instrument rules, what about the practical performance? Recently, Levin, Wieland and Williams [8] have, by simulations in different macro models, examined the performance of forecast-based instrument rules against other reasonable reaction functions. They find that its performance of forecast-based instrument rules are rather inferior and nonrobust, when evaluated according to (2.1) or the special case (2.2). This finding is not surprising, given the result in the present paper that an equilibrium with a forecast-based instrument rule corresponds to a discretionary equilibrium with a very different loss function, (3.2).[9]

All together, the forecast-based instrument rule with equilibrium forecasts seems quite problematic, in spite of its entrenched position in the QPM of the Bank of Canada [6] and the FPS of the Reserve Bank of New Zealand [4].[10]

[9] In a recent paper, Batini, Harrison and Millard [3] examine a forecast-based and other instrument rules in a model of an open economy. A forecast-based instrument rule of the form (1.1) with parameters similar to those used in Batini and Haldane [2] ($T = 5$ quarters, $f_\pi = 5$, $f_i = .5$) works very badly in that model. An optimized forecast-based instrument rule works well. However, the optimized horizon T is only one quarter. In the Rudebusch and Svensson [13] model, such an instrument rule works very badly. Thus, the results of Batini, Harrison and Millard [3] are hardly supportive of forecast-based instrument rules.

[10] In my review of the operation of monetary policy in New Zealand, [17], I suggest that the Reserve Bank of New Zealand consider alternatives to forecast-based instrument rules in the FPS, with reference to the problems reported here.

A The optimal equilibrium condition

If (1.1) is not an optimal first-order condition for the loss function (2.1), what is? What is the optimal first-order condition for (2.3) and how does it differ from (3.1)?

Consider an equilibrium optimal for (2.3) and let $\{\partial\pi_{t+\tau,t}/\partial i_t\}_{\tau=0}^{\infty}$ and $\{\partial x_{t+\tau,t}/\partial i_t\}_{\tau=0}^{\infty}$ be corresponding impulse responses. A potential first-order-condition is that the derivative of (2.3) with respect to an impulse to $i_{t,t}$ will be zero, giving

$$\sum_{\tau=0}^{\infty} \delta^{\tau} \left[\frac{\partial \pi_{t+\tau,t}}{\partial i_t} (\pi_{t+\tau,t} - \pi^*) + \lambda \frac{\partial x_{t+\tau,t}}{\partial i_t} x_{t+\tau,t} \right] = 0. \tag{A.1}$$

This is clearly different from (3.1), in that terms for *all* future periods appear and that *no* terms with explicit instrument rates appear.

However, a bit of caution in the interpretation of this first-order condition is warranted. It corresponds to an unanticipated one-time adjustment in the instrument rate in period t, which is a meaningful experiment under discretionary optimization each period, but not under commitment to a given reaction function. In the latter case, only *anticipated* one-time adjustments in the instrument rate in period t are allowed. This instead correspond to the first-order condition

$$\sum_{\tau=-\infty}^{\infty} \delta^{\tau} \left[\frac{\partial \pi_{t+\tau,t}}{\partial i_t} (\pi_{t+\tau,t} - \pi^*) + \lambda \frac{\partial x_{t+\tau,t}}{\partial i_t} x_{t+\tau,t} \right] = 0, \tag{A.2}$$

where the summation from $\tau = -\infty$ indicates optimality "in a timeless perspective" discussed in Woodford [22], Svensson and Woodford [19] and Svensson [18]. Then the impulse responses for $\tau < 0$ should be interpreted as the response to an anticipated future impulse.

Of course, the first-order condition (A.2) is hardly an operational "targeting rule" ("targeting rule" meaning a condition for the target variables). As discussed in Svensson and Woodford [19] and Svensson [18], for a common forward-looking Phillips curve (where current inflation is predetermined and one-period-ahead inflation is determined by a forward-looking Phillips curve), an equivalent and simpler optimal targeting rule can be formulated as

$$\pi_{t+1+\tau,t} - \pi^* = -\frac{\lambda}{\kappa}(x_{t+1+\tau,t} - x_{t+\tau,t}),$$

for $\tau \geq 0$, where, for $\tau = 0$, $x_{t,t}$ denotes $x_{t,t-1}$, the one-period-ahead output-gap forecast in period $t-1$, and κ is the slope of the short-run Phillips curve.

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