Price Level Targeting vs. Inflation Targeting: A Free Lunch?

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First version: July 1995
This version: August 1997

Abstract

Price level targeting (without base drift) and inflation targeting (with base drift) are compared, with persistence in output (generated by sticky prices, for instance). Counter to conventional wisdom, price level targeting results in *lower* short-run inflation variability than inflation targeting (if output is at least moderately persistent). Price level targeting also eliminates any average inflation bias. In case society has preferences corresponding to inflation targeting, it may nevertheless prefer to assign price level targeting to the central bank. Price level targeting thus appears to have more advantages than commonly acknowledged.

Keywords: Price stability, inflation targets, rules, discretion
JEL Classification Numbers: E42, E52, E58

*I thank George Akerlof, Alan Blinder, Guy Debelle, Jon Faust, Stanley Fischer, John Hassler, Dale Henderson, Michael Kiley, Miles Kimball, David Lebow, Frederic Mishkin, Torsten Persson, Carl Walsh, participants in seminars at the CEPR Money Workshop at IGIER, Milan, IIES, Institute for Graduate Studies, Geneva, NBER Summer Institute Monetary Economics Workshop, Tilburg University, University of California, Berkeley, and University of California, Santa Cruz. I also thank Mårten Blix for research assistance, and Maria Gil and Christina Lönnblad for secreterial and editorial assistance. Expressed views as well as any errors and obscurities are my own responsibility.

1 Introduction

"Price stability" is often recommended as a goal for monetary policy. Price stability has been interpreted in different ways, though. Price stability can be interpreted as *price level* stability, that is, a stationary price level with low variance. In practice, price stability has often been interpreted as low and stable *inflation*. As is well known, unless above-average inflation is followed by below-average inflation, this results in base drift of the price level. Base drift in the price level implies that the price level becomes non-trend-stationary, and the variance of the future price level increases without bounds with the forecast horizon. This is obviously rather far from literal price stability. I shall refer to a monetary policy regime as *price level targeting* or *inflation targeting,* depending upon whether the goal is a stable price level or a low and stable inflation rate, where the latter allows base drift of the price level.

In the real world, there are currently several monetary policy regimes with explicit or implicit inflation targeting (see Haldane (1995) and Leiderman and Svensson (1995)), but there are no regimes with explicit or implicit price level targeting. Sweden during the 1930s may so far be the only regime in history with explicit price level targeting (cf. Fisher (1934), Jonung (1979) and Black and Gavin (1990)).

Even if there are no current examples of price level target regimes, price level targeting has received increasing interest in the monetary policy literature, and several recent papers compare inflation targeting and price level targeting. A number of these papers are collected in Bank of Canada (1994), and Duguay (1994) summarizes these and some of the other papers and provides a thorough discussion of the issues involved; see also Fischer (1994) and Goodhart and Viñals (1994). Some papers compare inflation and price level targeting by simulating the effect of postulated reaction functions (Lebow, Roberts and Stockton (1992), Fillon and Tetlow (1994), Haldane and Salmon (1995)). Other papers compare the properties of postulated simple stochastic processes for inflation and the price level (Duguay (1994), Fischer (1994)). A frequent result, emerging as the conventional wisdom, is that the choice between price-level targeting and inflation targeting involves a trade-off between low-frequency price level variability on the one hand and high-frequency inflation and output variability on the other.[1] Thus, price level

[1] The result is emphasized in Lebow, Robert and Stockton (1992), Fischer (1994), and Haldane and Salmon (1995). In contrast, Fillon & Tetlow (1994) report that in their simulations, price level targeting results in *less* inflation variability but in more output variability than inflation targeting. No explanation is offered beyond the observation that the results indicate strong serial correlation of the price level. Duguay (1994) does not report the unconditional variance of one-period inflation rates in his examination of different processes for inflation and the price level, although for some of the parameters studied that variance is actually less under price level targeting (see the appendix to the present paper).

targeting has the advantage of reduced long-term variability of the price level. This should be beneficial for long-term nominal contracts and intertemporal decisions, but comes at the cost of increased short-term variability of inflation and output. The intuition is straightforward: In order to stabilize the price level under price level targeting, higher-than-average inflation must be succeeded by lower-than-average inflation. This should result in higher inflation variability than inflation targeting, since in the latter case, base level drift is accepted and higher-than-average inflation need only be succeeded by average inflation. Via nominal rigidities, the higher inflation variability should then result in higher output variability.[2]

Applying postulated monetary policy reaction functions, 'instrument rules', evokes the issue of whether these reaction functions are optimal for reasonable objective functions and constraints of the central bank, and whether they are consistent with the realistic situation when the central bank acts under discretion and commitment to an optimal or a simple second-best rule (like those in McCallum (1990) or Taylor (1993)) is not possible (cf. Laidler (1993)).[3] Similarly, applying postulated exogenous processes for inflation and the price level evokes the issue of whether these are consistent with a reasonable equilibrium.

The purpose of this paper is to compare price level and inflation targeting, but the paper departs from the previous literature on price level versus inflation targeting by considering the *endogenous* decision rules that result when the central bank has specific objectives associated with inflation targeting and price level targeting and acts under discretion. For comparison, the corresponding endogenous decision rules under commitment are also reported, although the focus is on the discretion case. The reaction functions are hence endogenous, given central bank objectives and constraints, including available commitment technology.

The paper follows Svensson (1997) in interpreting inflation targeting as implying not only an objective to stabilize inflation around an inflation target, but in practice also an objective

[2] Hall (1984, 1986) provides arguments for price stability. McCallum (1990) argues that price level targeting provides a relatively small gain in long run price predictability, since price level variability (for the U.S.) is already relatively small under inflation targeting. Gerlach (1993) interprets inflation targets as a 'target zone' for the price level. Balke and Emery (1994) examine what monetary policy rules are consistent with inflation and price level targeting (which they refer to as Weak and Strong Price Stability). Scarth (1994), Crawford and Dupasquier (1994), and Konieczny (1994) discuss various aspects of price targeting and inflation targeting.

Base drift in money supply is distinct from base drift in the price level. As shown by Walsh (1986), some degree of money supply base drift is warranted even with price level stability, if there are permanent shocks to money demand and output.

[3] McCallum (1995, 1996) has argued that the central bank can in practice choose the commitment policy even if no commitment technology is available. If McCallum's argument is accepted, this paper's main message is overturned. However, I believe that for his argument to be considered valid, McCallum has to provide a reasonable explicit model where his suggested outcome is an equilibrium. As far as I can see, his outcome is not subgame perfect (that is, consistent with backward induction) in existing standard models, absent a commitment mechanism. (In McCallum (1996) it is clear that he does not base his argument on so-called trigger-strategy equilibria; such equilibria normally suffer from indeterminacy and lack of a coordination mechanism for the private sector's trigger strategies due to the Folk Theorem.)

to stabilize output (or the output gap).[4] This is motivated by the existence of target bands in actual inflation targeting regimes, indicating that some short-term inflation variability may be acceptable due to imperfect control over inflation but perhaps also in order to dampen output fluctuations; the fact that no inflation targeting central bank seems to behave as if it wants to attain the inflation target at any cost (cf. Haldane (1995) and Leiderman and Svensson (1995)); and by wording in King (1995) that indicates that the inflation targeting Bank of England is not an "inflation nutter" with zero weight on output stabilization.[5] Price level targeting is consequently interpreted as including an objective to stabilize the price level around a price level target together with an objective to stabilize output (or the output gap).

The paper considers the realistic case when there is persistence in output movements. This persistence can arise in several ways, for instance due to imperfections in the labor market as in Lockwood and Phlippopoulos (1994), or from sticky prices in the so-called P-bar model, recently discussed in McCallum (1994) (in the appendix, I show that a variant of the P-bar model results in the expectational Phillips curve with output persistence used here).

The degree of persistence in output is indeed crucial for the results: Without persistence, a trivial trade-off between long-term price level variability and short-term inflation variability arises. With at least moderate persistence, counter to the conventional wisdom, there is *no* trade-off between price level variability and inflation variability. Price level targeting then results in *lower* inflation variability than inflation targeting. This result is due to the endogenous decision rule that results under discretion for different targets. Under inflation targeting, the decision rule is a linear feed-back rule for inflation on the output gap. Then the variance of inflation is proportional to the variance of the output gap. Under price level targeting, the decision rule is a linear feed-back rule for the *price level* on the output gap. Then inflation is a linear function of the *first difference* of the output gap. The variance of inflation is then proportional to the variance of the first difference of the output gap. With at least moderate persistence, the variance of the first difference of the output gap is less then the variance of the level of the output gap.

[4] Svensson (1997) argues that inflation targeting regimes should be interpreted as having in practice also output targets, and compares inflation targeting regimes to (1) Rogoff (1985) 'weight-conservative' central banks with more weight on inflation stabilization, (2) 'linear inflation contracts' proposed by Walsh (1995) and extended by Persson and Tabellini (1993), and (3) 'output targeting' regimes, both with and without persistence in output. For instance, without persistence, an optimal inflation target equal to the socially best inflation rate less any discretionary inflation bias is identical to a linear inflation contract and better than having a Rogoff 'weight-conservative' central bank.

[5] Very recently, Fischer (1996), King (1996) and Taylor (1996), at the August 1996 Jackson Hole conference sponsored by Federal Reserve Bank of Kansas City, all interpreted inflation targeting as also involving some output stabilization.

In addition, a price level target has the advantage of eliminating any average inflation bias that results under discretion, in case the output target exceeds the natural rate of output. Any average inflation bias is replaced by a harmless price level bias.

Finally, in the case when society prefers to minimize inflation variability rather than price level variability, it may still be better off by having a price-level-targeting central bank, if there is at least moderate output persistence: The variance of inflation will be lower, any average inflation bias will disappear, and with expectations incorporating price level targeting, output gap variability will be the same as under inflation targeting.[6]

Section 2 presents the model with an inflation-targeting central bank. Section 3 introduces a price-level-targeting central bank. Section 4 evaluates having a price-level-targeting central bank in the case when society has preferences corresponding to inflation targeting. Section 5 concludes. The appendix presents technical details, including some results on exogenous inflation and price level processes.

2 Inflation targeting

The treatment of inflation targeting under persistence follows Svensson (1997), which in turn builds on the recent extension of the analysis of rules and discretion in monetary policy to the case of persistence in Lockwood and Philippopoulos (1994), Jonsson (1995) and Lockwood, Miller and Zhang (1995).

The short-run Phillips curve is

$$y_t = \rho y_{t-1} + \alpha(\pi_t - \pi_t^e) + \varepsilon_t, \tag{2.1}$$

where y_t is the (log) output gap in period t, α and ρ are constants ($\alpha > 0$ and $0 \leq \rho < 1$), $\pi_t = p_t - p_{t-1}$ is the (log of the gross) inflation rate, p_t is the (log) price level, π_t^e denotes inflation expectations in period $t-1$ of the inflation rate in period t, and ε_t is an i.i.d. temporary supply shock with mean 0 and variance σ^2. The private sector has rational expectations. That is,

$$\pi_t^e = \mathrm{E}_{t-1}\pi_t, \tag{2.2}$$

where E_{t-1} denotes expectations conditional upon information available in period $t-1$, which includes the realization of all variables up to and including period $t-1$, as well as the constant parameters of the model.

[6] The consequences of downward nominal rigidity and nonnegative nominal interest rates are discussed in the concluding section.

The short-run Phillips curve can be interpreted and motivated in several ways. It is identical to the Phillips curve used in Lucas (1973), where it is motivated by imperfect information about the general price level. More realistically, it may refer to a situation in which nominal wages for period t are set one period in advance, based on expectations in period $t-1$, without knowing the supply shock ε_t in period t. The autoregressive term then arises, for instance, as in the wage setting model in Lockwood and Philippopoulos (1994), where trade unions set nominal wages one period in advance, disregarding non-union workers' preferences and only taking into account union members' preferences for real wages and employment, and where union membership depends on previous employment.

However, as shown in the appendix, the Phillips curve with output persistence is also consistent with a variant the so-called P-bar model of sticky prices first proposed by Grossman (1974) and more recently discussed in McCallum (1994). Then there are permanent supply shocks that make the natural output level (the capacity level) a random walk. The price level is sticky and inflation is determined by the lagged output gap (the difference between aggregate demand and the capacity level), expected inflation in the 'equilibrium' price level (corresponding to the natural output level), actual inflation in the equilibrium price level, and a temporary supply shock. (The effect of the permanent supply shock on the output gap vanishes and only the temporary supply shock enters in (2.1).)

Thus (2.1) and (2.2) represent the constraints facing the central bank. What about the central bank's objectives? As in Svensson (1997), Fischer (1996), King (1996) and Taylor (1996), I interpret inflation targeting as stabilizing inflation around a given (long-run) inflation target, π^* (say 2 percent per year), as well as stabilizing the output gap around an output gap target, y^*. This can be represented by an intertemporal loss function for the central bank given by

$$V = \mathrm{E}_0 \left[\sum_{t=1}^{\infty} \beta^{t-1} L_t \right], \tag{2.3}$$

with the "period" loss function

$$L_t = \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right], \tag{2.4}$$

where $\lambda > 0$ is the relative weight on output gap stabilization.

The output gap target y^* is taken to be nonnegative, $y^* \geq 0$. A zero output gap target, $y^* = 0$, can be interpreted as in Taylor (1996) and Svensson (1996) as a situation when there is *no* long-run output target, in the sense that the long-run output target is not subject to choice but given by the capacity level of output. A positive output gap target, $y^* > 0$, can be interpreted as a situation in which distortions in the economy, for instance in the labor market, cause the socially preferred output level to exceed the natural output level, which in turn affects the central bank's loss function due to political pressure or other circumstances. A positive output gap target introduces an average benefit from inflation surprises and causes an average inflation bias under discretion. For the purpose of this paper, it is not important whether the output gap target for monetary policy is (rationally) zero, or (irrationally) positive.

The central bank is, for simplicity, assumed to have perfect control over the inflation rate π_t. It sets the inflation rate in each period after having observed the current supply shock ε_t. Although the current supply shock is observed by both the central bank and the private sector, the assumption behind the Phillips curve (2.1) that some prices or wages are set in advance and predetermined by previous expectations makes monetary policy effective.[7]

2.1 Commitment

The optimal rule under commitment is reported for comparison. It can be derived as the solution to the problem

$$V^*(y_{t-1}) = \min_{\pi_t(\varepsilon_t, y_{t-1}), \pi_t^e(y_{t-1})} \mathrm{E}_{t-1} \left\{ \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right] + \beta V^*(y_t) \right\} \tag{2.5}$$

subject to (2.1) and (2.2). The lagged output gap enters as a state variable. Here π_t may depend on both the supply shock ε_t and the lagged output gap y_{t-1}, whereas π_t^e may only depend on y_{t-1}. The indirect loss function $V^*(y_{t-1})$ will be quadratic and can be written

$$V^*(y_{t-1}) = \gamma_0^* + \gamma_1^* y_{t-1} + \frac{1}{2} \gamma_2^* y_{t-1}^2. \tag{2.6}$$

It is shown in the appendix that the optimal rule is

$$\pi_t = \pi^* - b^* \varepsilon_t \tag{2.7}$$

where

$$b^* = \frac{(\lambda + \beta \gamma_2^*) \alpha}{1 + (\lambda + \beta \gamma_2^*) \alpha^2} = \frac{\lambda \alpha}{1 + \lambda \alpha^2 - \beta \rho^2} \tag{2.8}$$

and

$$\gamma_2^* = \frac{\lambda \rho^2}{1 - \beta \rho^2}. \tag{2.9}$$

[7] As shown in the appendix, the results are not affected if the central bank uses money supply as an instrument and money supply affects aggregate demand. The results are also the same if the central bank uses an interest rate as its instrument, and aggregate demand is affected by the interest rate (cf. Rogoff (1985) and McCallum (1994)). A control error on the price level will, however, affect the results somewhat, as explained below.

The output gap will then fulfill

$$y_t = \rho y_{t-1} + (1 - \alpha b^*)\varepsilon_t. \tag{2.10}$$

We see that the optimal inflation response to supply shocks is larger under persistence ($\rho > 0$, $\gamma_2^* > 0$) than without ($\rho = \gamma_2^* = 0$). Since the current output gap changes affect the future output gap, stabilizing the output gap becomes more important; hence inflation is allowed to fluctuate more. Note that inflation only depends on the new information that has arrived after the private sector formed its expectations; any dependence on previous information known by the private sector just goes into expected inflation, which adds to the loss function without affecting the output gap; cf. Persson and Tabellini (1993).

The results are summarized in *Table 1*, the column for Commitment. Conditional and unconditional expected inflation equal the inflation target, rows (5) and (6). The conditional and unconditional variance of inflation are equal and given in rows (7) and (8).

The future price level is a random walk with drift,

$$p_T = p_t + (T - t)\,\pi^* - b^* \sum_{\tau=t+1}^{T} \varepsilon_\tau, \quad T > t,$$

and its conditional variance will hence be increasing in the horizon, row (11). The unconditional variance of the price level, row (12), is hence unbounded.

Long term inflation will be

$$\frac{p_T - p_t}{T - t} = \pi^* - b^* \frac{\sum_{\tau=t+1}^{T} \varepsilon_\tau}{T - t}, \quad T > t,$$

with conditional and unconditional expectation equal to the inflation target, rows (14) and (15). The conditional variance of long term inflation will be decreasing in the horizon, row (16), and equal to the unconditional variance, row (17).

2.2 Discretion

Under discretion, the decision problem of the central bank can be written

$$\hat{V}(y_{t-1}) = \mathrm{E}_{t-1} \min_{\pi_t} \left\{ \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda\,(y_t - y^*)^2 \right] + \beta \hat{V}(y_t) \right\}, \tag{2.11}$$

where the minimization in period t is subject to (2.1) but is done for given inflation expectations π_t^e. The central bank thus no longer internalizes the effect of its decisions on inflation expectations, although it takes into account that changes in the current output gap will affect current expectations of future inflation (this is incorporated in $\hat{V}(y_t)$). The indirect loss function can be written

$$\hat{V}(y_{t-1}) = \hat{\gamma}_0 + \hat{\gamma}_1 y_{t-1} + \frac{1}{2}\hat{\gamma}_2 y_{t-1}^2. \tag{2.12}$$

In the appendix, it is shown that the decision rule and the output gap fulfill[8]

$$\pi_t = \hat{a} - \frac{\hat{b}}{1-\alpha\hat{b}} y_t = \hat{a} - \frac{\hat{b}}{1-\alpha\hat{b}} \rho y_{t-1} - \hat{b}\varepsilon_t, \tag{2.13}$$

$$y_t = \rho y_{t-1} + (1-\alpha\hat{b})\varepsilon_t. \tag{2.14}$$

The constants are given by[9]

$$\hat{a} = \pi^* + \lambda\alpha y^* - \beta\alpha\hat{\gamma}_1, \quad \hat{b} = \frac{(\lambda + \beta\hat{\gamma}_2)\,\alpha}{1 + (\lambda + \beta\hat{\gamma}_2)\,\alpha^2}, \tag{2.15}$$

where

$$\hat{\gamma}_1 = -\frac{\lambda y^* \left[1 + (\lambda + \beta\hat{\gamma}_2)\,\alpha^2\right]\rho}{1 - \beta\rho\left[1 + (\lambda + \beta\hat{\gamma}_2)\,\alpha^2\right]} \leq 0, \tag{2.16}$$

$$\hat{\gamma}_2 = \frac{1 - \beta\rho^2 - 2\lambda\beta\alpha^2\rho^2 - \sqrt{(1-\beta\rho^2)^2 - 4\lambda\beta\alpha^2\rho^2}}{2(\beta\alpha\rho)^2} > 0. \tag{2.17}$$

As explained in the appendix, an existence condition must be fulfilled.

The results under discretion are summarized in Table 1, the column for Discretion. The decision rule can be written as a feedback rule on the current output gap, or as a function of the lagged output gap and the current supply shock. Without persistence, that is, for $\rho = 0$, we have $\hat{\gamma}_2 = \gamma_2^* = 0$. Then the inflation response to supply shocks under discretion is the same as the optimal rule, $\hat{b} = b^*$. With persistence, we have $\hat{\gamma}_2 > \gamma_2^*$ (see the appendix), and by comparing (2.15) and (2.8) we see that, under discretion, there is a stabilization bias in that the inflation response to supply shocks is larger than the optimal rule,

$$\hat{b} > b^*.$$

Since under discretion the future inflation bias depends on the current output gap, it becomes even more important to stabilize the output gap, which requires a stronger inflation response. Thus, conditional and unconditional output gap variability is lower under discretion than under commitment, rows (2) and (3) in Table 1.

[8] The equilibrium concept is a Markov-perfect equilibrium where trigger strategies are not allowed and actions depend on history only via the lagged state variable, y_{t-1} (cf. Lockwood and Philippopoulos (1994)).

[9] If $y^* = 0$, the decision rule has $\hat{\gamma}_1 = 0$, $\hat{a} = \pi^*$.

Conditional expected inflation is given in row (5). We see that the inflation bias, $\mathrm{E}_t\pi_{t+1}-\pi^*$, depends on the lagged output gap and is hence state-dependent. If the output gap target is zero, there is no average inflation bias. If the output gap target is positive, the average inflation bias,

$$\mathrm{E}\left[\pi_t\right]-\pi^*=\lambda\alpha y^*-\beta\alpha\hat{\gamma}_1,$$

is positive and larger than the inflation bias $\lambda\alpha y^*$ without output gap persistence.

The conditional and unconditional variance of inflation is higher under discretion, rows (7) and (8), since the inflation rate is a linear function of output rather than of the supply shock.

The future price level is an I(1) process that fulfills

$$p_T=p_t+\sum_{\tau=t+1}^{T}\pi_\tau=p_t+(T-t)\hat{a}-\frac{\hat{b}}{1-\alpha\hat{b}}\sum_{\tau=t+1}^{T}y_\tau,$$

and long-term inflation will be

$$\frac{p_T-p_t}{T-t}=\hat{a}-\frac{\hat{b}}{1-\alpha\hat{b}}\frac{\sum_{\tau=t+1}^{T}y_\tau}{T-t},$$

Thus, expectations and variances of the future price level and long-term inflation in rows (14)-(17) will depend on the expectations and variances of the sum and average of future output gaps. These are reported in Table 2.

Table 1. Inflation targeting

		Commitment	Discretion
(1)	y_t	$\rho y_{t-1}+(1-\alpha b^*)\varepsilon_t$	$\rho y_{t-1}+(1-\alpha\hat{b})\varepsilon_t$
(2)	$\mathrm{Var}_t y_{t+1}$	$(1-\alpha b^*)^2\sigma^2$	$(1-\alpha\hat{b})^2\sigma^2$
(3)	$\mathrm{Var}\left[y_t\right]$	$\frac{(1-\alpha b^*)^2\sigma^2}{1-\rho^2}$	$\frac{(1-\alpha\hat{b})^2\sigma^2}{1-\rho^2}$
(4)	π_t	$\pi^*-b^*\varepsilon_t$	$\hat{a}-\frac{\hat{b}}{1-\alpha\hat{b}}y_t$
(5)	$\mathrm{E}_t\pi_{t+1}$	π^*	$\hat{a}-\frac{\hat{b}}{1-\alpha\hat{b}}\rho y_t$
(6)	$\mathrm{E}\left[\pi_t\right]$	π^*	$\hat{a}$
(7)	$\mathrm{Var}_t\pi_{t+1}$	$b^{*2}\sigma^2$	$\hat{b}^2\sigma^2$
(8)	$\mathrm{Var}\left[\pi_t\right]$	$b^{*2}\sigma^2$	$\frac{\hat{b}^2\sigma^2}{1-\rho^2}$
(9)	p_t	$p_{t-1}+\pi^*-b^*\varepsilon_t$	$p_{t-1}+\hat{a}-\frac{\hat{b}}{1-\alpha\hat{b}}y_t$
(10)	p_T	$p_t+(T-t)\pi^*-b^*\sum_{\tau=t+1}^T\varepsilon_\tau$	$p_t+(T-t)\hat{a}-\frac{\hat{b}}{1-\alpha\hat{b}}\sum_{\tau=t+1}^T y_\tau$
(11)	$\mathrm{Var}_t p_T$	$(T-t)b^{*2}\sigma^2$	$\left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2\mathrm{Var}_t\sum_{\tau=t+1}^T y_\tau$
(12)	$\mathrm{Var}\left[p_t\right]$	∞	∞
(13)	$\frac{p_T-p_t}{T-t}$	$\pi^*-b^*\frac{\sum_{\tau=t+1}^T\varepsilon_\tau}{T-t}$	$\hat{a}-\frac{\hat{b}}{1-\alpha\hat{b}}\frac{\sum_{\tau=t+1}^T y_\tau}{T-t}$
(14)	$\mathrm{E}_t\frac{p_T-p_t}{T-t}$	π^*	$\hat{a}-\frac{\hat{b}}{1-\alpha\hat{b}}\frac{1-\rho^{T-t}}{1-\rho}\rho\frac{y_t}{T-t}$
(15)	$\mathrm{E}\left[\frac{p_T-p_t}{T-t}\right]$	π^*	$\hat{a}$
(16)	$\mathrm{Var}_t\frac{p_T-p_t}{T-t}$	$b^{*2}\frac{\sigma^2}{T-t}$	$\left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2\frac{\mathrm{Var}_t\sum_{\tau=t+1}^T y_\tau}{(T-t)^2}$
(17)	$\mathrm{Var}\left[\frac{p_T-p_t}{T-t}\right]$	$b^{*2}\frac{\sigma^2}{T-t}$	$\left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2\frac{\mathrm{Var}\left[\sum_{\tau=t+1}^T y_\tau\right]}{(T-t)^2}$

Table 2. Expectation and variance of future output gaps under discretion

(1)	y_T	$=$	$\rho^{T-t}y_t+\sum_{\tau=t+1}^T\rho^{T-\tau}(1-\alpha\hat{b})\varepsilon_\tau$
(2)	$\sum_{\tau=t+1}^T y_\tau$	$=$	$\frac{1-\rho^{T-t}}{1-\rho}\rho y_t+\sum_{\tau=t+1}^T\frac{1-\rho^{T-\tau+1}}{1-\rho}(1-\alpha\hat{b})\varepsilon_\tau$
(3)	$\mathrm{E}_t\sum_{\tau=t+1}^T y_\tau$	$=$	$\frac{1-\rho^{T-t}}{1-\rho}\rho y_t$
(4)	$\mathrm{Var}_t y_T$	$=$	$\left(1-\rho^{2(T-t)}\right)\frac{(1-\alpha\hat{b})^2\sigma^2}{1-\rho^2}$
(5)	$\mathrm{Var}\left[y_t\right]$	$=$	$\frac{(1-\alpha\hat{b})^2\sigma^2}{1-\rho^2}$
(6)	$\mathrm{Var}_t\sum_{\tau=t+1}^T y_\tau$	$=$	$\left[(T-t)-2\frac{1-\rho^{T-t}}{1-\rho}\rho+\frac{1-\rho^{2(T-t)}}{1-\rho^2}\rho^2\right]\frac{1-\rho^2}{(1-\rho)^2}\mathrm{Var}\left[y_t\right]$
(7)	$\mathrm{Var}\left[\sum_{\tau=t+1}^T y_\tau\right]$	$=$	$\left\{\left[(T-t)-2\frac{1-\rho^{T-t}}{1-\rho}\rho+\frac{1-\rho^{2(T-t)}}{1-\rho^2}\rho^2\right]\frac{1-\rho^2}{(1-\rho)^2}+\left(\frac{1-\rho^{T-t}}{1-\rho}\rho\right)^2\right\}\mathrm{Var}\left[y_t\right]$
(8)	y_T-y_t	$=$	$-(1-\rho^{T-t})y_t+\sum_{\tau=t+1}^T\rho^{T-\tau}(1-\alpha\hat{b})\varepsilon_\tau$
(9)	$\mathrm{Var}\left[y_t-y_{t-1}\right]$	$=$	$2(1-\rho)\mathrm{Var}\left[y_t\right]$
(10)	$\mathrm{Var}\left[y_T-y_t\right]$	$=$	$2(1-\rho^{T-t})\mathrm{Var}\left[y_t\right]$

3 Price level targeting

The Phillips curve (2.1) can be written

$$y_t = \rho y_{t-1} + \alpha(p_t - p_t^e) + \varepsilon_t, \tag{3.1}$$

since $\pi_t - \pi_t^e = p_t - p_t^e$, where p_t^e denotes the expectations in period $t-1$ of the (log) price level in period t. The private sector's rational expectations imply

$$p_t^e = \mathrm{E}_{t-1} p_t. \tag{3.2}$$

A price-level-targeting central bank is assumed to have the period loss function

$$L_t = \frac{1}{2}\left[(p_t - p_t^*)^2 + \lambda(y_t - y^*)^2\right], \tag{3.3}$$

where p_t^* is the (log) price level target. In order to be consistent with the inflation target of an inflation-targeting central bank, the price level target fulfills

$$p_t^* = p_{t-1}^* + \pi^*. \tag{3.4}$$

The previous assumption that the central bank has perfect control over inflation implies that it has perfect control over the price level. It sets the price level in each period after having observed the current supply shock ε_t.[10]

3.1 Commitment

Under commitment to an optimal rule the decision problem is

$$V^*(y_{t-1}) = \min_{p_t(\varepsilon_t, y_{t-1}), p_t^e(y_{t-1})} \mathrm{E}_{t-1}\left\{\frac{1}{2}\left[(p_t - p_t^*)^2 + \lambda\,(y_t - y^*)^2\right] + \beta V^*(y_t)\right\} \tag{3.5}$$

[10] Nominal income targeting has been examined for instance in Bean (1983), in several contributions in Bryant, Hooper and Mann (1993), in Henderson and McKibbin (1993), in McCallum (1990), and more recently in Hall and Mankiw (1994). One has to distinguish between targeting the level and the growth rate of nominal income. Nominal income *level* targeting would in the present framework correspond to

$$L_t^Y = \frac{1}{2}\left[Y_t - Y_t^*\right]^2$$

where Y_t and Y_t^* are (the log of) nominal income and its target, $Y_t = p_t + y_t$, $Y_t^* = p_t^* + y^*$, and y_t and y^* are (the log of) real output and its target. Then,

$$L_t^Y = \frac{1}{2}\left[Y_t - Y_t^*\right]^2 = \frac{1}{2}\left[(p_t - p_t^*)^2 + (y_t - y^*)^2\right] + (p_t - p^*)(y_t - y^*).$$

Thus, nominal income level targeting is not exactly equal to price level targeting with $\lambda = 1$; the cross term enters as well. The difference is, of course, that nominal income level targeting implies a constant unitary marginal rate of substitution between the price level and the employment rate, regardless of the levels of these variables.

subject to (3.1) and (3.2). The price level p_t may depend on the lagged output gap and the current supply shock; expectations p_t^e depend on the lagged output gap only.

The decision problem is identical to that of an inflation-targeting central bank under commitment, (2.5), except that p_t and p_t^* replace π_t and π^*. Thus the form of the indirect loss function is unchanged. With the same reasoning as above, the optimal decision rule is

$$p_t = p_t^* - b^* \varepsilon_t, \tag{3.6}$$

with b^* given by (2.8). The output gap will then fulfill (2.10).

The result is summarized in *Table 3*, the column for Commitment. The future price level is no longer a random walk with drift but trend-stationary and given by

$$p_T = p_T^* - b^* \varepsilon_T = p_t + (T-t)\pi^* - b^* \left(\varepsilon_T - \varepsilon_t\right),$$

with conditional and unconditional expectation equal to $p_T^* = p_t^* + (T-t)\pi^*$, and constant conditional and unconditional variance, row (11) and (12).

Inflation fulfills

$$\pi_t = p_t - p_{t-1} = \pi^* - b^*(\varepsilon_t - \varepsilon_{t-1}).$$

The condition expected inflation is no longer constant, row (5). The unconditional variance of inflation is twice the conditional variance, row (7) and (8).

Long-term inflation is given by

$$\frac{p_T - p_t}{T-t} = \pi^* - b^* \frac{\varepsilon_T - \varepsilon_t}{T-t}$$

The conditional expectation of the long-term inflation rate is given in row (14). The conditional and unconditional variance is decreasing by the square of the horizon, row (16) and (17).

3.2 Discretion

Under discretion, the decision problem of the central bank can be written

$$\hat{V}(y_{t-1}) = \mathrm{E}_{t-1} \min_{p_t} \left\{ \frac{1}{2} \left[(p_t - p_t^*)^2 + \lambda \left(y_t - y^*\right)^2 \right] + \beta \hat{V}(y_t) \right\}, \tag{3.7}$$

where the minimization in period t is subject to (2.1) but is done for given price level expectations p_t^e. Thus the central bank no longer internalizes the effect of its decisions on price level expectations, although it takes into account that changes in the current output gap will affect current expectations of future price levels (this is incorporated in $\hat{V}(y_t)$).

Except for the change in variables from π_t to p_t, the decision problem is the same as under inflation targeting. Thus, the indirect loss function will be the same as under inflation targeting. By the same argument as above, the decision rule fulfills

$$p_t = \hat{a}_t - \frac{\hat{b}}{1 - \alpha\hat{b}} y_t = \hat{a}_t - \frac{\hat{b}}{1 - \alpha\hat{b}} \rho y_{t-1} - \hat{b}\varepsilon_t, \tag{3.8}$$

with

$$\hat{a}_t = p_t^* + \lambda\alpha y^* - \beta\alpha\hat{\gamma}_1, \tag{3.9}$$

where $\hat{b}$ is given by (2.15), $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are given by (2.16) and (2.17), and the same existence condition is fulfilled. The output gap will behave as (2.14).

With persistence, the price level response to supply shocks is larger under discretion than under commitment. Since, under discretion, the future price level bias depends on the current output gap, it becomes even more important to stabilize the output gap. This requires a stronger price level response. The price level under price level targeting behaves precisely as the inflation rate under inflation targeting, with an average (if the output gap target is positive) and a state-contingent *price level* bias instead of an inflation bias.

The results are summarized in Table 3, the column for Discretion. The future price level is

$$p_T = \hat{a}_T - \frac{\hat{b}}{1 - \alpha\hat{b}} y_T.$$

and depends only on the future output gap. The conditional and unconditional variance are reported in row (11) and (12).

Inflation will be given by

$$\pi_t = p_t - p_{t-1} = \pi^* - \frac{\hat{b}}{1 - \alpha\hat{b}} (y_t - y_{t-1}), \tag{3.10}$$

where I have used (3.4). We see that there is no average inflation bias under price level targeting, row (6), although there is a state-contingent inflation bias, row (5). The conditional variance of inflation is the same as under inflation targeting, row (7), whereas the unconditional variance is different, row (8).

Indeed, comparing inflation under inflation targeting and price level targeting, we note that inflation under inflation targeting is a linear function of the output gap (Table 1, row (4)), whereas under price level targeting it is a linear function of the *first difference* of the output gap (Table 2, row (4)). The unconditional variance of these are by Table 2 related as

$$\mathrm{Var}\left[y_t - y_{t-1}\right] = 2(1 - \rho)\mathrm{Var}\left[y_t\right].$$

Since the unconditional variance of the first difference of the output gap is lower than the unconditional variance of the output gap if $\rho > \frac{1}{2}$, it follows that the unconditional variance of inflation is lower under price level targeting if the output gap is at least moderately persistent. If $y^* = 0$, $\rho > \frac{1}{2}$ is both necessary and sufficient for a lower variance of inflation under price level targeting; if $y^* > 0$, $\rho > \frac{1}{2}$ is sufficient but not necessary.

Long-term inflation is

$$\frac{p_T - p_t}{T - t} = \pi^* - \frac{\hat{b}}{1 - \alpha\hat{b}} \frac{y_T - y_t}{T - t},$$

and depends on the average difference between the future and current output gap, $\frac{y_T - y_t}{T-t}$, whereas under inflation targeting it depends on the average sum of future output gaps, $\frac{\sum_{\tau=t+1}^{T} y_\tau}{T-t}$. The conditional and unconditional variances are reported in Table 3, rows (16) and (17), cf. Table 2.

Table 3. Price level targeting

		Commitment	Discretion
(1)	y_t	$\rho y_{t-1} + (1 - \alpha b^*)\varepsilon_t$	$\rho y_{t-1} + (1 - \alpha\hat{b})\varepsilon_t$
(2)	$\mathrm{Var}_t y_{t+1}$	$(1 - \alpha b^*)^2\sigma^2$	$(1 - \alpha\hat{b})^2\sigma^2$
(3)	$\mathrm{Var}[y_t]$	$\frac{(1-\alpha b^*)^2\sigma^2}{1-\rho^2}$	$\frac{(1-\alpha\hat{b})^2\sigma^2}{1-\rho^2}$
(4)	π_t	$\pi^* - b^*(\varepsilon_t - \varepsilon_{t-1})$	$\pi^* - \frac{\hat{b}}{1-\alpha\hat{b}}(y_t - y_{t-1})$
(5)	$\mathrm{E}_t\pi_{t+1}$	$\pi^* + b^*\varepsilon_t$	$\pi^* + \frac{\hat{b}}{1-\alpha\hat{b}}(1-\rho)y_t$
(6)	$\mathrm{E}[\pi_t]$	π^*	π^*
(7)	$\mathrm{Var}_t\pi_{t+1}$	$b^{*2}\sigma^2$	$\hat{b}^2\sigma^2$
(8)	$\mathrm{Var}[\pi_t]$	$2b^{*2}\sigma^2$	$\frac{2\hat{b}^2\sigma^2}{1+\rho}$
(9)	p_t	$p_t^* - b^*\varepsilon_t$	$\hat{a}_t - \frac{\hat{b}}{1-\alpha\hat{b}}y_t$
(10)	p_T	$p_T^* - b^*\varepsilon_T$	$\hat{a}_T - \frac{\hat{b}}{1-\alpha\hat{b}}y_T$
(11)	$\mathrm{Var}_t p_T$	$b^{*2}\sigma^2$	$\frac{1-\rho^{2(T-t)}}{1-\rho^2}\hat{b}^2\sigma^2$
(12)	$\mathrm{Var}[p_t]$	$b^{*2}\sigma^2$	$\frac{\hat{b}^2\sigma^2}{1-\rho^2}$
(13)	$\frac{p_T-p_t}{T-t}$	$\pi^* - b^*\frac{\varepsilon_T-\varepsilon_t}{T-t}$	$\pi^* - \frac{\hat{b}}{1-\alpha\hat{b}}\frac{y_T-y_t}{T-t}$
(14)	$\mathrm{E}_t\frac{p_T-p_t}{T-t}$	$\pi^* + b^*\frac{\varepsilon_t}{T-t}$	$\pi^* + \frac{\hat{b}}{1-\alpha\hat{b}}\frac{(1-\rho^{T-t})y_t}{T-t}$
(15)	$\mathrm{E}\left[\frac{p_T-p_t}{T-t}\right]$	π^*	π^*
(16)	$\mathrm{Var}_t\frac{p_T-p_t}{T-t}$	$b^{*2}\frac{\sigma^2}{(T-t)^2}$	$\left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2\frac{\mathrm{Var}_t y_T}{(T-t)^2}$
(17)	$\mathrm{Var}\left[\frac{p_T-p_t}{T-t}\right]$	$2b^{*2}\frac{\sigma^2}{(T-t)^2}$	$\left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2\frac{\mathrm{Var}[y_T-y_t]}{(T-t)^2}$

4 Price level targeting even if society has inflation target preferences?

Section 2 and 3 above have examined the equilibria that result if the central bank targets inflation or the price level. So far I have not said anything about what the social preferences for monetary policy might be. Deriving the objectives for monetary policy from a social welfare function related to individual agents' preferences over consumption and leisure is beyond the scope of this paper. In this section, I will instead answer a much easier question: Suppose that social preferences for monetary policy simply correspond to either inflation targeting, (2.3) and (2.4), or price level targeting, (2.3) and (3.3). Suppose, furthermore, that society can assign any of these loss functions (but no other) to a central bank that has no commitment technology and acts under discretion. Which of the two loss functions should society assign to the central bank?

If social preferences correspond to price level targeting, it is obvious that it is better to assign price level targeting to the central bank. An inflation-targeting central bank would result in the same output gap behavior, but the base drift in the price level would make the price level non-trend-stationary, with price level variability increasing without bound with the horizon. In addition, if the output gap target is positive, the price level would on average grow faster than the price level target.

If social preferences correspond to inflation targeting, would it be better to assign inflation targeting to the central bank? The answer is no, if there is at least moderate output gap persistence. The reason why price-level targeting is better is, intuitively, that it (i) causes less inflation variability and (ii) results in the same output gap behavior. This is enough to make price level targeting better. If the output gap target is positive, price level targeting has an additional benefit since it (iii) eliminates any average inflation bias.

A rigorous argument, which compares the resulting social indirect loss functions, is reported in the appendix.

This result can be further illuminated by a direct comparison of the decision rules. Under inflation targeting the optimal decision rule under commitment is (2.7). Due to (2.10), it can be written

$$\pi_t = \pi^* - \frac{b^*}{1 - \alpha b^*}(y_t - \rho y_{t-1}). \tag{4.1}$$

An inflation-targeting central bank under discretion delivers the decision rule

$$\pi_t = \hat{a} - \frac{\hat{b}}{1 - \alpha \hat{b}} y_t, \tag{4.2}$$

where $\hat{a} \geq \pi^*$ and $\hat{b} > b^*$. A price-level-targeting central bank under discretion delivers the decision rule

$$\pi_t = \pi^* - \frac{\hat{b}}{1 - \alpha\hat{b}}(y_t - y_{t-1}). \tag{4.3}$$

Clearly, under discretion a price-level targeting central bank may deliver a better approximation to the optimal decision rule (4.1) than an inflation-targeting one: The coefficient $\frac{\hat{b}}{1-\alpha\hat{b}}$ is the same under both kinds of targeting (although larger than under commitment). The output gap behavior is the same. With enough output gap persistence, the first difference of the output gap, $y_t - y_{t-1}$, is a better approximation to the unanticipated change in the output gap, $y_t - \rho y_{t-1}$, than just the output gap, y_t. This is enough to make a price-level targeting central bank preferable. If the output gap target is positive, there is the additional benefit of no average inflation bias under price level targeting.[11]

This comparison of decision rules also reveals that a price-level-targeting central bank under discretion does not deliver the optimal rule for inflation targeting under commitment. Svensson (1997) examines how modified inflation targets can improve the discretionary equilibrium with persistence in output and compares with Rogoff (1985) 'conservative' central banks and with Walsh (1995)-Persson and Tabellini (1993) linear inflation contracts.

5 Conclusions

According to an emerging, although not completely unanimous (cf. Dillon and Fellow (1994)) conventional wisdom, the choice between price level targeting and inflation targeting involves a trade-off between (i) low-frequency price level variability and (ii) high-frequency inflation and output variability. This conventional wisdom arises from the use of exogenous reaction functions or exogenous inflation and price level processes, which may or may not be consistent with objectives and constraints (including commitment technologies) faced by central banks. In contrast, this paper examines price level and inflation targeting by deriving endogenous decision rules and equilibrium price level and inflation processes, when central banks have been assigned price level or inflation targets and, realistically, act under discretion and face persistent output movements.

In this framework, price level targeting naturally results in lower low-frequency price level variability than inflation targeting. However, if output persistence is at least moderate, it also

[11] Gavin and Stockman (1991) provide a different argument why price level targeting might dominate inflation targeting even if social preferences correspond to inflation targeting: it may reduce the central bank's incentive to create inflation for special interests and blame it on random events.

results in lower high-frequency inflation variability, counter to conventional wisdom. The reason is that under inflation targeting inflation depends on the output gap, whereas under price level targeting inflation depends on the *change* in the output gap; with sufficient persistence, the change in the output gap is less variable than the output gap itself.

If the output target is higher than the natural output level, price level targeting has the additional advantage of eliminating the average inflation bias that then results under inflation targeting.

In case society's preferences correspond to price level targeting, price level targeting is clearly better than inflation targeting, since the latter results in a non-trend-stationary price level and, when there is an inflation bias, in a price level that increasingly deviates from the target price level. In case society's preferences correspond to inflation targeting, because of the reduced inflation variability, it is *still* better for society to assign a price level target to the central bank (if the output gap persistence is at least moderate). (The elimination of any average inflation bias is an additional benefit.) This result can also be understood with reference to the optimal rule under commitment. Under commitment and inflation targeting, inflation depends only on the new information that has arrived after private sector expectations were formed, in this case the supply shock. Under discretion and inflation targeting, inflation depends on the output gap; under price level targeting inflation depends on the change in the output gap; when the output gap is persistent, the latter is a better approximation to the supply shock than the former.

The paper has demonstrated the importance of output persistence for the results and, I hope, the benefits of deriving endogenous decision rules for assigned targets rather than using postulated reaction functions, when comparing inflation targeting and price level targeting.

In the model used here price level targeting and inflation targeting result in the same output variability, since both regimes result in the same conditional one-period variance of the price level and the inflation rate (although the unconditional variability of one-period inflation, and the conditional more-than-one-period variance of the price level and inflation rate, are lower under price level targeting), and only the unanticipated part of one-period price movements affect output.

However, if nominal wages are downwardly rigid, anticipated negative inflation (deflation) would increase real wages and increase output. This may increase output variability; in particular it may reduce *average* output. The effect has been studied by Lebow, Roberts and Stockton (1992), Crawford and Dupasquier (1994), Fillon and Tetlow (1994) and Akerlof, Dickens and

Perry (1996). For given inflation variability, the effect depends on the average inflation rate, regardless of whether there is price level or inflation targeting. The effect is hence an argument for a positive inflation target under inflation targeting and a price level target that increases at a steady rate during price level targeting, since that would reduce the frequency of deflation. However, the reduced variability of inflation under price level targeting still seems to be an argument in favor of price level targeting. Productivity growth will in any case reduce the effect. For the United States, Lebow, Stockton and Wascher (1995) report empirical evidence that indicate little downward rigidity and very small aggregate output effects of reducing U.S. inflation to zero, and in simulations Fillon and Tetlow (1994) also report small output effects. Akerlof, Dickens and Perry (1996) find relatively larger output and unemployment effects when inflation is reduced to zero. They assume that the degree of downward nominal rigidity observed in periods with relatively high positive inflation remains the same in a zero-inflation environment. Any degree of downward nominal rigidity is, however, likely to be endogenous and regime dependent and hence decrease with less inflation.[12] Even with the assumption of unchanged downward nominal rigidity, Akerlof, Dickens and Perry find only a small effect on output and employment of reducing inflation to 2 percent per year.

Nonnegative nominal interest rates have also been used as an argument for a positive inflation rate, since low or negative inflation could then result in too high real interest rates, and in particular prevent monetary policy from being sufficiently expansionary in recessions (Summers (1991)). But, Lebow (1993) shows that monetary policy can still be expansionary by using other instruments than interest rates on government bonds and bills, if these interest rates occasionally fall to zero. In simulations, Fuhrer and Madigan (1994) find very small effects on output from nonnegative nominal interest rates. The problem will be smaller, if future real interest rates are generally higher than in the 1960s and the 1970s. For a given average inflation rate, the reduced inflation variability under price level targeting once more seems to speak in favor of price level targeting.

In any case, to the extent that downwardly rigid nominal wages and nonnegative nominal interest rates imply a positive average inflation rate, there is no principle difficulty with a price level target which increases at a steady rate, since that does not reduce the predictability of the price level.

The parameters of the Phillips curve (the slope, the degree of persistence, and the variance of

[12] For instance, an increasingly common way to circumvent downward nominal wage rigidity in my home country is to add a flexible (and, on average positive) non-negative bonus, to a downwardly rigid wage.

supply shocks) might not be invariant to a shift from inflation targeting to price level targeting. It is not obvious, though, whether the parameters are likely to change and if so, in what direction, especially since conditional variances (and average inflation in case the output target equals the natural rate) are the same in the two regimes. Clearly a more elaborate analysis with explicit microfoundations of the Phillips curve, is then required.

Will random walk measurement errors of the price level provide an argument against price level targeting? No, for if there are such measurement errors, there will be an unavoidable random walk component to the 'true' price level, but inflation targeting will add another random walk component, making the variance of the price level still higher under inflation targeting than under price level targeting.

What is the effect of control errors? Suppose there are i.i.d. control errors, η_t, on the price level, with variance σ_η^2. Under inflation targeting, this will add σ_η^2 to the variance of inflation. Under price level targeting, this will be added twice to the variance of inflation, which means that the degree of persistence must be somewhat higher (than 0.5), in order to make the inflation variance less under price level targeting (unless the variance due to control errors is so large as to dominate all other sources of variability).

Do social preferences correspond to inflation targeting, price level targeting, or something else? Deriving the objectives for monetary policy from some social welfare function over private agents' preferences is definitely beyond the scope of this paper[13]. One issue that needs to be dealt with in such an undertaking is what the social benefits of reduced long-term uncertainty of the price level are. This seems to be an under-researched area (see Konieczny (1994) and Duguay (1994) for discussion). There are obvious informational and computational benefits of a stable, or at least predictable, unit of account for intertemporal decisions and for decisions that occur relatively infrequently. Although these benefits are obvious, they are difficult to asses quantitatively. Standard economic theory is certainly at a disadvantage when assessing such costs, since it relies on the assumption of unbounded computational capacity of agents. I believe that we have to some extent become so used to a randomly increasing price level that we have grown blind to the information and computation costs it imposes. It has been argued that the analogy to length and other physical units is revealing: Suppose that the meter or the foot were to be randomly reduced each year. We could certainly live in such a world; we would only have to keep track of which year meter or foot things were measured in, and we could carry a card in

[13] As an example of this in a different context, when monetary policy can be seen as part of an optimal taxation problem, see Chari, Christiano and Kehoe's (1996) examination of Friedman's zero interest rate rule.

our wallets with the appropriate conversion factors. We could certainly live in such a world; but it would no doubt be a considerable struggle. For some reason, we have come to accept such a state of affairs in the economic sphere.

Reduced long-term uncertainty would obviously reduce the uncertainty associated with long-term nominal contracts, like long-term nominal bonds. But if the cost of such uncertainty is significant, why is it not circumvented by indexation? One possibility is that the information and computational cost of indexation is itself substantial; the fact that citizens seem to shift to foreign currency as a unit of account only when domestic inflation goes above 20-30 percent per year has been quoted as evidence that those costs may be quite substantial (Konieczny (1994)). More work on formal models of the costs of long-term price level uncertainty would be very welcome.

As noted by Konieczny (1994), some of these ideas were very well put a long time ago:

> If there is anything in the world which ought to be stable it is money, the measure of everything which enters the channels of trade. What confusion would there not be in a state where weights and measures frequently changed? On what basis and with what assurance would one person deal with another, and which nations would come to deal with people who lived in such disorder? (François Le Blanc (1690), *Traité Historique de Monnayes de France,* Paris, quoted by Einaudi (1953, p. 233).)

Appendix

A A P-bar model and the expectational Phillips curve with persistence

Consider a variant of the so-called P-bar model recently discussed in McCallum (1994),

$$
\begin{aligned}
\Delta p_t &= \tilde{\lambda}\left(\bar{p}_{t-1} - p_{t-1}\right) + \tilde{\alpha}\Delta\bar{p}_t + (1-\tilde{\alpha})\,\Delta\bar{p}_{t|t-1} - \tilde{\varepsilon}_t && \text{(A.1)}\\
y_t^d &= m_t - p_t && \text{(A.2)}\\
\bar{p}_t &= m_t - \bar{y}_t && \text{(A.3)}\\
\Delta\bar{y}_t &= e_t && \text{(A.4)}\\
y_t &= y_t^d - \bar{y}_t, && \text{(A.5)}
\end{aligned}
$$

where $\Delta p_t = p_t - p_{t-1}$, p_t is the (log) nominal price level, $\bar{p}_t$ is the 'equilibrium' (or 'natural') (log) price level that would result if actual output would equal the natural output level, $0 < \tilde{\lambda} \leq 1$, $0 < \tilde{\alpha} < 1$, $\Delta\bar{p}_{t|t-1} = \mathrm{E}_{t-1}\Delta\bar{p}_t$, y_t^d is (log) aggregate demand, m_t is (log) money supply (the central bank's instrument), $\bar{y}_t$ is the (log) capacity level (the natural rate of output), $\tilde{\varepsilon}_t$ and e_t are i.i.d. shocks with zero means, and y_t is the output gap. This variant differs from the one discussed and interpreted in McCallum (1994) by allowing for an influence of current inflation in the equilibrium prices on current actual inflation ($\tilde{\alpha} > 0$). It also includes a temporary supply shock, $\tilde{\varepsilon}_t$, as suggested by McCallum (1994, p. 254). The shock e_t is a permanent shock to capacity output, which makes the natural rate a random walk.

We can write (A.1) as

$$
\begin{aligned}
\Delta p_t &= \tilde{\lambda} y_{t-1} + \tilde{\alpha}\left(\Delta m_t - \Delta\bar{y}_t\right) + (1-\tilde{\alpha})\left(\Delta m_{t|t-1} - \Delta\bar{y}_{t|t-1}\right) - \tilde{\varepsilon}_t \\
&= \tilde{\lambda} y_{t-1} + \tilde{\alpha}\Delta m_t + (1-\tilde{\alpha})\,\Delta m_{t|t-1} - \tilde{\alpha} e_t - \tilde{\varepsilon}_t, && \text{(A.6)}
\end{aligned}
$$

where I have used

$$
\begin{aligned}
\bar{p}_t - p_t &= y_t^d - \bar{y}_t = y_t && \text{(A.7)}\\
\Delta\bar{p}_t &= \Delta m_t - \Delta\bar{y}_t && \text{(A.8)}\\
\Delta\bar{y}_{t|t-1} &= 0.
\end{aligned}
$$

Hence, from (A.7), (A.8) and (A.6),

$$
\Delta y_t = \Delta\bar{p}_t - \Delta p_t
$$

$$= -\tilde{\lambda} y_{t-1} + (1-\tilde{\alpha})\left(\Delta m_t - \Delta m_{t|t-1}\right) - (1-\tilde{\alpha}) e_t + \tilde{\varepsilon}_t \tag{A.9}$$

$$y_t = \left(1-\tilde{\lambda}\right) y_{t-1} + (1-\tilde{\alpha})\left(\Delta m_t - \Delta m_{t|t-1}\right) - (1-\tilde{\alpha}) e_t + \tilde{\varepsilon}_t. \tag{A.10}$$

I want to derive (2.1) from (A.1)-(A.5). From (A.6) we have

$$\Delta p_t - \Delta p_{t|t-1} = \tilde{\alpha}\left(\Delta m_t - \Delta m_{t|t-1}\right) - \tilde{\alpha} e_t - \tilde{\varepsilon}_t$$

$$\Delta m_t - \Delta m_{t|t-1} = \frac{1}{\tilde{\alpha}}\left(\Delta p_t - \Delta p_{t|t-1}\right) + e_t + \frac{1}{\tilde{\alpha}}\tilde{\varepsilon}_t. \tag{A.11}$$

Now I can use (A.10) and (A.11) to eliminate $\Delta m_t - \Delta m_{t|t-1}$;

$$\begin{aligned} y_t &= \left(1-\tilde{\lambda}\right) y_{t-1} + (1-\tilde{\alpha})\left[\frac{1}{\tilde{\alpha}}\left(\Delta p_t - \Delta p_{t|t-1}\right) + e_t + \frac{1}{\tilde{\alpha}}\tilde{\varepsilon}_t\right] - (1-\tilde{\alpha}) e_t + \tilde{\varepsilon}_t \\ &= \left(1-\tilde{\lambda}\right) y_{t-1} + \frac{1-\tilde{\alpha}}{\tilde{\alpha}}\left(\Delta p_t - \Delta p_{t|t-1}\right) + \frac{1}{\tilde{\alpha}}\tilde{\varepsilon}_t. \end{aligned} \tag{A.12}$$

Define

$$\rho = 1 - \tilde{\lambda} \tag{A.13}$$

$$\alpha = \frac{1-\tilde{\alpha}}{\tilde{\alpha}} \tag{A.14}$$

$$\varepsilon_t = \frac{1}{\tilde{\alpha}}\tilde{\varepsilon}_t. \tag{A.15}$$

Then (A.12) is identical to (2.1).

Hence, this variant of the P-bar model, (A.1)-(A.5), is in practice equivalent to the expectational Phillips curve with output gap persistence, (2.1). We can think of the central bank using m_t as an instrument and controlling p_t via (A.6), which then determines y_t via (2.1), or we can, equivalently, think of the central bank as directly controlling p_t in (2.1). The disturbance e_t to the natural output level vanishes from (A.12); the disturbance $\tilde{\varepsilon}_t$ in (A.1) is needed to get a disturbance in (2.1).

B Inflation targeting

B.1 Commitment to an optimal rule

The first order conditions with respect to π_t and π_t^e result in

$$(\pi_t - \pi^*) + \lambda\alpha(y_t - y^*) + \beta\alpha V_y^*(y_t) - \mathrm{E}_{t-1}\left[\lambda\alpha(y_t - y^*) + \beta\alpha V_y^*(y_t)\right] = 0, \tag{B.1}$$

where the Lagrange multiplier of (2.2) has been eliminated.

Taking expectations at $t-1$ of (B.1) gives

$$\mathrm{E}_{t-1}\pi_t = \pi^*, \tag{B.2}$$

the expected inflation rate equals the socially best inflation rate and is independent of output. Substitution of (2.1), (2.2), (B.2) and (2.6) into (B.1) results in the decision rule

$$\pi_t = \pi^* - b^*\varepsilon_t \tag{B.3}$$

with

$$b^* = \frac{(\lambda + \beta\gamma_2^*)\alpha}{1 + (\lambda + \beta\gamma_2^*)\alpha^2}. \tag{B.4}$$

The output gap will then fulfill

$$y_t = \rho y_{t-1} + (1 - \alpha b^*)\varepsilon_t. \tag{B.5}$$

In order to find b^*, γ_2^* has to be determined. The coefficients γ_1^* and γ_2^* can be identified by substituting (B.3) and (B.5) into (2.5). Together with (2.6) this results in

$$\gamma_1^* = -\frac{\lambda y^* \rho}{1 - \beta\rho} \leq 0 \quad \text{and} \quad \gamma_2^* = \frac{\lambda\rho^2}{1 - \beta\rho^2} > 0. \tag{B.6}$$

Using this in (B.4) results in

$$b^* = \frac{\lambda\alpha}{1 + \lambda\alpha^2 - \beta\rho^2}. \tag{B.7}$$

B.2 Discretion

The first order condition will be

$$\pi_t - \pi^* + \lambda\alpha(y_t - y^*) + \beta\alpha\hat{V}_y(y_t) = \pi_t - \pi^* + (\lambda + \beta\hat{\gamma}_2)\alpha y_t - (\lambda y^* - \beta\hat{\gamma}_1)\alpha = 0, \tag{B.8}$$

where I have used (2.12). The marginal loss of increased inflation expectations have vanished from the first order condition.

Taking expectations of (B.8) gives

$$\mathrm{E}_{t-1}\pi_t = \pi^* + (\lambda y^* - \beta\hat{\gamma}_1)\alpha - (\lambda + \beta\hat{\gamma}_2)\alpha\rho y_{t-1}. \tag{B.9}$$

Combining (2.1), (2.2), (B.8) and (B.9) gives a feedback rule of the form

$$\pi_t = \hat{a} - \frac{\hat{b}}{1 - \alpha\hat{b}} y_t, \tag{B.10}$$

with

$$\hat{a} = \pi^* + (\lambda y^* - \beta\hat{\gamma}_1)\alpha \quad \text{and} \quad \hat{b} = \frac{(\lambda + \beta\hat{\gamma}_2)\alpha}{1 + (\lambda + \beta\hat{\gamma}_2)\alpha^2}. \tag{B.11}$$

The output gap will be given by (2.14).

In order to determine $\hat{\gamma}_1$ and $\hat{\gamma}_2$, I substitute (B.10) and (2.14) for π_t and y_t in (2.11). Using (2.12) to identify the coefficient for y_{t-1}^2 results in

$$\hat{\gamma}_2 = (\lambda + \beta\hat{\gamma}_2)\rho^2 + (\lambda + \beta\hat{\gamma}_2)^2\alpha^2\rho^2. \tag{B.12}$$

This is a second-degree equation in $\hat{\gamma}_2$, which hence has two potential roots. The equation has real roots if and only if the first existence condition

$$\lambda \leq \lambda_1 \equiv \frac{\left(1 - \beta\rho^2\right)^2}{4\beta\alpha^2\rho^2} \tag{B.13}$$

holds. Only the smaller solution, (2.17), is relevant (see Lockwood & Philippopoulos (1994) and Svensson (1997)).

If the second term on the right-hand side of (B.12) were zero, $\hat{\gamma}_2$ would equal γ_2^*, cf. (2.9). Since the term is positive, $\hat{\gamma}_2 > \gamma_2^*$.

Identification of the coefficient for y_{t-1}, $\hat{\gamma}_1$, results in (2.16). In order to ensure that there is a finite solution to $\hat{\gamma}_1$, the second existence condition

$$\beta\rho\left[1 + (\lambda + \beta\hat{\gamma}_2)\alpha^2\right] < 1 \tag{B.14}$$

must hold. The condition has a natural interpretation: The expression on the left hand side of the inequality is the discounted total increase in output in period t of a unit increase in output in period $t-1$, when inflation in period t is held constant. The total effect consists of the direct effect, ρ, and the indirect effect via reduced inflation expectations, $\frac{\partial y_t}{\partial \pi_t^e}\frac{\partial \pi_t^e}{\partial y_{t-1}}$, cf. (B.9). If this discounted effect is above unity, the present value of the effect in all future periods will be unbounded.

With (2.17) one can show that (B.14) is equivalent to

$$\lambda < \lambda_2 \equiv \frac{(1 - \beta\rho)(1 - \rho)}{\beta\alpha^2\rho}. \tag{B.15}$$

It is shown in Svensson (1997) that for some parameter values (B.15) is more binding than (B.13). More precisely, the complete existence condition is (i) for $\frac{1}{2} < \rho < 1$ and $0 < \beta < \frac{2\rho-1}{\rho^2}$,

$\lambda \leq \lambda_1$, (ii) for $\frac{1}{2} \leq \rho < 1$ and $\beta = \frac{2\rho-1}{\rho^2}$, $\lambda < \lambda_1 = \lambda_2$, and (iii) for $0 < \rho < 1$ and $\frac{2\rho-1}{\rho^2} < \beta < 1$, $\lambda < \lambda_2 < \lambda_1$. If $y^* = 0$, only (B.13) is relevant.[14]

Identification of $\hat{\gamma}_0$ results in

$$\hat{\gamma}_0 = \frac{1}{1-\beta}\frac{1}{2}\left\{(\hat{a}-\pi^*)^2 + \lambda y^{*2} + \left[\hat{b}^2 + (\lambda + \beta\hat{\gamma}_2)(1-\alpha\hat{b})^2\right]\sigma^2\right\}. \tag{B.16}$$

C Price level targeting with inflation targeting preferences?

The equilibria resulting from either an inflation-targeting or price-level-targeting central bank will be evaluated with a social loss function corresponding to inflation targeting.

With an inflation-targeting central bank, the relevant social indirect loss function is the same as defined in the decision problem (2.11), $\hat{V}(y_{t-1})$, and given by (2.12), with the coefficients $\hat{\gamma}_1$ and $\hat{\gamma}_2$ given by (2.16) and (2.17). The coefficient $\hat{\gamma}_0$ is given by (B.16).

With a price-level-targeting central bank, the relevant social indirect loss function, denoted by $V^p(y_{t-1})$, is defined as

$$V^p(y_{t-1}) = \mathrm{E}_{t-1}\left\{\frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda(y_t - y^*)^2\right] + \beta V^p(y_t)\right\}, \tag{C.1}$$

where (3.10) and (2.14) are substituted for π_t and y_t. This value function will be quadratic and can be written

$$V^p(y_{t-1}) = \gamma_0^p + \gamma_1^p y_{t-1} + \frac{1}{2}\gamma_2^p y_{t-1}^2, \tag{C.2}$$

where the coefficients γ_0^p, γ_1^p and γ_2^p remain to be determined.

Hence, the difference between the two social indirect loss functions is

$$V^p(y_{t-1}) - \hat{V}(y_{t-1}) = (\gamma_0^p - \hat{\gamma}_0) + (\gamma_1^p - \hat{\gamma}_1)y_{t-1} + \frac{1}{2}(\gamma_2^p - \hat{\gamma}_2)y_{t-1}^2. \tag{C.3}$$

Let me start with the first term on the right-hand side of (C.3). Identification of the constant γ_0^p in (C.1) and (C.2) results, after some algebra, in

$$\gamma_0^p = \frac{1}{1-\beta}\frac{1}{2}\left\{\lambda y^{*2} + \left[\hat{b}^2 + (\lambda + \beta\gamma_2^p)(1-\alpha\hat{b})^2\right]\sigma^2\right\}. \tag{C.4}$$

From (B.16) we then have

$$\gamma_0^p - \hat{\gamma}_0 = \frac{1}{1-\beta}\frac{1}{2}\left\{-(\hat{a}-\pi^*)^2 + \beta(\gamma_2^p - \hat{\gamma}_2)(1-\alpha\hat{b})^2\sigma^2\right\}. \tag{C.5}$$

[14] The conditions (B.14) and (B.15) do not appear in the analysis of Lockwood and Philippopoulos (1994), since they assume that $y^* = 0$.

If α in (2.1) equals unity (as in Lockwood and Philippopoulos (1994) and in Lockwood, Miller and Zhang (1995)), the existence conditions appear rather restrictive. If $\beta = 0.95$ and $\rho = 0.4$ (0.8), we have $\frac{2\rho-1}{\rho^2} = -1.25$ (0.4), so (B.15) applies. Then $\lambda_2 = 0.98$ (0.06), respectively. If α instead equals 0.2, the corresponding λ_2 values are 25 times larger, that is, 24.5 (1.58). The corresponding values for λ_1 are 1.18 (0.06) for $\alpha = 1$, and 29.6 (1.58) for $\alpha = 0.2$.

The first term on the right-hand side is nonpositive. It obviously arises because the price level target equilibrium has no average inflation bias. The second term depends on the difference between the coefficients γ_2^p and $\hat{\gamma}_2$, that is, the convexity of the indirect loss function. Identification of γ_2^p in (C.1) and (C.2) gives

$$\gamma_2^p = \frac{\lambda\rho^2 + \left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2 (1-\rho)^2}{1-\beta\rho^2}. \tag{C.6}$$

In order to facilitate comparison, by (B.11) and (B.12), $\hat{\gamma}_2$ can be written as[15]

$$\hat{\gamma}_2 = \frac{\lambda\rho^2 + \left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2 \rho^2}{1-\beta\rho^2}. \tag{C.7}$$

Hence,

$$\gamma_2^p - \hat{\gamma}_2 = \frac{\left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2 [(1-\rho)^2 - \rho^2]}{1-\beta\rho^2} = \frac{\left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2 (1-2\rho)}{1-\beta\rho^2}. \tag{C.8}$$

The difference between γ_2^p and $\hat{\gamma}_2$ is negative when $\rho > \frac{1}{2}$, since the inflation rate is less sensitive to lagged output under price level targeting if $\rho > \frac{1}{2}$. A given level of y_{t-1} in period $t-1$ will give rise to a squared inflation term in period t equal to $\left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2 (1-\rho)^2 y_{t-1}^2$ under a price level target and equal to $\left(\frac{\hat{b}}{1-\alpha\hat{b}}\right)^2 \rho^2 y_{t-1}$ under an inflation target (the present value of a future sequence of such terms requires discounting by $\beta\rho^2$).

Finally, let me look at the second term in (C.3), corresponding to the linear term in the value functions. Identifying the linear term in (C.1) and (C.2) gives

$$\gamma_1^p = -\frac{\lambda y^* \rho}{1-\beta\rho}. \tag{C.9}$$

In order to facilitate comparison, I use (B.11), (B.11) and (2.16) with some algebra to rewrite $\hat{\gamma}_1$ in terms of the average inflation bias,[16]

$$\hat{\gamma}_1 = -\frac{\lambda y^* \rho + (\hat{a} - \pi^*)\frac{\hat{b}}{1-\alpha\hat{b}}\rho}{1-\beta\rho}. \tag{C.10}$$

[15] Note that by (B.11) $(\lambda + \beta\hat{\gamma}_2)\alpha = \frac{\hat{b}}{1-\alpha\hat{b}}$. Use this in the second term on the right-hand side of (B.12).

[16] Write (2.16) as

$$\hat{\gamma}_1 = -\frac{\lambda y^* \rho + \lambda\alpha y^* c}{1-\beta\rho-\beta\alpha c},$$

where $c = (\lambda + \beta\hat{\gamma}_2)\alpha\rho = \frac{\hat{b}}{1-\alpha\hat{b}}\rho$. Rewrite this as

$$\hat{\gamma}_1 = -\frac{\lambda y^* \rho + (\lambda\alpha y^* - \beta\alpha\hat{\gamma}_1)\, c}{1-\beta\rho}$$

and use (B.11).

Hence

$$\gamma_1^p - \hat{\gamma}_1 = \frac{(\hat{a} - \pi^*)\frac{\hat{b}}{1-\alpha\hat{b}}\rho}{1 - \beta\rho} \geq 0. \tag{C.11}$$

This difference is positive, when the output gap target is positive. Then an increase in y_{t-1} reduces the loss function more under inflation targeting: the resulting reduction in π_t is more beneficial when the average inflation bias is positive under inflation targeting.

The unconditional mean of (C.3) is

$$\mathrm{E}\left[V^p(y_{t-1}) - \hat{V}(y_{t-1})\right] = \gamma_0^p - \hat{\gamma}_0 + \frac{1}{2}(\gamma_2^p - \hat{\gamma}_2)\mathrm{Var}\left[y_{t-1}\right]. \tag{C.12}$$

This is strictly negative for $\rho > \frac{1}{2}$.

D Simple inflation targeting and price level targeting processes

Suppose *inflation targeting* results in the AR(1) process for inflation

$$\pi_t = h\pi_{t-1} + \eta_t, \tag{D.1}$$

where $|h| < 1$ and η_t is i.i.d. with $\mathrm{E}[\eta_t] = 0$ and $\mathrm{Var}[\eta_t] = s^2$. The unconditional variance of inflation under inflation targeting, denoted $\mathrm{Var}[\pi_t]_\pi$, fulfills

$$\mathrm{Var}\left[\pi_t\right]_\pi = \frac{s^2}{1 - h^2}. \tag{D.2}$$

The price level has a unit root,

$$p_t = p_{t-1} + \pi_t, \tag{D.3}$$

and its unconditional variance is unbounded.

Suppose *price level targeting* results in the AR(1) process for the price level

$$p_t = kp_{t-1} + \eta_t,$$

where $|k| < 1$. The unconditional variance of the price level, denoted $\mathrm{Var}[p_t]_p$, is then

$$\mathrm{Var}\left[p_t\right]_p = \frac{s^2}{1 - k^2}.$$

The corresponding inflation process is

$$\pi_t = p_t - p_{t-1} = -(1 - k)p_{t-1} + \eta_t.$$

The unconditional variance of inflation under price level targeting, $\mathrm{Var}[\pi_t]_p$, is

$$\mathrm{Var}\left[\pi_t\right]_p = (1 - k)^2\mathrm{Var}\left[p_t\right]_p + s^2 = \frac{2s^2}{1 + k}. \tag{D.4}$$

The difference between the unconditional variance of inflation under price level targeting and inflation targeting is

$$\mathrm{Var}\,[\pi_t]_p - \mathrm{Var}\,[\pi_t]_\pi = \left(\frac{2}{1+k} - \frac{1}{1-h^2}\right) s^2 = \frac{1-2h^2-k}{(1-h^2)(1+k)} s^2.$$

Hence,

$$\mathrm{Var}\,[\pi_t]_p < \mathrm{Var}\,[\pi_t]_\pi \quad \text{if and only if} \quad k > 1 - 2h^2. \tag{D.5}$$

We see that if $h = k$, we have $\mathrm{Var}\,[\pi_t]_p < \mathrm{Var}\,[\pi_t]_\pi$ if and only if $h = k > \frac{1}{2}$.

Fischer (1994, Figure 2.4 and Footnote 45) compares (D.2) and (D.4) with $h = 0$ and $k = 0.5$, for which case $k < 1 - 2h^2$ and $\mathrm{Var}[\pi_t]_p = \frac{4}{3}s^2 > \mathrm{Var}[\pi_t]_\pi = s^2$; the inflation variance is higher under price level targeting.

Duguay (1994) examines the processes (D.1) and (D.3) for different values of h and k. Typical values used are $h = 0.5$ and 0.7 (inflation targeting such that 75% of the adjustment of inflation towards the target is achieved in 2 and 4 periods (years), respectively), $k = 0.7$ (price level targeting where 75% of the adjustment of the price level towards the target is achieved in 4 periods (years)), and $s^2 = 1$ (when π and p are measured in %/year and %, respectively, that is, scaled by 100).[17] Let me use these values and compute the unconditional variance of inflation. For these values, $k > 1 - 2h^2$, the variance is less under price level targeting, and we get $\mathrm{Var}[\pi_t]_\pi = 1.33$ and 1.95, respectively, and $\mathrm{Var}[\pi_t]_p = 1.18$. Now the variance of inflation is lower under price level targeting.

Duguay (1994) does not report this unconditional standard deviation of one-period inflation; instead he reports the conditional standard deviation of the price level and the average inflation rate, $\sqrt{\mathrm{Var}_t p_T}$ and $\sqrt{\mathrm{Var}_t \frac{p_T - p_t}{T-t}} = \frac{\sqrt{\mathrm{Var}_t p_T}}{T-t}$, for different time horizons $T - t$. *Tables A1* and *A2* summarize some results for the processes (D.1) and (D.3).

What univariate processes for inflation and the price level do inflation targeting and price level targeting under discretion result in? Under inflation targeting (2.13) implies $y_t = \frac{1-\alpha\hat{b}}{\hat{b}}(\pi_t - \hat{a})$. Using this in (2.14) results in

$$\pi_t = (1-\rho)\hat{a} + \rho\pi_{t-1} + \hat{b}\varepsilon_t. \tag{D.6}$$

Thus, disregarding the constant, inflation targeting corresponds to the process (D.1) with $h = \rho$. Under price level targeting (3.8) implies $y_t = \frac{1-\alpha\hat{b}}{\hat{b}}(p_t - \hat{a}_t)$. Using this in (2.14) gives

$$\tilde{p}_t = \rho\tilde{p}_{t-1} + \hat{b}\varepsilon_t, \tag{D.7}$$

[17] My notation differs from Duguay's. My h is his β, and my k is his $1 - \alpha$.

where $\tilde{p}_t = p_t - \hat{a}_t$. Thus, the process for $\tilde{p}_t$ corresponds to the process (D.3) with $k = \rho$. The corresponding inflation process will be

$$\pi_t = \pi^* - (1-\rho)(p_{t-1} - \hat{a}_{t-1}) + \hat{b}\varepsilon_t. \tag{D.8}$$

Thus, for $h = k = \rho > \frac{1}{2}$ we get the result that $\mathrm{Var}\left[\pi_t\right]_p < \mathrm{Var}\left[\pi_t\right]_\pi$.

Table A1. Inflation targeting

(1)	π_t	$=$	$h\pi_{t-1} + \eta_t$
(2)	π_T	$=$	$h^{T-t}\pi_t + \sum_{\tau=t+1}^{T} h^{T-\tau}\eta_\tau$
(3)	p_T	$=$	$p_t + \sum_{\tau=t+1}^{T} \pi_\tau$
(4)	$\mathrm{Var}_t\pi_T$	$=$	$\left(1 - h^{2(T-t)}\right)\frac{s^2}{1-h^2}$
(5)	$\mathrm{Var}_t p_T$	$=$	$\left[(T-t) - 2\frac{1-h^{T-t}}{1-h}h + \frac{1-h^{2(T-t)}}{1-h^2}h^2\right]\frac{s^2}{(1-h)^2}$
(6)	$\mathrm{Var}\left[\pi_t\right]_\pi$	$=$	$\frac{s^2}{1-h^2}$
(7)	$\mathrm{Var}\left[p_t\right]_\pi$	$=$	∞

Table A2. Price level targeting

(1)	p_t	$=$	$kp_{t-1} + \eta_t$
(2)	π_t	$=$	$-(1-k)p_{t-1} + \eta_t$
(3)	p_T	$=$	$k^{T-t}p_t + \sum_{\tau=t+1}^{T} k^{T-\tau}\eta_\tau$
(4)	π_T	$=$	$-(1-k)p_{T-1} + \eta_T$
(5)	$\mathrm{Var}_t p_T$	$=$	$\left(1 - k^{2(T-t)}\right)\frac{s^2}{1-k^2}$
(6)	$\mathrm{Var}_t\pi_{t+1}$	$=$	s^2
(7)	$\mathrm{Var}_t\pi_T$	$=$	$\left[(1-k)^2\left(1 - k^{2(T-t-1)}\right) + 1 - k^2\right]\frac{s^2}{1-k^2} \quad (T \geq 2)$
(7)	$\mathrm{Var}\left[p_t\right]_p$	$=$	$\frac{s^2}{1-k^2}$
(9)	$\mathrm{Var}\left[\pi_t\right]_p$	$=$	$\frac{2s^2}{1+k}$

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