Recent and Prospective Developments in Monetary Policy

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Questions

- How to implement optimal monetary policy?
- How to reconcile theory and practice in monetary policy?
Outline

Optimal policy

- Traditional, optimal policy function
- Projections, not policy functions
- Forecast targeting (mean)
- Judgment
- Non-certainty equivalence: Distribution forecast targeting

Possible technical problems

- Determinacy (Woodford)
- Commitment in a timeless perspective
Outline

- Decision process
  - Staff presents feasible alternatives to MPC
  - MPC chooses optimal projections
- Announcing optimal projections
- Measures of resource utilization
- Using explicit loss functions
Optimal monetary policy: Standard approach

- Linear model (decision in period $t$, $X_t$ given, $\tau \geq 0$)

\[
\begin{bmatrix}
X_{t+\tau+1} \\
HE_{t+\tau}x_{t+\tau+1}
\end{bmatrix}
= A \begin{bmatrix}
X_{t+\tau} \\
x_{t+\tau}
\end{bmatrix}
+ Bi_{t+\tau} + \begin{bmatrix}
C \\
0
\end{bmatrix} \varepsilon_{t+\tau+1}
\]

- $X_t$ predetermined variables, $x_t$ forward-looking variables, $i_t$ policy instrument(s), $\varepsilon_t$ zero-mean i.i.d. shocks

- Determination of $X_{t+1}$, given $X_t$, $x_t$, $i_t$, $\varepsilon_{t+1}$:

\[X_{t+1} = A_{11}X_t + A_{12}x_t + B_1i_t + C\varepsilon_{t+1}\]

- Determination of $x_t$, given $E_tHx_{t+1}$, $X_t$, $i_t$:

\[E_tHx_{t+1} = A_{21}X_t + A_{22}x_t + B_2i_t\]
Optimal monetary policy: Standard approach

- **Quadratic intertemporal loss function**

\[ L_t = E_t \sum_{\tau=0}^{\infty} (1 - \delta) \delta^\tau L_{t+\tau} \]

**Period loss**

\[ L_{t+\tau} = Y'_{t+\tau} \Lambda Y_{t+\tau} \]

**Target variables**

\[ Y_{t+\tau} = D \begin{bmatrix} X_{t+\tau} \\ x_{t+\tau} \\ i_{t+\tau} \end{bmatrix} \]

- **Flexible inflation targeting**

\[ L_t = (\pi_t - \pi^*)^2 + \lambda_y (y_t - \bar{y}_t)^2 + \lambda_i (i_t - i_{t-1})^2 \]

\[ y_t - \bar{y}_t \text{ output gap, measure or resource utilization} \]
Optimal monetary policy: Standard approach

- Optimal policy function (given $X_t, \Xi_{t-1}$)
  \[
i_t = F_i \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}
  \]

- Solution (given $X_t, \Xi_{t-1}, \tau \geq 0$)
  \[
  \begin{bmatrix}
  x_{t+\tau} \\
  i_{t+\tau}
  \end{bmatrix}
  = F_x \begin{bmatrix} F_x \\ F_i \end{bmatrix} \begin{bmatrix} X_{t+\tau} \\ \Xi_{t+\tau-1} \end{bmatrix}
  \]
  \[
  \begin{bmatrix}
  X_{t+\tau+1} \\
  \Xi_{t+\tau}
  \end{bmatrix}
  = M \begin{bmatrix} X_{t+\tau} \\ \Xi_{t+\tau-1} \end{bmatrix} + \begin{bmatrix} C \\ 0 \end{bmatrix} \Xi_{t+\tau+1}
  \]

- History dependence: Determination of $\Xi_{t-1}$
  \[
  \Xi_{t-1} = M_{\Xi X} X_{t-1} + M_{\Xi \Xi} \Xi_{t-2} = \sum_{\tau=1}^{\infty} M_{\Xi \Xi}^{\tau-1} M_{\Xi X} X_{t-\tau}
  \]
Optimal monetary policy: Projections

- Projection (mean): Notation: $z^t \equiv \{z_{t+\tau,t}\}_{\tau=0}^\infty$

- Optimal (mean) projections ($\hat{X}^t, \hat{x}^t, \hat{i}^t, \hat{Y}^t$) (conditional on current state of economy, model of transmission mechanism, loss function)

\[
\begin{bmatrix}
\hat{i}_{t+\tau,t} \\
\hat{x}_{t+\tau,t}
\end{bmatrix} = \begin{bmatrix} F_i \\ F_x \end{bmatrix} \begin{bmatrix} \hat{X}_{t+\tau,t} \\ \hat{\Xi}_{t+\tau-1,t} \end{bmatrix}, \quad \hat{Y}_{t+\tau,t} = D \begin{bmatrix} \hat{X}_{t+\tau,t} \\ \hat{x}_{t+\tau,t} \\ i_{t+\tau,t} \end{bmatrix}
\]

\[
\begin{bmatrix}
\hat{X}_{t+\tau+1,t} \\
\hat{\Xi}_{t+\tau,t}
\end{bmatrix} = M \begin{bmatrix} \hat{X}_{t+\tau,t} \\ \hat{\Xi}_{t+\tau-1,t} \end{bmatrix}, \quad \hat{X}_{t,t} = X_t
\]

- Current state of economy, estimate $X_{t|t}$

\[
\hat{X}_{t,t} = X_{t|t} \quad (\text{instead of} \quad \hat{X}_{t,t} = X_t)
\]

Details: Svensson-Woodford 03, “Indicator Variables for Optimal Policy”
Feasible set of projections (conditional on (estimate of) current state of economy, model of transmission mechanism)

\[(X^t, x^t, i^t, Y^t) \in \mathcal{T}(X_{t|t})\]

Projection model

\[
\begin{bmatrix}
X_{t+\tau+1,t} \\
Hx_{t+\tau+1,t}
\end{bmatrix} = A \begin{bmatrix}
X_{t+\tau,t} \\
x_{t+\tau,t}
\end{bmatrix} + Bi_{t+\tau,t}, \quad X_{t,t} = X_{t|t}. \tag{1}
\]

Feasible set of projections

\[\mathcal{T}(X_{t|t}) \equiv \{(X^t, x^t, i^t, Y^t) \mid (X^t, x^t, i^t, Y^t) \text{ satisfy (1)}\}\]
Optimal monetary policy: Projections

- Intertemporal loss function over projections (no $E_t$, no $1 - \delta$)

$$\mathcal{L}(Y^t; \Lambda) \equiv \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau,t}$$

Period loss

$$L_{t+\tau,t} \equiv Y'_{t+\tau,t} \Lambda Y_{t+\tau,t}$$

- Optimal projection (given $\Lambda$)

$$(\hat{X}^t, \hat{x}^t, \hat{i}^t, \hat{Y}^t) = \text{arg min} \{ \mathcal{L}(Y^t; \Lambda) \mid (X^t, x^t, i^t, Y^t) \in T(X_t|t) \}$$

- Efficient set of projections (for set of $\Lambda$):
  Vary $\Lambda$ in $\mathcal{L}(Y^t; \Lambda)$ to generate set of efficient projections
Finite-horizon approximation:
Reach steady state in finite time $T > 0$
System of finite-dimensional equations
Arbitrarily close approximation
Computational advantage?
Details:
- Svensson IJCB 05: “Monetary Policy with Judgment: Forecast Targeting”
- Svensson-Tetlow IJCB 05: “Optimal Policy Projections”
Choose instrument-rate projection \( (i^t) \) so forecasts of target variables \( (Y^t) \) “look good”

\[
\hat{i}^t = \arg \min \{ L(Y^t; \Lambda) | (X^t, x^t, i^t, Y^t) \in T(X_t|t) \}
\]

Look good: “Inflation forecast approaching inflation target, output-gap forecast approaching zero, good compromise between stabilizing inflation and the output gap”

Certainty equivalence (linear model, quadratic loss, additive uncertainty):
Mean forecast targeting
Deviation $z_t$ (add factors)

\[
\begin{bmatrix}
X_{t+\tau+1} \\
Hx_{t+\tau+1|t+\tau}
\end{bmatrix} = A \begin{bmatrix}
X_{t+\tau} \\
x_{t+\tau}
\end{bmatrix} + Bi_{t+\tau} + \begin{bmatrix}
z_{t+1} \\
0
\end{bmatrix}
\]

Assume moving-average stochastic process of finite order $T > 0$

\[
z_{t+1} = \varepsilon_{t+1} + \sum_{j=1}^{T} \varepsilon_{t+1,t+1-j},
\]

Innovation $\tilde{\varepsilon}_t \equiv (\varepsilon'_t, \varepsilon''_t)' \equiv (\varepsilon'_t, \varepsilon'_{t+1,t}, ..., \varepsilon'_{t+T,t})'$ is a zero-mean i.i.d. random $(T + 1)n_x$-vector
Judgment $z^t = \{z_{t+\tau}, t\}_{\tau=0}^{\infty}$

$z_{t+\tau}, t \equiv E_t z_{t+\tau} = \varepsilon_{t+\tau}, t + z_{t+\tau}, t-1$.

$\varepsilon_{t+\tau}, t = z_{t+\tau}, t - z_{t+\tau}, t-1$ innovation in period $t$ to the previous judgment $z_{t+\tau}, t-1$

Dynamics of deviation $z_t$ and judgment $z^{t+1}$

$$
\begin{bmatrix}
  z_{t+1} \\
  z_{t+1}^{t+1}
\end{bmatrix} = A_z
\begin{bmatrix}
  z_t \\
  z_t
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_t \\
  \varepsilon^{t+1}
\end{bmatrix},
$$

(2)

Replace $X_{t|t}$ by $(X'_{t|t}, z^{t'})'$ to incorporate judgment (expand vector of predetermined variables)

Add (2) to model

Details in

- Svensson IJCB 05: “Monetary Policy with Judgment: Forecast Targeting”
- Svensson-Tetlow IJCB 05: “Optimal Policy Projections”
Non-certainty equivalence: Distribution forecast targeting

- The above for certainty equivalence, mean projections sufficient
- Non-certainty equivalence (nonlinear model, nonquadratic loss, or nonadditive uncertainty)
- Interpret $(X^t, x^t, i^t, Y^t), X_{t|t}$ as probability distributions, $T(X_{t|t})$ set of feasible probability distributions
- $\mathcal{L}(Y^t; \Lambda)$ loss function over distributions
- Optimal projections $(\hat{X}^t, \hat{x}^t, \hat{i}^t, \hat{Y}^t)$: Optimal probability distributions
- Details for MJLQ: Svensson-Williams 07
  “Monetary Policy with Model Uncertainty: Distribution Forecast Targeting,”
  “Bayesian and Adaptive Optimal Policy under Model Uncertainty”
Possible technical problems: Determinacy

- Problem (Woodford): Given instrument-rate path, possible indeterminacy (wrong eigenvalue configuration)
- Solution: Out-of-equilibrium commitment

\[
\hat{i}_{t,t} = F_i \begin{bmatrix} X_{t|t} \\ \Xi_{t-1} \end{bmatrix}
\]

\[
i_t = \hat{i}_{t,t} + f_{ix}(x_t - \hat{x}_{t,t})
\]

Choose \( f_{ix} \) so right eigenvalue configuration

\[
i_t = \hat{i}_{t,t} + \alpha \left\{ (\pi_t - \pi^*) + \frac{\lambda}{\kappa} \left[ (y_t - \bar{y}_t) - (y_{t-1} - \bar{y}_{t-1}) \right] \right\}
\]

\[
i_t = \hat{i}_{t,t} + \alpha (\pi_t - \pi_{t,t})
\]

Details: Svensson-Woodford 05, “Implementing Optimal Policy through Inflation-Forecast Targeting”
Possible technical problems: Commitment in a timeless perspective

- How to implement commitment in a timeless perspective
- Modified loss function

\[ \tilde{L}(Y^t; \Lambda; \Xi_{t-1}) \equiv L(Y^t; \Lambda) + \Xi'_{t-1} \frac{1}{\delta} H(x_{t,t} - x_{t,t-1}) \]

Details: Svensson-Woodford 05
- Initial \( \Xi_{t-1} \) when no previous optimization?
Possible technical problems: Commitment in a timeless perspective - initial multipliers

1. Assume optimal policy in the past

\[ \mathbb{E}_{t-1} = M_{EX} X_{t-1} + M_{EE} \mathbb{E}_{t-2} \approx \sum_{\tau=1}^{T} M_{EE}^{\tau-1} M_{EX} X_{t-\tau} \]
2. Assume any systematic policy in the past, not necessary optimal (use first-order condition to determine $\Xi_{t-1}$)

\[
\bar{A}' \begin{bmatrix} \tilde{\xi}_{s+1|t} \\ \Xi_s \end{bmatrix} = \frac{1}{\delta} \bar{H}' \begin{bmatrix} \tilde{\xi}_s \\ \Xi_{s-1} \end{bmatrix} + \bar{W} \begin{bmatrix} X_s \\ x_s \\ i_s \end{bmatrix}
\]

\[
\begin{bmatrix} \tilde{\xi}_{t-1} \\ \Xi_{t-1} \end{bmatrix} = \bar{B} \begin{bmatrix} \tilde{\xi}_{t-2} \\ \Xi_{t-2} \end{bmatrix} + \sum_{s=0}^{\infty} \bar{F}^s \Phi \bar{W} \begin{bmatrix} X_{t-1+s|t-1} \\ x_{t-1+s|t-1} \\ i_{t-1+s|t-1} \end{bmatrix}.
\]

Choosing the instrument-rate projection

- **Choice object**
  - Instrument-rate projection $i^t = \{i_{t+\tau, t}\}_{\tau=0}^T (T \geq 12 \text{ qtrs});$ not explicit policy function
  - Current instrument rate $i_{t,t}$ (small) part of the decision
  - Implicitly
    $$i_{t,t} = F_{iX}X_t + F_{i\Xi}\Xi_{t-1} + F_{iz}z^t$$
Choosing the instrument-rate projection: Decision process

- Staff presents feasible alternatives to MPC
  - Illustrate $T_t(X_{t|t}, z^t)$ for given $X_{t|t}, z^t$; choice set of MPC; alternative $(Y^t, i^t)$
  - Efficient set of $T_t(X_{t|t}, z^t)$ for given $X_{t|t}, z^t$
  - Report $L(Y^t; \Lambda; \Xi_{t-1})$ for alternative $\Lambda$
  - Scenarios for alternative $X_{t|t}, z^t$
  - Simulations to show distribution (probability bands)
  - Several iterations staff-MPC

- MPC chooses optimal projection $(\hat{Y}^t, \hat{i}^t)$
Baseline scenario in *Monetary Policy Report 1/07*

- **Key policy rate**
- **Output gap**
- **CPI-ATE**
- **CPI**

Sources: Statistics Norway and Norges Bank
Figure 1. Repo rate with uncertainty bands
Per cent, quarterly averages

Source: The Riksbank
**Example: Alternative instrument-rate projections**

**Chart 3.5a** 3-month money market rate in the baseline scenario and in alternative scenarios with high and low interest rates. Quarterly figures. 04 Q1 – 08 Q4

**Source:** Norges Bank

**Chart 3.5b** Projections for the CPI-ATE and the output gap in the baseline scenario and in alternative scenarios with high and low interest rates. Quarterly figures. Per cent. 04 Q1 – 08 Q4

**Sources:** Statistics Norway and Norges Bank
Different interest rate scenarios

**Repo rate**

<table>
<thead>
<tr>
<th>Per cent</th>
<th>04</th>
<th>05</th>
<th>06</th>
<th>07</th>
<th>08</th>
<th>09</th>
<th>10</th>
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<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>4.5</td>
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<tr>
<td>Higher interest rate</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Lower interest rate</td>
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<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
</tr>
</tbody>
</table>

**CPIX**

<table>
<thead>
<tr>
<th>Annual percentage change</th>
<th>04</th>
<th>05</th>
<th>06</th>
<th>07</th>
<th>08</th>
<th>09</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main scenario</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>Higher interest rate</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Lower interest rate</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>4.5</td>
</tr>
</tbody>
</table>

**GDP growth**

<table>
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<tr>
<th>Annual percentage change</th>
<th>04</th>
<th>05</th>
<th>06</th>
<th>07</th>
<th>08</th>
<th>09</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
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<td>3.0</td>
<td>2.5</td>
<td>2.0</td>
<td>1.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Higher interest rate</td>
<td>3.5</td>
<td>3.0</td>
<td>2.5</td>
<td>2.0</td>
<td>1.5</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Lower interest rate</td>
<td>4.0</td>
<td>3.5</td>
<td>3.0</td>
<td>2.5</td>
<td>2.0</td>
<td>1.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Estimated output gaps**

<table>
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<tr>
<th>Percentage deviation from the HP trend</th>
<th>04</th>
<th>05</th>
<th>06</th>
<th>07</th>
<th>08</th>
<th>09</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main scenario</td>
<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
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<tr>
<td>Higher interest rate</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>Lower interest rate</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Note. Broken lines represent the Riksbank’s forecast.

Sources: Statistics Sweden and the Riksbank
Choosing the instrument-rate projection: Possible problems

- Deciding on a path vs. a point
- Previously RBNZ 97, Norges Bank 05
- Riksbank 07: New: 6 board members
- If it works for 6, it should work for 9, 12, 19, ...
- How to decide?
  - Majority voting
  - Median path, iterating (Svensson 03, 06, 07)
  - Simpler: Limit number of alternatives: Main scenario + one or two alternatives
  - Interaction staff-MPC to determine these alternatives
Announcing the instrument-rate projection

- Best way to manage expectations
- Expectations about entire instrument-rate path \( (i^t) \) matters, not current instrument rate \( (i_{t,t}) \)
- Natural part of package of inflation and output(-gap) forecasts \( (\pi^t, y^t - \bar{y}^t, i^t) \)
- Most transparent
Market misunderstanding: Commitment or forecast conditional on current information?

- Not problem in New Zealand or Norway
- Emphasize uncertainty and conditional nature: Probability bands
Announce the instrument-rate projection; not the policy function

- Policy function too complicated
  - Arguments $X_t$ and $\Xi_{t-1}$ (alternatively current and lagged shocks): High-dimensional, complicated. Also $z^t$!

- Private sector only needs mean forecasts of inflation, output, and instrument rate (Svensson-Woodford 05)
- MPC only needs graphs to see policy alternatives
- Leave policy function implicit
Implementing flexible inflation targeting

- Stabilizing both inflation gap and resource utilization (output gap)

\[ L_t = (\pi_t - \pi^*)^2 + \lambda_y (y_t - \bar{y}_t)^2 + \lambda_{\Delta i} (i_t - i_{t-1})^2 \]

- Hierarchical vs. dual mandate
  - Long-run means (1st moments): Hierarchical
    - Inflation target subject to choice; potential output subject to estimation
  - Variability (2nd moments): Dual
    - Stabilize both inflation gap and output gap
Measures of resource utilization

- Potential output, output gap
- Use all info (employment, unemployment, capacity utilization, flexprice output) to estimate and forecast potential output
- Potential vs. efficient (socially optimal) output
- Gap between efficient (socially optimal) and potential output: Constant or variable?
Flexible inflation targeting: Explicit loss function

- Welfare based?
  - No
  - Riksbank’s mandate is flexible inflation targeting (price stability + stabilizing real economy), not general welfare
- Welfare measures: Model-dependent, partial
- Welfare not operational as objective for central banks

- Simple traditional loss function (flexible inflation targeting)

\[ L_t = (\pi_t - \pi^*)^2 + \lambda_y (y_t - \bar{y}_t)^2 + \lambda_{\Delta i} (i_t - i_{t-1})^2 \]

- Instrument-rate smoothing?
- Voting about parameters
- Experiment internally before going public
- Staff reports losses for projections
Conclusions

- Think of MPCs as choosing projections rather than the current instrument rate
- Optimal monetary policy: The optimal projection in the set of feasible projections (rather than the optimal policy function; too complicated)
- Mean forecast targeting goes a long way
- Distribution forecast targeting is possible
- Judgment can be incorporated systematically and with discipline
- Determinacy is not problem for given instrument-rate projections
- Commitment in a timeless perspective can be implemented
Conclusions

- A committee can decide on an optimal interest-rate projection.
- The number of alternative interest-rate projections can be restricted.
- Misunderstanding of interest-rate projections as commitments is not a problem.
- It is sufficient to announce optimal instrument-rate projections; neither necessary nor feasible to announce the optimal policy function.
Conclusions

- Hierarchical vs. dual mandate: A red herring
- Measures and estimation of resource utilization need work
- Explicit loss functions can be used
- MPCs can vote on loss-function parameters
- Interest-rate smoothing problematic
- Experiment internally before going public
Extra slides
Simple New-Keynesian model

Nominal and real interest rate

\[ i_t = r_t + \pi_{t+1|t} \]

Output gap and real-interest-rate gap

\[ y_t - \bar{y}_t = y_{t+1|t} - \bar{y}_{t+1|t} - \sigma(r_t - \bar{r}_t) \]
\[ = -\sigma \sum_{h=0}^{\infty} (r_{t+h|t} - \bar{r}_{t+h|t}) \]

Inflation

\[ \pi_t - \bar{\pi} = \delta(\pi_{t+1|t} - \bar{\pi}) + \kappa(y_t - \bar{y}_t) + z_t \]
\[ = \kappa \sum_{j=0}^{\infty} \delta^j(y_{t+j|t} - \bar{y}_{t+j|t}) + \sum_{j=0}^{\infty} \delta^jz_{t+\tau+j|t} \]
\[ = -\sigma \kappa \sum_{j=0}^{\infty} \delta^j \sum_{h=0}^{\infty} (r_{t+j+h|t} - \bar{r}_{t+j+h|t}) + \sum_{j=0}^{\infty} \delta^jz_{t+\tau+j|t} \]
Extra slides
Simple New-Keynesian model

Period loss function

\[ L_t = (\pi_t - \bar{\pi})^2 + \lambda_y (y_t - \bar{y}_t) + \lambda_{\Delta i} (i_t - i_{t-1})^2 \]

Intertemporal loss function

\[ \mathcal{L}_t = \sum_{\tau=0}^{\infty} (1 - \delta) \delta^\tau L_{t+\tau | t} \]
Extra slides
Simple New-Keynesian projection model

Decision period \( t \), horizon \( \tau \geq 0 \)
Exogenous projections \( \bar{y}^t, \bar{r}_t \); endogenous projections \( y^t, \pi^t, r^t, i^t \)
 Monetary-policy stance (output gap, Wicksell): \( r^t - \bar{r}^t \)

\[
i_{t+\tau,t} = r_{t+\tau,t} + \pi_{t+1+\tau,t}
\]

\[
y_{t+\tau,t} - \bar{y}_{t+\tau,t} = y_{t+1+\tau,t} - \bar{y}_{t+1+\tau,t} - \sigma (r_{t+\tau,t} - \bar{r}_{t+\tau,t})
= -\sigma \sum_{h=0}^{\infty} (r_{t+\tau+h,t} - \bar{r}_{t+\tau+h,t})
\]

\[
\pi_{t+\tau,t} - \bar{\pi} = \delta (\pi_{t+1+\tau,t} - \bar{\pi}) + \kappa (y_{t+\tau,t} - \bar{y}_{t+\tau,t}) + z_{t+\tau,t}
= \kappa \sum_{j=0}^{\infty} \delta^j (y_{t+\tau+j,t} - \bar{y}_{t+\tau+j,t}) + \sum_{j=0}^{\infty} \delta^j z_{t+\tau+j,t}
= -\sigma \kappa \sum_{j=0}^{\infty} \delta^j \sum_{h=0}^{\infty} (r_{t+\tau+j+h,t} - \bar{r}_{t+\tau+j+h,t}) + \sum_{j=0}^{\infty} \delta^j z_{t+\tau+j,t}
\]
Period loss $L_{t+\tau,t}$

$$L_{t+\tau,t} = (\pi_{t+\tau,t} - \bar{\pi})^2 + \lambda_y (y_{t+\tau,t} - \bar{y}_{t+\tau,t}) + \lambda_{\Delta i} (i_{t+\tau,t} - i_{t-1+\tau,t})^2$$

Intertemporal loss $\mathcal{L}_t$

$$\mathcal{L}_t = \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau,t}$$