Open-Economy Inflation Targeting

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Abstract

The paper extends previous analysis of closed-economy inflation targeting to a small open economy with forward-looking aggregate supply and demand with some microfoundations, and with stylized realistic lags in the different transmission channels for monetary policy. The paper compares targeting of CPI and domestic inflation, strict and flexible inflation targeting, and inflation-targeting reaction functions and the Taylor rule. The optimal monetary policy response to several different shocks is examined. Flexible CPI-inflation targeting stands out as successful in limiting not only the variability of CPI inflation but also the variability of the output gap and the real exchange rate. Somewhat counter to conventional wisdom, negative productivity supply shocks and positive demand shocks have similar effects on inflation and the output gap, and induce similar monetary policy responses. The model gives limited support for a so-called monetary conditions index, MCI, of the monetary-policy impact on aggregate demand, but the impact on inflation is too complex to be captured by any single index. The index differs from currently used indices in combing (1) a long rather than a short real interest rate with the real exchange rate and (2) expected future values rather than current values. Because of (2), the index is not directly observable and verifiable to external observers.

JEL Classification: E52, E58, F41.

"Why does the Bank make things so complicated? Why doesn't it just follow the Taylor rule?" [Interruption by a distinguished macro economist at an American university, when the author was presenting Bank of Sweden's approach to inflation targeting.]

1 Introduction

During the 1990's, several countries (New Zealand, Canada, U.K., Sweden, Finland, Australia and Spain) have shifted to a new monetary policy regime, inflation targeting. This regime is characterized by (1) an explicit quantitative inflation target, either an interval or a point target, where the center of the interval or the point target currently varies across countries from 1.5 to 2.5 percent per year, (2) an operating procedure that can be described as "inflation-forecast targeting", namely the use of an internal conditional inflation forecast as an intermediate target variable, and (3) a high degree of transparency and accountability.[1]

The operating procedure can be described as inflation-forecast targeting in the following sense: The central bank's internal conditional inflation forecast (conditional upon current information, a specific instrument path, the bank's structural model(s), and judgemental adjustments of model forecasts with the use of extra-model information) is used as an intermediate target variable. An instrument path is selected that results in a conditional inflation forecast in line with a(n explicit or implicit) target for the inflation forecast (for instance, at a particular horizon, the forecast for inflation at a particular horizon equals, or is sufficiently close to, the quantitative inflation target). This instrument path then constitutes the basis for the current instrument setting.

This operating procedure is, in some sense, a necessary consequence of the lags in the transmission of monetary policy and the bank's imperfect control of inflation. In order to implement inflation targeting efficiently, an inflation-targeting central bank must have a forward-looking perspective, and must construct conditional inflation forecasts in order to decide upon the current instrument setting.[2]

The above operating procedure implies that all relevant information is used in conducting monetary policy. It also implies that there is no explicit instrument rule, that is, the current

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[1] See, for instance, Leiderman and Svensson [32], Haldane [25], [26], Mayes and Riches [33], McCallum [34], Svensson [52], [53], Freedman [22], and Bernanke and Mishkin [5].

[2] As is emphasized in Svensson [52] and [53], it is important that the forecast is the central bank's *internal structural* forecast, and not an external forecast or market expectation. If the central bank instead lets the instrument react to market expectations in a mechanical way, there may be instability, nonuniqueness or nonexistence of equilibria, as has been shown by Woodford [65] and further discussed in Bernanke and Woodford [6]

instrument setting is not a prescribed explicit function of current information. Nevertheless, the procedure results in an endogenous reaction function, which expresses the instrument as a function of the relevant information. The reaction function will, in general, not be a Taylor-type rule (where a Taylor-type rule denotes a reaction function rule that is a linear function of current inflation and output only),[3] except in the special case when current inflation and output are sufficient statistics for the state of the economy. Typically, it will depend on much more information; indeed, on anything affecting the central bank's conditional inflation forecast. Especially for an open economy, the reaction function will also depend on foreign variables, for instance foreign inflation, output and interest rates, since these have domestic effects.

Furthermore, the reaction function is generally not only a function of the gap between the inflation forecast, the intermediate target variable, and the inflation target. In the literature, "targeting" and "intermediate targets" are frequently associated with a particular *information restriction* for the reaction function, namely that the instrument must only depend on the gap between the intermediate target variable and the target level (and lags of this gap).[4] I find this information restriction rather unwarranted. In any case, "targeting variable x" is in this paper (as in Rogoff [45], Walsh [63], Svensson [52] and [53], and Rudebusch and Svensson [48]) used in the sense of "*setting a target* for variable x". Thus, "having an intermediate target" means "using all relevant information to bring the intermediate target variable in line with the target."[5]

Finally, inflation-targeting regimes are characterized by a high degree of transparency and accountability. Inflation-targeting central banks regularly issue "Inflation Reports," explaining and motivating their policy to the general public. In New Zealand, the Reserve Bank Governor's performance is being evaluated, and his job is potentially at risk, if inflation exceeds 3 percent per year or falls below 0. In the U.K., the Chancellor of Exchequer recently announced that, if inflation deviates more than 1 percentage point from the inflation target of 2.5 percent, the Governor of the Bank of England shall explain in an open letter why the divergence has occurred and what steps the Bank is taking to deal with it.

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[3] For the Taylor rule, cf. [58], the instrument is a short nominal interest rate and its deviation from a long-run mean equals the sum of 1.5 times the deviation of current inflation from an inflation target and 0.5 times the percentage deviation of current output from the natural output level.

[4] See, for instance, Bryant, Hooper and Mann [10], Judd and Motley [27] and McCallum [35].

[5] For instance, the information-restriction interpretation of "inflation targeting" would have the bizarre implication that the instrument must only respond to deviations of inflation from its target, and to nothing else. Such a policy is extremely inefficient, as is demonstrated in Rudebusch and Svensson [48]. Furthermore, it has nothing to do with real-world inflation targeting, as is obvious from the large literature. Finally, even if only inflation enters the loss function, as in "strict" inflation targeting, the appropriate corresponding instrument rule responds to both inflation and output, as demonstrated by Svensson [52].

In [52], I attempt to clarify the role of conditional inflation forecasts in the central bank's implementation as well as the public's monitoring of inflation targeting. In [53], I extend the analysis of inflation targeting to (1) the appropriate monetary-policy response to different shocks, (2) the role of additional monetary-policy goals (like output stabilization and interest-rate smoothing)[6], and (3) the consequences of model uncertainty. I show that an appropriate way of responding to shocks is to examine how they affect the conditional inflation forecast (the intermediate target variable) and then to adjust the instrument so as to bring the conditional inflation forecast back in line with its target. The case when the only concern of the central bank is to stabilize the inflation, is called "strict" inflation targeting; the situation when the central bank also puts some weight on output-stabilization, interest-rate smoothing, or some other goal is called "flexible" inflation targeting. Flexible inflation targeting, as well as concern about model uncertainty, generally have similar consequences: The appropriate policy is then generally less activist, meaning that the instrument is generally less adjusted to a given shock,[7] and inflation should (from a position away from the target) be brought more gradually in line with the inflation target. Thus, the target path for the conditional inflation forecast approaches the inflation target more slowly, and the horizon at which the inflation forecast equals the inflation target is longer.[8]

All real-world inflation-targeting economies are quite open economies with free capital mobility, where shocks originating in the rest of the world are important, and where the exchange rate plays a prominent role in the transmission mechanism of monetary policy. Nevertheless, the analysis in [52] and [53] and most previous formal work on inflation targeting deal with closed economies.[9] The main purpose of this paper is to extend the formal analysis of inflation targeting to a small open economy where the exchange rate and shocks from the rest of the world are important for conducting monetary policy. Another purpose is to incorporate recent advances in the modelling of forward-looking aggregate supply and demand. Most of the previous work on inflation targeting has used simple representations of aggregate supply and demand that more or less disregard forward-looking aspects.[10]

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[6] The case of output stabilization is also examined in the earlier paper, [52].

[7] Cf. Brainard [9]

[8] Svensson [54] provides a general and informal discussion of strict vs. flexible inflation targeting, including arguments why all inflation-targeting central banks, including the Reserve Bank of New Zealand, in practice pursue flexible rather than strict inflation targeting.

[9] For formal work that deals with open-economy aspects of inflation targeting, see, for instance, Blake and Westaway [7], Nadal-De Simone, Dennis and Redward [38] and Nadal-De Simone [37] and Persson and Tabellini [42]. Persson and Tabellini [42] (footnote 14) briefly discuss targeting of CPI inflation vs. domestic inflation.

[10] A notable exception is Bernanke and Woodford [6]. See also Svensson [53], section 7.

Including the exchange rate in the discussion of inflation targeting has several important consequences. *First*, the exchange rate allows additional channels for the transmission of monetary policy. In a closed economy, standard transmission channels include an aggregate demand channel and an expectations channel. With the aggregate demand channel, monetary policy affects aggregate demand, with a lag, via its effect on the short real interest rate (and possibly on the availability of credit). Aggregate demand then affects inflation, with another lag, via an aggregate supply equation (a Phillips curve). The expectations channel allows monetary policy to affect inflation expectations which, in turn, affect inflation, with a lag, via wage and price setting behavior.

In an open economy, the real exchange rate will affect the relative price between domestic and foreign goods, which in turn will affect both domestic and foreign demand for domestic goods, and hence contribute to the aggregate-demand channel for the transmission of monetary policy. There is also a *direct* exchange rate channel for the transmission of monetary policy to inflation, in that the exchange rate affects domestic currency prices of imported final goods, which enter the consumer price index (CPI) and hence CPI inflation. Typically, the lag of this direct exchange rate channel is considered to be shorter than that of the aggregate demand channel. Hence, by inducing exchange rate movements, monetary policy can affect CPI inflation with a shorter lag. Finally, there is an additional exchange rate channel to inflation: The exchange rate will affect the domestic currency prices of imported intermediate inputs. Eventually, it will also affect nominal wages via the effect of the CPI on wage-setting. In both cases, it will affect the cost of domestically produced goods, and hence domestic inflation (inflation in the prices of domestically produced goods).

Second, as an asset price, the exchange rate is inherently a forward-looking and expectations-determined variable. This contributes to making forward-looking behavior and the role of expectations essential in monetary policy.

Third, some foreign disturbances will be transmitted through the exchange rate, for instance, changes in foreign inflation, foreign interest rates and foreign investors' foreign-exchange risk premium. Disturbances to foreign demand for domestic goods will directly affect aggregate demand for domestic goods.

Thus, this paper will attempt to construct a small open economy model, with particular emphasis on the exchange rate channels in monetary policy, in order to model the effect on the equilibrium of domestic and foreign disturbances and the appropriate monetary-policy response

to these disturbances under inflation targeting.

1.1 Issues

Several particular issues will be discussed. First, all inflation-targeting countries have chosen to target CPI inflation, or some measure of underlying inflation that excludes some components from the CPI, for instance, costs of credit services. None of them has chosen only to target domestic inflation (either inflation in the domestic component of the CPI, or GDP inflation). One difference between CPI inflation and domestic inflation is that the direct exchange rate channel is more prominent in the former case. I will try to characterize the differences between these two targeting cases.

Second, under strict inflation targeting (when stabilizing inflation around the inflation target is the only objective for monetary policy; the terminology follows Svensson [53]) the direct exchange rate channel offers a potentially effective inflation stabilization at a relatively short horizon. Such ambitious inflation targeting may require considerable activism in monetary policy (activism in the sense of frequent adjustments of the monetary policy instrument), with the possibility of considerable variability in macro variables other than inflation. In contrast, flexible inflation targeting (when there are additional objectives for monetary policy, for instance output stabilization), may allow less activism and possibly less variability in macro variables other than inflation. Consequently, I will attempt to characterize the differences between strict and flexible inflation targeting.

Third, I will try to characterize the appropriate monetary policy response to domestic and foreign shocks, and especially the appropriate response to exchange rate movements, under different forms of inflation targeting. In this context, the Taylor rule offers a focal point for discussing reaction functions, and it is, in practice, increasingly used as a reference point in practical monetary policy discussions.[11] Consequently, I will compare the reaction functions arising under inflation targeting in an open economy to the Taylor rule, particularly in order to judge what guidance the Taylor rule provides in a small open economy.

Fourth, several inflation-targeting central banks use so-called monetary policy indices, MCIs, which combine a short interest rate and the exchange rate in an index supposed to measure the impact of monetary policy on aggregated demand, inflation or both.[12] The model presented will

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[11] Several of the papers for the NBER Conference on Monetary Policy Rules, January 1998, consider Taylor-type instrument rules, for instance, Taylor [60]. Clarida, Gali and Gertler [12] estimate forward-looking Taylor-type rules for several countries.

[12] See Ericsson and Kerbeshian [17] for a bibliography on MCIs.

be used for some brief comments on the role of MCIs.

The results of my study indicate that strict CPI-inflation targeting indeed implies a vigorous use of the direct exchange rate channel to stabilize CPI inflation at a short horizon. This results in considerable variability of the real exchange rate and other variables. In contrast, flexible CPI-inflation targeting ends up stabilizing CPI-inflation at a longer horizon, and thereby also stabilizes real exchange rates and other variables to a significant extent. In comparison with the Taylor rule, the reaction functions under inflation targeting in an open economy responds to more information than does the Taylor rule. In particular, the reaction function for CPI-inflation targeting deviates substantially from the Taylor rule, with significant direct responses to foreign disturbances. With regard to the monetary-policy response to different shocks, counter to conventional wisdom, the optimal responses to positive demand shocks and negative supply shocks are very similar.

With regard to the role of MCIs, the model presented gives some limited support for an MCI that measures the monetary-policy impact on aggregate demand to some extent. However, counter to current practice, this MCI combines the real exchange rate with a long real interest rate rather than with a short real interest rate. Furthermore, the MCI combines the expected future real exchange rate and the expected future long real interest rate, rather than the current rates, and thus it is not directly observable and verifiable to external observers. Finally, the MCI only refers to the impact on aggregate demand. The monetary-policy impact on inflation, which is transmitted via several different channels with different lags, is too complex to be summarized by any single index.

Section 2 presents the model, section 3 compares the different cases of targeting, section 4 discusses MCIs and section 5 presents the conclusions. Appendices A-F contain some technical details.

2 The model

Comparing and discussing targeting of CPI inflation and domestic inflation, as well as strict and flexible inflation targeting, requires a flexible model allowing a variety of loss functions for the central bank. I consider the case of a small rather than a large open economy, which is also the actual situation for most economies with inflation targeting.[13]

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[13] Strictly speaking, the economy is small in the world asset market and in the market for foreign goods, but not in the world market for its output.

Lags and imperfect control of inflation are crucial aspects of monetary policy, which should be explicitly taken into account in formal models of inflation targeting, as emphasized in Svensson [52]. As discussed in the Introduction above, the exchange rate introduces additional channels for monetary policy, with different lags. Finally, forward-looking expectations are crucial to exchange rate determination and may be important in aggregate supply and aggregate demand.[14] Thus, these seem to be the minimum building blocks that must be incorporated in order to discuss inflation targeting in an open economy.

2.1 A simple model of a small open economy

The model has an aggregate supply equation (Phillips curve) of the form

$$\pi_{t+2} = \alpha_\pi \pi_{t+1} + (1 - \alpha_\pi)\pi_{t+3|t} + \alpha_y[y_{t+2|t} + \beta_y(y_{t+1} - y_{t+1|t})] + \alpha_q q_{t+2|t} + \varepsilon_{t+2}. \qquad (2.1)$$

Here, for any variable x, $x_{t+\tau|t}$ denotes $E_t x_{t+\tau}$, that is, the rational expectation of $x_{t+\tau}$ in period $t+\tau$, conditional on the information available in period t. Furthermore, π_t denotes domestic (log gross) inflation in period t. Domestic inflation is measured as the deviation of log gross domestic inflation from a constant mean, which equals the constant inflation target. Since the central bank's loss function to be specified assumes that any output target is equal to the natural output level, there will be no average inflation bias (deviation of average inflation from the inflation target). Hence, average inflation will coincide with the constant inflation target. The variable y_t is the output gap, defined as

$$y_t \equiv y_t^d - y_t^n, \qquad (2.2)$$

where y_t^d is (log) aggregate demand and y_t^n is the (log) natural output level. The latter is assumed to be exogenous and stochastic and follows

$$y_{t+1}^n = \gamma_y^n y_t^n + \eta_{t+1}^n, \qquad (2.3)$$

where the coefficient γ_y^n fulfills $0 \leq \gamma_y^n < 1$ and η_{t+1}^n is a serially uncorrelated zero-mean shock to the natural output level (a "productivity" shock). The variable q_t is the (log) real exchange rate, defined as

$$q_t \equiv s_t + p_t^* - p_t, \qquad (2.4)$$

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[14] Ball [4] follows a different strategy, when incorporating exchange rates in an open-economy model of inflation targeting. He retains the backward-looking model presented in Svensson [52] and used in Ball [3], and adds an equation for the exchange rate. In order to retain the backward-looking nature of the model, the exchange rate equation lacks an expectation term and will then generally violate exchange rate parity and non-arbitrage.

where p_t is the (log) price level of domestic(ally produced) goods, p_t^* the (log) foreign price level (measured as deviations from appropriate constant trends), and s_t denotes the (log) exchange rate (measured as the deviation from a constant trend, the difference between the domestic inflation target and the mean of foreign inflation; the real exchange rate will be stationary in equilibrium).[15] The term ε_{t+2} is a zero-mean i.i.d. inflation shock (a "cost-push" shock). Thus, we have two distinct "supply" shocks, namely a productivity shock and a cost-push shock. The coefficients α_π, α_y, β_y and α_q are constant and positive; furthermore α_π and β_y are smaller than unity.

The supply function is derived in appendix C, with some microfoundations. Aside from the open-economy aspects, this function is similar to the aggregate supply function given in Svensson [53], section 7, although the more rigorous derivation here (along the lines of Woodford [66] and Rotemberg and Woodford [46]) has resulted in a somewhat different dating of the variables on the right side in (2.1). Inflation depends on lagged inflation and previous expectations of the output gap and future inflation. It is similar to a Calvo-type [11] Phillips curve in that inflation depends upon expectations of future inflation. It is similar to the Fuhrer and Moore [23] Phillips curve in that inflation depends on both lagged inflation and expected future inflation. However, it is assumed that domestic inflation is predetermined two periods in advance, in order to have a two-period lag in the effect of monetary policy on domestic inflation (and hence a longer lag than for the output gap, see below). The term including $q_{t+2|t}$ in (2.1) represents the effect of expected costs of imported intermediate inputs (or resulting wage compensation).

Let ω be the share of imported goods in the CPI.[16] Then CPI inflation, π_t^c, fulfills[17]

$$\pi_t^c = (1-\omega)\pi_t + \omega\pi_t^f = \pi_t + \omega(q_t - q_{t-1}). \tag{2.5}$$

Here π_t^f denotes domestic-currency inflation of imported foreign goods, which fulfills

$$\pi_t^f = p_t^f - p_{t-1}^f = \pi_t^* + s_t - s_{t-1} = \pi_t + q_t - q_{t-1},$$

where

$$p_t^f = p_t^* + s_t \tag{2.6}$$

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[15] Since there are no nontraded goods, the real exchange rate is also the terms of trade.

[16] The share of imported goods in the CPI is approximately constant for small deviations around a steady state. It is exactly constant if the utility function over domestic and imported goods has a constant elatisticity of substitution equal to unity (that is, is a Cobb-Douglas utility function), as is actually assumed below.

[17] Since there is no interest-rate component in the CPI, it is best interpreted as CPIX; that is, CPI inflation (and domestic inflation) are exclusive of any credit service costs.

is the (log) domestic-currency price of imported foreign goods, and $\pi_t^* = p_t^* - p_{t-1}^*$ is foreign inflation. That is, I assume that there is no lag in the pass-through of import costs to domestic prices of imported goods.

Aggregate demand for domestically produced goods is given by the aggregate demand equation (expressed in terms of the output gap, (2.2)),

$$y_{t+1} = \beta_y y_t - \beta_\rho \rho_{t+1|t} + \beta_y^* y_{t+1|t}^* + \beta_q q_{t+1|t} - (\gamma_y^n - \beta_y) y_t^n + \eta_{t+1}^d - \eta_{t+1}^n, \qquad (2.7)$$

where y_t^* is (log) foreign output, all coefficients are constant and nonnegative, with $0 \leq \beta_y < 1$, and η_{t+1}^d is a zero-mean i.i.d. demand shock. The variable ρ_t is defined as

$$\rho_t \equiv \sum_{\tau=0}^{\infty} r_{t+\tau|t}, \qquad (2.8)$$

where r_t, the (short domestic-good) real interest rate (measured as the deviation from a constant mean, the natural real interest rate), fulfills

$$r_t \equiv i_t - \pi_{t+1|t}, \qquad (2.9)$$

where i_t is the (short) nominal interest rate (measured as the deviation from the sum of the inflation target and the natural real interest rate). The nominal interest rate is the instrument of the central bank.[18]

Thus, the variable ρ_t is the sum of current and expected future (deviations of) real interest rates. This sum always converges in the equilibria examined below (recall that everything is measured as deviations from constant means). The variable ρ_t is (under the expectations hypothesis) related to (the deviations from the mean of) a long real zero-coupon bond rate: Consider the real rate r_t^T with maturity T. Under the expectations hypothesis, it fulfills

$$r_t^T = \frac{1}{T} \sum_{\tau=0}^{T} r_{t+\tau|t}.$$

Hence, for a long (but finite) maturity T, the variable ρ_t is approximately the product of the long real rate and its maturity,

$$\rho_t \approx T r_t^T. \qquad (2.10)$$

The aggregate demand is predetermined one period in advance. It depends on lagged expectations of accumulated future real interest rates, foreign output and the real exchange rate. The

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[18] The variable ρ_t fulfills $\rho_t = \sum_{\tau=0}^{\infty} r_{t+\tau|t}^c - \omega q_t$, where $r_t^c \equiv i_t - \pi_{t+1|t}^c = r_t - \omega(q_{t+1|t} - q_t)$ is the CPI real interest rate. Hence, we can express ρ_t in terms of r_t^c rather than r_t (the derivation in appendix A actually starts from an Euler condition in terms of r_t^c). Since $\pi_{t+1|t}$ is a predetermined state variable, whereas $\pi_{t+1|t}^c$ is forward-looking, I find it is more practical to use r_t rather than r_t^c.

aggregate demand equation is derived, with some microfoundations, and discussed in further detail in appendix A.[19]

The exchange rate fulfills the interest parity condition

$$i_t - i_t^* = s_{t+1|t} - s_t + \varphi_t,$$

where i_t^* is the foreign nominal interest rate and φ_t is the foreign-exchange risk premium. In order to eliminate the non-stationary exchange rate, I use (2.4) to rewrite this as the real interest parity condition

$$q_{t+1|t} = q_t + i_t - \pi_{t+1|t} - i_t^* + \pi_{t+1|t}^* - \varphi_t. \tag{2.11}$$

I assume that foreign inflation, foreign output and the foreign-exchange risk premium follow stationary univariate AR(1) processes,

$$\pi_{t+1}^* \;=\; \gamma_\pi^* \pi_t^* + \varepsilon_{t+1}^* \tag{2.12}$$

$$y_{t+1}^* \;=\; \gamma_y^* y_t^* + \eta_{t+1}^* \tag{2.13}$$

$$\varphi_{t+1} \;=\; \gamma_\varphi \varphi_t + \xi_{\varphi,t+1}, \tag{2.14}$$

where the coefficients are nonnegative and less than unity, and the shocks are zero-mean i.i.d. Furthermore, I assume that the foreign interest rate follows a Taylor-type rule, that is, that it is a linear function of foreign inflation and output,

$$i_t^* = f_\pi^* \pi_t^* + f_y^* y_t^* + \xi_{it}^*, \tag{2.15}$$

where the coefficients are constant and positive, and ξ_{it}^* is a zero-mean i.i.d. shock. These specifications of the exogenous variables are chosen for simplicity; obviously the exogenous variables may be cross-correlated in more general ways without causing any difficulties, and additional variables can be introduced to represent the state of the rest of the world.

Note that ρ_t and q_t are closely related. By (2.8)–(2.11) we have (assuming $\lim_{\tau\to\infty} q_{t+\tau|t} = 0$)

$$
\begin{aligned}
q_t \;&=\; -\sum_{\tau=0}^{\infty} r_{t+\tau|t} + \sum_{\tau=0}^{\infty}(i_{t+\tau|t}^* - \pi_{t+1+\tau|t}^* + \varphi_{t+\tau|t}) \\
&=\; -\rho_t + \sum_{\tau=0}^{\infty}(i_{t+\tau|t}^* - \pi_{t+1+\tau|t}^* + \varphi_{t+\tau|t}). \tag{2.16}
\end{aligned}
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[19] There is an obvious similarity to the closed-economy aggregate demand function of Fuhrer and Moore [23], except that a *lagged* long real coupon-bond rate enters in their function.

By (2.12)–(2.15), we have (exploiting the sum of a geometric series)

$$
\begin{aligned}
\sum_{\tau=0}^{\infty}(i^*_{t+\tau|t} - \pi^*_{t+1+\tau|t}) &= i^*_t + \sum_{\tau=1}^{\infty} i^*_{t+\tau|t} - \sum_{\tau=1}^{\infty} \pi^*_{t+\tau|t} \\
&= i^*_t + f^*_\pi \frac{\gamma^*_\pi}{1 - \gamma^*_\pi}\pi^*_t + f^*_y \frac{\gamma^*_y}{1 - \gamma^*_y}y^*_t - \frac{\gamma^*_\pi}{1 - \gamma^*_\pi}\pi^*_t \\
&= i^*_t + \frac{(f^*_\pi - 1)\gamma^*_\pi}{1 - \gamma^*_\pi}\pi^*_t + \frac{f^*_y \gamma^*_y}{1 - \gamma^*_y}y^*_t;
\end{aligned}
\tag{2.17}
$$

hence,

$$
\rho_t = - q_t + i^*_t + \frac{(f^* - 1)\gamma^*_\pi}{1 - \gamma^*_\pi}\pi^*_t + \frac{f^*_y \gamma^*_y}{1 - \gamma^*_y}y^*_t + \frac{1}{1 - \gamma_\varphi}\varphi_t.
\tag{2.18}
$$

As shown in appendix B, the variable ρ_t can be interpreted as the negative of an infinite-horizon market discount factor, that is, the present value of domestic goods infinitely far into the future.

In summary, the model consists of the aggregate supply equation, (2.1), the CPI equation, (2.5), the aggregate demand equation, (2.7), the definitions of the sum of current and expected future real interest rates and the real interest rate, (2.8) and (2.9), real interest-rate parity, (2.11), and the equations for the exogenous variables: foreign inflation and output, the foreign-exchange risk premium and the foreign interest rate, (2.12)–(2.15).

The timing and the lags have been selected to provide realistic relative lags for the transmission of monetary policy. Consider a change in the instrument i_t in period t. Current domestic inflation and the output gap are predetermined. Domestic inflation in period $t + 1$ is also predetermined; hence so are domestic inflation expectations, $\pi_{t+1|t}$. Thus, the short real interest rate, r_t, is immediately affected, as are the forward-looking variables, the real exchange rate, q_t, the sum of expected current and future real interest rates, ρ_t, and the expected domestic inflation in period $t + 3$, $\pi_{t+3|t}$. Current CPI inflation is by (2.5) affected by the current real exchange rate (this is the direct exchange rate channel). The aggregate demand in period $t + 1$, y_{t+1}, is by (2.7) affected via the instrument's effect on the expected real exchange rate, $q_{t+1|t}$, (part of the exchange rate channel) and the sum of expected future real interest rates, $\rho_{t+1|t}$, (the aggregate demand channel). Domestic inflation in period $t + 2$, π_{t+2}, is by (2.1) affected by the instrument via the expected real exchange rate depreciation $(q_{t+2|t} - q_{t+1|t})$ (the remaining part of the exchange rate channel), via the output gap in period $t + 1$ (the aggregate demand channel), and by domestic-inflation expectations, $\pi_{t+3|t}$ (the inflation-expectations channel).

Thus, there is no lag in the monetary policy effect on CPI inflation, a one-period lag in the effect on aggregate demand, and a two-period lag in the effect on domestic inflation. Both

11

VAR evidence and practical central-bank experience indicate that there is a shorter lag for CPI inflation and aggregate demand than for domestic inflation.[20] [21]

2.2 The loss function

I assume that the central bank's loss function is the unconditional expectation, $\mathrm{E}[L_t]$, of a period loss function given by

$$L_t = \mu_\pi^c \pi_t^{c2} + \mu_\pi \pi_t^2 + \lambda y_t^2 + \mu_i i_t^2 + \nu_i \left(i_t - i_{t-1}\right)^2, \tag{2.19}$$

where all weights are nonnegative. Thus, the loss function is

$$\mathrm{E}[L_t] = \mu_\pi^c \mathrm{Var}[\pi_t^c] + \mu_\pi \mathrm{Var}[\pi_t] + \lambda \mathrm{Var}[y_t] + \mu_i \mathrm{Var}[i_t] + \nu_i \mathrm{Var}[i_t - i_{t-1}], \tag{2.20}$$

that is, the weighted sum of the corresponding unconditional variances. The first two terms correspond to CPI-inflation targeting and domestic-inflation targeting, respectively. The third term corresponds to output-gap stabilization, the fourth to instrument or nominal interest-rate stabilization, and the fifth to instrument or nominal interest-rate smoothing.[22]

"Strict CPI-inflation targeting" corresponds to μ_π^c positive and all other weights are equal to zero. "Flexible CPI-inflation targeting" allows other positive weights, for instance λ , μ_i or ν_i. "Domestic" inflation targeting rather than "CPI" inflation targeting has μ_π positive weight rather than μ_π^c. Thus, the decision problem for the bank is to choose the instrument, i_t, conditional upon the information available in period t, so as to minimize (2.20).

The loss function (2.20) can be seen as the (scaled) limit of the intertemporal loss function

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}, \tag{2.21}$$

when the discount factor δ, fulfilling $0 < \delta < 1$, approaches unity (see appendix E for details).

2.3 State-space form

It is shown in appendix D that the model can be written in a convenient state-space form. Let X_t and Y_t denote the (column) vectors of predetermined state variables and goal variables,

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[20] See for instance Cushman and Zha [15].

[21] Since I regard the short interest rate as the instrument of monetary policy, it is not necessary to explicitly introduce money. Nevertheless, a demand for money can, of course, be introduced in a number of ways, in which case the central bank simply supplies the money demanded at the selected level of the interest rate.

[22] The flexibility of the model allows us to include any variable of interest in the period loss function. One could, for instance, include terms $\mu_r r_t^2$, $\nu_r \left(r_t - r_{t-1}\right)^2$, $\mu_s s_t^2$, $\nu_s (s_t - s_{t-1})^2$, $\mu_q q_t^2$ and $\nu_q (q_t - q_{t-1})^2$, corresponding to stabilization and smoothing of real interest rates and nominal and real exchange rates.

respectively, let x_t denote the (column) vector of forward-looking variables, and let v_t denote the (column) vector of innovations to the predetermined state variables,

$$
\begin{aligned}
X_t &= \left(\pi_t, y_t, \pi_t^*, y_t^*, i_t^*, \varphi_t, y_t^n, q_{t-1}, i_{t-1}, \pi_{t+1|t}\right)' \\
Y_t &= \left(\pi_t^c, \pi_t, y_t, i_t, i_t - i_{t-1}\right)' \\
x_t &= \left(q_t, \rho_t, \pi_{t+2|t}\right)' \\
v_t &= \left(\varepsilon_t, \eta_t^d - \eta_t^n, \varepsilon_t^*, \eta_t^*, f_\pi^* \varepsilon_t^* + f_y^* \eta_t^* + \xi_{it}^*, \xi_{\varphi t}, \eta_t^n, 0, 0, \alpha_\pi \varepsilon_t + \alpha_y \beta_y (\eta_t^d - \eta_t^n)\right)',
\end{aligned}
$$

where $'$ denotes the transpose. Let $Z_t = (X_t', x_t')'$ be the vector of the predetermined state variables and the forward-looking variables. Denote the dimensions of X_t, x_t, Y_t and Z_t by n_1, n_2, n_3 and $n = n_1 + n_2$, respectively ($n_1 = 10$, $n_2 = 3$, $n_3 = 5$). Then the model can be written

$$
\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = AZ_t + Bi_t + B^1 i_{t+1|t} + \begin{bmatrix} v_{t+1} \\ 0 \end{bmatrix} \tag{2.22}
$$

$$
Y_t = C_Z Z_t + C_i i_t \tag{2.23}
$$

$$
L_t = Y_t' K Y_t, \tag{2.24}
$$

where A is an $n \times n$ matrix; B and B^1 are $n \times 1$ column vectors; C_Z is an $n_3 \times n$ matrix; C_i is an $n_3 \times 1$ column vector; and K is an $n_3 \times n_3$ diagonal matrix with the diagonal

$$
(\mu_\pi^c, \mu_\pi, \lambda, \mu_i, \nu_i)
$$

and with all off-diagonal elements equal to zero (see appendix D for details).

2.4 The solution

Except for the term $B^1 i_{t+1|t}$, the model is a standard linear stochastic regulator problem with rational expectations and forward-looking variables (the standard problem is solved in Oudiz and Sachs [41], Backus and Driffill [2], and Currie and Levine [14], and applied in Svensson [50]). Appendix E shows how the extra term $B^1 i_{t+1|t}$ is handled.

With forward-looking variables, there is a difference between the case of discretion and the case of commitment to an optimal rule, as discussed in the above references. In the discretion case, the forward-looking variables will be linear functions of the predetermined variables,

$$
x_t = H X_t,
$$

where the $n_2 \times n_1$ matrix H is endogenously determined. The optimal reaction function will be a linear function of the predetermined variables,

$$i_t = fX_t, \tag{2.25}$$

where the $1 \times n_1$ row vector f is endogenously determined.

In the commitment case, the optimal policy and the forward-looking variables also depend on the shadow prices of the forward-looking variables. Only the discretion solution is considered here. See appendix E for details of the solution.

The dynamics of the economy are then described by

$$X_{t+1} = M_{11}X_t + v_{t+1} \tag{2.26}$$

$$x_t = HX_t \tag{2.27}$$

$$i_t = fX_t \tag{2.28}$$

$$Y_t = (C_{Z1} + C_{Z2}H + C_if)X_t, \tag{2.29}$$

where the $n \times n$ matrix M is given by

$$M \equiv (I - B^1F)^{-1}(A + BF),$$

where $F = (f, 0...0)$ is the $1 \times n$ row vector where n_2 zeros are inserted at the end of f, and where the matrices

$$M = \begin{bmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{bmatrix}, \; C_Z = \begin{bmatrix} C_{Z1} \\ C_{Z2} \end{bmatrix}$$

are partitioned according to X_t and x_t.

3 Results on optimal policies

3.1 Model parameters

In this version of the paper, no attempt is made to calibrate or estimate the model. The parameters are simply selected to be a priori not unreasonable. The numerical results are therefore only indicative.

The following parameters are selected: In the aggregate supply equation, (2.1): $\alpha_\pi = 0.6$, $\alpha_y = (1 - \alpha_\pi)\tilde{\alpha}_y$ where $\tilde{\alpha}_y = 0.2$, $\alpha_q = (1 - \alpha_\pi)\tilde{\alpha}_q$ where $\tilde{\alpha}_q = 0.025$, and $\sigma_\varepsilon^2 = 1$ (the last parameter is the variance of the cost-push shock). In the CPI equation, (2.5): $\omega = 0.3$. In the

aggregate demand equation, (2.7): $\beta_y = 0.8$, $\beta_y^* = (1 - \beta_y)\tilde{\beta}_y^*$ where $\tilde{\beta}_y^* = 0.27$, $\beta_\rho = (1 - \beta_y)\tilde{\beta}_\rho$ where $\tilde{\beta}_\rho = 0.35$, $\beta_q = (1 - \beta_y)\tilde{\beta}_q$ where $\tilde{\beta}_q = 0.195$, $\gamma_y^n = 0.96$, $\sigma_d^2 = 1$ and $\sigma_n^2 = 0.5$ (σ_d^2 and σ_n^2 are the variance of the demand and supply shocks, η_t^d and η_t^n, respectively). In the equations for the exogenous variables, (2.3) and (2.12)–(2.15): $\gamma_\pi^* = \gamma_y^* = \gamma_\varphi = 0.8$, $f_\pi^* = 1.5$, $f_y^* = 0.5$ and $\sigma_{\varepsilon^*}^2 = \sigma_{\eta^*}^2 = \sigma_{\xi\varphi}^2 = \sigma_{\xi i^*}^2 = 0.5$ (the coefficients in the equation for the foreign interest rate conform to the Taylor rule).[23]

3.2 Targeting cases and Taylor rules

The different cases of monetary-policy targeting are defined by the weights in the loss function. The four targeting cases (combinations of positive weights) to be examined are displayed in table 1. The case of strict CPI-inflation targeting does not converge unless a small weight on interest smoothing is added; for uniformity, the weight ν_i is set equal to 0.01 for all targeting cases. In addition, two versions of the Taylor rule are included, corresponding to whether the instrument responds to domestic inflation or to CPI inflation.

Table 1. Targeting cases and Taylor rules

1. Strict domestic-inflation targeting	$\mu_\pi = 1$, $\nu_i = 0.01$
2. Flexible domestic-inflation targeting	$\mu_\pi = 1$, $\nu_i = 0.01$, $\lambda = 0.5$
3. Strict CPI-inflation targeting	$\mu_\pi^c = 1$, $\nu_i = 0.01$
4. Flexible CPI-inflation targeting	$\mu_\pi^c = 1$, $\nu_i = 0.01$, $\lambda = 0.5$
5. Taylor rule, domestic inflation	$i_t = 1.5\pi_t + 0.5y_t$
6. Taylor rule, CPI inflation	$i_t = 1.5\pi_t^c + 0.5y_t$

3.3 Summary results on reaction functions

The coefficients in the reaction functions, the elements of the row vectors f that correspond to the four optimal reaction functions and the two Taylor rules, are summarized for the six cases in table 2.

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[23] Behind these parameters are the underlying parameters (see appendices A and C) $\alpha = 0.5$, $\vartheta = 1.25$, $\tilde{\omega} = 0.8$, $\xi = \frac{(1-\alpha)(1-\alpha\delta)}{\alpha(1+\tilde{\omega}\vartheta)} = 0.25$, $\gamma = 0.1$, $\tilde{\alpha}_q = \xi\gamma = 0.025$, $\tilde{\alpha}_y = \xi\tilde{\omega} = 0.2$, $\kappa = 1 - \omega = 0.7$, $\sigma = 0.5$, $\theta = 1$, $\theta^* = 2$, $\omega^* = 0.15$, $\tilde{\beta}_\rho = \kappa\sigma = 0.35$, $\bar{\beta}_y^* = 0.9$, $\tilde{\beta}_y^* = (1-\kappa)\bar{\beta}_y^* = 0.27$, $\tilde{\beta}_q = (1-\kappa)\theta^*\omega^* - \kappa(\sigma - \theta)\omega = 0.195$.

Table 2. Reaction-function coefficients

Case	π_t	y_t	$\pi_{t+1|t}$	π_t^*	y_t^*	i_t^*	φ_t	y_t^n	q_{t-1}	i_{t-1}	q_t
1. Strict domestic	0.00	0.27	2.43	0.14	0.11	0.00	0.20	0.02	0.00	0.62	-
2. Flexible domestic	0.00	1.39	1.42	0.17	0.14	0.00	0.24	0.07	0.00	0.53	-
3. Strict CPI	0.02	−0.01	−2.28	−0.79	0.01	1.00	1.01	0.01	−0.01	0.00	-
4. Flexible CPI	0.72	−0.26	−0.69	−0.47	0.15	0.97	1.41	0.28	−0.22	0.01	-
5. Taylor, domestic	1.50	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-
6. Taylor, CPI	1.50	0.50	0.00	0.00	0.00	0.00	0.00	0.00	−0.45	0.00	0.45

Let me initially make some general comments about the reaction functions. First, the Taylor rule (table 2, rows 5 and 6) makes the instrument depend on current inflation (domestic or CPI) and the output gap only, with the coefficients 1.5 and 0.5, respectively. In this model, the Taylor rule for CPI inflation (row 6) has the property that the reaction function depends on a forward-looking variable, q_t (since CPI inflation by (2.5) fulfills $\pi_t^c = \pi_t + \omega(q_t - q_{t-1})$).

Second, the reaction functions for domestic-inflation targeting look somewhat similar to the Taylor rule for domestic inflation, except that (1) they depend on expected domestic inflation $\pi_{t+1|t}$ (which is predetermined) rather than current domestic inflation, (2) the coefficients differ from that of the Taylor rule, and (3) they also depend on other state variables. The reason for (1) is that by (2.1) expected domestic inflation two periods ahead (the shortest horizon at which domestic inflation is affected by the instrument) does not depend on current domestic inflation but on (the predetermined) expected domestic inflation one period ahead. The reaction functions for domestic-inflation targeting are intuitive in that strict inflation targeting (with no weight on output-gap stabilization) has a smaller coefficient on the output gap and a larger coefficient on expected domestic inflation than flexible inflation targeting. The coefficients on expected inflation and (for flexible domestic-inflation targeting) on the output gap are larger than those of the Taylor rule; however, optimal Taylor-type rules (that is, linear reaction functions with optimized coefficients on current inflation and the output gap and all other coefficients equal to zero) are often found to have somewhat larger coefficients than 1.5 and 0.5 (cf. Rudebusch and Svensson [48] and other papers in Taylor [61]). Hence, (2) is not so surprising. With regard to (3), it is natural that optimal reaction functions depend on several of the state variables; it is somewhat surprising that the coefficients are so small, except the coefficient for i_{t-1}. On the other hand, it is somewhat surprising that that coefficient is so large, since the weight ν_i is only 0.01.

Third, the reaction functions for CPI-inflation targeting look very different from the Taylor rule. The negative coefficients on expected domestic inflation and on the output gap stand out. We can, of course, not draw specific conclusions from the actual numerical values of the output-gap coefficients, since the model's parameters have not been calibrated or estimated. Nevertheless, a sizeable negative coefficient on the output gap and expected domestic inflation is certainly a stark contrast to the Taylor rule. Also, the coefficients on the foreign interest rate and the foreign exchange risk premium are relatively large, about one, rather than zero.

The reason for the coefficients for strict CPI-inflation targeting is that by (2.5) the exchange rate channel gives the central bank a possibility to stabilize CPI inflation completely. Suppose expected CPI inflation is equal to zero, which gives

$$\pi^c_{t+1|t} = \pi_{t+1|t} + \omega(q_{t+1|t} - q_{t|t}) = 0, \tag{3.1}$$

that is,

$$q_{t+1|t} - q_t = -\frac{1}{\omega}\pi_{t+1|t}. \tag{3.2}$$

Furthermore, by (2.11) the instrument fulfills

$$\begin{aligned}
i_t &= \pi_{t+1|t} + q_{t+1|t} - q_t + i^*_t - \pi^*_{t+1|t} + \varphi_t \\
&= -\frac{1-\omega}{\omega}\pi_{t+1|t} + i^*_t - \gamma^*_\pi \pi^*_t + \varphi_t,
\end{aligned} \tag{3.3}$$

where I have used (3.2) and (2.12). This is indeed the reaction function displayed in table 2 for strict CPI-inflation targeting (row 3), except that it is slightly modified since $\nu_i > 0$ and the central bank smooths the instrument to a small extent.

Flexible CPI-inflation targeting increases the coefficient on current domestic inflation from zero to positive, and reduces the coefficient on the output gap from zero to negative. At first, this seems counterintuitive, and we must look at the corresponding impulse responses below to understand this.

Fourth, we note that current CPI inflation, π^c_t, does not enter in the reaction function, due to the fact that it is not an independent state variable, but a linear combination of the state variables. Indeed, since π^c_t and i_t are both linear combinations of the state variables,

$$\begin{aligned}
\pi^c_t &= (1-\omega)\pi_t + \omega(\pi_t + q_t - q_{t-1}) \equiv aX_t \\
i_t &= fX_t,
\end{aligned}$$

the reaction function can, of course, be expressed as a (non-unique) function of π_t^c and the state variables, for instance

$$i_t = \kappa \pi_t^c + (f - \kappa a) X_t,$$

for any arbitrary coefficient κ.

Fifth, we note that the current real exchange rate, q_t, does not enter for the optimal reaction function. The reaction function is a function of predetermined variables only, not of any forward-looking variables. The lagged real exchange rate, q_{t-1}, is a state variable, though, and does enter in some of the reaction functions. Note that since q_t is a linear function of the state variables,

$$q_t = H_1.X_t \tag{3.4}$$

where H_1. denotes the first row of matrix H, we can, of course (as above for CPI inflation), write the reaction function as a (non-unique) function of q_t,

$$i_t = \kappa q_t + (f - \kappa H_1.)X_t$$

for any arbitrary coefficient κ.

Sixth, the reaction function is generally not of the form frequently used in the literature,[24]

$$\Delta i_t \equiv i_t - i_{t-1} = bX_t,$$

for some row vector b (where b_9, the coefficient for i_{t-1}, is zero). That is, the reaction functions are generally not such that the change in the instrument depends on the state variables (other than i_{t-1}), that is, the coefficient on i_{t-1} is not equal to minus one. In the cases in table 2, the lagged interest rate enters only because there is a small weight on interest smoothing, $\nu_i > 0$.

3.4 Discussion of targeting cases

Selected unconditional standard deviations are reported for the six cases in table 3.[25]

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[24] See, for instance, Williams [64].

[25] The nominal exchange rate is nonstationary, so its unconditional standard deviation is unbounded.

Table 3. Unconditional standard deviations

Targeting case	π_t^c	π_t	y_t	q_t	i_t	r_t
1. Strict domestic-inflation	2.00	1.25	1.91	9.82	3.23	2.62
2. Flexible domestic-inflation	2.66	1.51	1.51	10.12	3.46	2.96
3. Strict CPI-inflation	0.04	2.00	3.62	13.79	4.41	6.05
4. Flexible CPI-inflation	1.09	1.32	1.96	6.73	2.50	2.41
5. Taylor rule, domestic	2.13	1.59	1.74	8.13	2.45	1.35
6. Taylor rule, CPI	1.84	1.66	1.77	8.26	2.54	1.82

Figures 3.1–3.6 report impulse responses for the six cases (units are percent or percent per year). In each figure, column 1 reports the impulse responses to a cost-push shock to domestic inflation, π_t, in period 0 ($\varepsilon_0 = 1$), cf. (2.1) and (D.1). This shock also affects the domestic inflation expected for period 1, $\pi_{1|0}$, by $\alpha_\pi \varepsilon_0$, cf. (D.2).

Column 2 reports impulse responses to a demand shock to the output gap, y_t, in period 0 ($\eta_0^d = 1$), cf. (2.7). This shock also affects the inflation expected for period 1, $\pi_{1|0}$, by $\alpha_y \beta_y \eta_0^d$, cf. (D.2). Column 3 reports the impulse responses to a *negative* productivity shock ($\eta_0^n = -1$). This shock implies a *positive* shock to the output gap ($-\eta_0^n = 1$), cf. (2.7), and to inflation expected for period 1, $\pi_{1|0}$ ($-\alpha_y \beta_y \eta_0^d = \alpha_y \beta_y > 0$), cf. (2.1). Column 4 reports impulse responses to a shock to foreign inflation, π_t^*, in period 0 ($\varepsilon_0^* = 1$). This shock also implies a shock $f_\pi^* \varepsilon_0^* = 1.5$ to the foreign interest rate, i_t^*, due to the assumption that the foreign interest rate follows the Taylor rule, cf. (2.15). Column 5 reports impulse responses to a shock to the foreign exchange risk premium, φ_t, in period 0 ($\xi_{\varphi 0} = 1$). In column 6, shocks to the interest rate set $i_t = 1$ for the first 4 periods, $t = 0, ..., 3$ (the shocks at $t = 1, 2, 3$ are anticipated in period 0).

3.4.1 Strict domestic-inflation targeting

For a cost-push shock to domestic inflation in period 0 ($\varepsilon_0 = 1$), domestic inflation increases to 1 in period 0 and to $\alpha_\pi = 0.6$ in period 2 (figure 3.1, column 1, row 2). There is a strong monetary policy response: a large increase in the nominal interest rate (row 4). As a result, the real interest rate rises (row 5), and there is a large appreciation of the real exchange rate (row 6; note that the vertical scale is smaller than for the other rows). As a result, the output gap contracts, and domestic inflation falls and reaches its target level after about 6 periods.

The shock to domestic inflation leads (for constant real exchange rate) to an equal shock to CPI inflation, cf. (2.5). However, the large real appreciation causes import prices to fall such

19

that the net effect is an initial fall in CPI inflation. The real depreciation that follows, and the shock to domestic inflation in period 1, cause a sizeable increase in CPI inflation in period 1.

For a demand shock ($\eta_0^d = 1$) (column 2), we see a somewhat smaller increase in the nominal and real interest rate, and a smaller real appreciation. As a result, the output gap falls back to zero in about three periods, and then undershoots a little. Domestic inflation is insulated to a large extent.

In column 3, we see that a negative productivity shock has effects which are remarkably similar to those of a positive demand shock. It increases the output gap (row 3) and increases domestic inflation in period 1 (row 2), and leads to a similar monetary policy response (row 4). This is of course due to the symmetric way in which demand and productivity shocks enter in the aggregate supply and demand functions (when the latter are expressed in terms of the output gap), (2.1) and (2.7). The impulse responses are not identical, though, since the shock to the natural output level has a persistent effect on the output gap, cf. the fifth term on the right side of (2.7), since $\gamma_y^n - \beta_y = 0.16$ with the parameters I have chosen. In particular, the response of the real exchange rate is much more persistent than its response to a demand shock (row 6, columns 2 and 3), since the persistent fall in aggregate demand due to the persistent productivity shock (when the output gap has closed) requires a persistent (but not permanent) appreciation of the real exchange rate. If β_y and γ_y^n were equal, the impulse responses would be equal for demand and supply shocks.

For shocks to foreign inflation and the foreign exchange risk premium (columns 4 and 5), monetary policy almost perfectly insulates domestic inflation (row 2).

In table 3, row 1, we see that the resulting variability of domestic inflation is relatively low, whereas the variability of CPI inflation and the output gap is relatively high. The variability of the real exchange rate is particularly high.

3.4.2 Flexible domestic-inflation targeting

For a shock to domestic inflation (figure 3.2, column 1), the increase in the nominal and real interest rates, and the real appreciation, under flexible domestic-inflation targeting, are more moderate than for strict domestic-inflation targeting. As a result, the output gap falls much less, and domestic inflation returns to the target more gradually. The output gap is stabilized to a greater extent than for strict domestic-inflation targeting.

For a demand shock (column 2), there is a larger monetary policy contraction, and the output

gap is stabilized further than for strict domestic-inflation targeting. As a consequence, there is more variability in CPI inflation. For shocks to foreign inflation and the foreign-exchange risk premium (column 4 and 5), monetary policy more or less cancels the effects on both domestic inflation and the output gap.

Compared to the case of strict domestic-inflation targeting, as we noted in table 2 (row 2), the reaction function has a lower coefficient for expected domestic inflation, and a higher coefficient for the output gap. The variability of both domestic and CPI inflation is higher (table 3, row 2), and the variability of the output gap is lower. The variability of both the real exchange rate and the real interest rate are higher.

3.4.3 Strict CPI-inflation targeting

For a shock to domestic inflation ($\varepsilon_0 = 1$), CPI inflation is almost completely insulated under strict CPI-inflation targeting, even though the same shock (for a given real exchange rate) by (2.5) hits CPI inflation. As noted above, cf. (3.1), the reason is that the central bank uses the direct real exchange-rate channel and makes real exchange-rate depreciation cancel the effect of domestic inflation on CPI inflation. This is apparent in row 6 in figure 3.3. An initial real appreciation is followed by further appreciation, and only when domestic inflation has fallen below zero does the real exchange rate start to depreciate. This requires a rather sophisticated management of the real interest rate. Recall that for a zero foreign real interest rate and foreign exchange risk premium, the real interest rate is the expected real rate of depreciation. Consequently, the real interest rate (row 5) must be proportional to the negative of domestic inflation lead by one period (column 2). Thus, the real interest rate must be negative (relative to its constant mean) while domestic inflation is positive the next period, and vice versa. This requires the reaction function for the nominal interest rate discussed above, (3.3). In particular, the initial response to the inflation shock is to reduce the nominal interest rate.

Similarly, for a demand shock, monetary policy must ensure that the real interest rate is proportional to the negative of expected domestic inflation lead by one period. This requires a negative nominal interest rate (relative to its constant mean) for the first five periods.

It is apparent from figure 3.3 and table 3, row 3, that strict CPI-inflation targeting, although successful in stabilizing CPI inflation, ends up causing large variability in domestic inflation, the output gap, and, particularly, the real exchange rate and the real interest rate.

3.4.4 Flexible CPI-inflation targeting

Strict CPI-inflation targeting causes considerable output-gap variability. Under flexible CPI-inflation targeting, with some weight on output-gap stabilization, that output-gap variability must be reduced. As a result, CPI inflation can no longer be completely insulated from shocks. For a shock to domestic inflation (figure 3.4, column 1), CPI inflation gradually returns to its target in about seven periods. The nominal interest rate is increased, the real interest rate is initially almost unchanged and then increases. The real exchange rate appreciates somewhat and then gradually depreciates towards its steady state value. The output gap contracts somewhat, and domestic inflation falls gradually towards the target.

For a demand shock, output returns to the steady state level in about four periods. Except in the initial period, there is a more contractionary monetary policy response, with the nominal interest rate becoming positive after period 1. Compared to strict CPI-inflation targeting, there is much less fluctuation, and hardly any cycles, in nominal and real interest rates, the real exchange rate, and output.

It remains to be understood why the output gap coefficient goes from zero to negative, from strict to flexible CPI-inflation targeting. Look at column 2 in figure 3.4, that is, the response to a demand shock. In order to stabilize CPI inflation, it is necessary to generate an expected real depreciation between periods 0 and 1, in order to counter the positive effect on CPI inflation in period 1 from the domestic inflation in period 1 caused by the demand shock. Therefore, the real and hence the nominal interest rate must be reduced in period 0. The increase in expected domestic inflation is moderate, and much smaller than the impulse to the output gap. A negative coefficient on the output gap is an efficient way of achieving the desired fall in the nominal interest rate, although this appears somewhat counterintuitive.

As in the other targeting cases, the response to a negative productivity shock, column 3, is remarkably similar to that of a demand shock, except that the response to the real exchange rate is much more persistent than for a demand shock, for the same reasons as noted above for strict domestic-inflation targeting.

With regard to variability (table 3, row 4), we see that variability is higher for CPI inflation, but lower for domestic inflation, the output gap, the real exchange rate, the instrument, and the real interest rate. Flexible CPI-inflation targeting allows for a less activist monetary policy, which brings about lower variability in these variables.

3.4.5 Taylor rules

The Taylor rule responding to domestic inflation results in a smooth return of domestic inflation, the output gap and the real exchange rate for both inflation and demand shocks (figure 3.5, columns 1 and 2, rows 2 and 3). The response of CPI inflation is a bit jagged. CPI inflation and the real exchange rate react strongly to shocks to the foreign exchange risk premium and to foreign inflation (as is the case for domestic-inflation targeting); this is not surprising, since the Taylor rule does not directly respond to these variables.

In table 3, row 5, we see that the variability of domestic inflation and the output gap is moderate, whereas it is large for CPI inflation and the real exchange rate.

The Taylor rule responding to CPI inflation results in a smoother response of CPI inflation (figure 3.6, row 1). We see in table 3, row 6, that the variability of CPI inflation is smaller, the variability of the real interest rate is larger, and that the variability of the other variables are somewhat larger.[26]

3.5 Conclusions from the comparison of targeting cases

From this comparison of the different targeting cases and the two versions of the Taylor rule, it appears that the flexible domestic-inflation targeting successfully stabilizes both domestic inflation and the output gap, although the variability of CPI inflation and the real exchange rate is high. Strict domestic-inflation targeting naturally stabilizes domestic inflation more, but increases the variability of the output gap and the real exchange rate.

Strict CPI-inflation targeting highlights the consequences of vigorous use of the direct exchange rate channel to stabilize CPI inflation. This vigorous use results in very high variability of the real exchange rate, and high variability of the other variables. Concern about the stability of the other variables is obviously a good reason not to try to fulfill the CPI inflation target at a very short horizon.

In contrast, flexible CPI-inflation targeting ends up causing low to moderate variability in all variables. The low variability of the real exchange rate (relative to the other cases) demonstrates that CPI-inflation targeting may involve a considerable amount of real exchange rate stabilization, as long as the ambition to stabilize CPI inflation is checked by some concern for output-gap stabilization. With some implicit social loss function that values stability in several

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[26] I am grateful to Akila Weeranpana, who discovered and corrected a programming error of mine for the Taylor rule for CPI inflation. Due to this error, very high variability for all variables were incorrectly reported for that Taylor rule in previous versions of this paper.

variables, flexible CPI-inflation targeting in an open economy may be an attractive alternative.[27]

The Taylor rule for domestic inflation stabilizes the variables less than the inflation-targeting cases, with the exception of the nominal and real interest rate. This might indicate that the coefficients are relatively low. The Taylor rule is, of course, not efficient, in two senses: (1), as emphasized by Ball [3], the coefficients are not optimal among the class of reaction functions responding only to current inflation and the output gap, and (2), as emphasized by Svensson [52] and [53], it disregards information not captured by current inflation and the output gap. For an open economy, such information is likely to be quite important. In this model, this is indicated by the impulse responses to shocks to foreign inflation and the foreign exchange risk premium. On the other hand, the Taylor rule for either domestic inflation or CPI inflation does not create particularly high variability in any variable except the real exchange rate. Hence, it appears somewhat robust; perhaps surprisingly robust.

4 The role of MCIs

So called monetary conditions indices, MCIs, have been discussed in the literature and are also used in practice. An MCI is supposed to measure the impact of monetary policy in some sense. Standard MCIs are linear combinations of a short real interest rate and the negative real exchange rate, usually with weights between 1:2 and 1:3.[28]

The present model is set up such that there is a potential MCI, I_t^{MC}, with respect to the effect on aggregate demand, namely

$$I_t^{\mathrm{MC}} \equiv \rho_{t+1|t} - \frac{\beta_q}{\beta_\rho} q_{t+1|t}. \tag{4.1}$$

Then aggregate demand by (A.4) obeys

$$y_{t+1|t}^d = \beta_y y_t^d - \beta_\rho I_t^{\mathrm{MC}} + \beta_y^* y_{t+1|t}^*,$$

and one might be tempted to say that I_t^{MC} "summarizes the impact of monetary policy on (next-period) aggregate demand," $y_{t+1|t}^d$.

By (2.8), we can write this index

$$I_t^{\mathrm{MC}} = \sum_{\tau=1}^{\infty} r_{t+\tau|t} - \frac{\beta_q}{\beta_\rho} q_{t+1|t}.$$

---

[27] Note that, due to (2.5), the term $\mu_\pi^c \pi_t^{c2}$ in the period loss function is equal to $\mu_\pi^c \pi^2 + \mu_\pi^c \omega^2 (q_t - q_{t-1})^2 + 2\mu_\pi^c \pi_t (q_t - q_{t-1})$.

[28] Ericsson and Kerbeshian [17] present a bibliography on MCIs.

24

Hence, it combines accumulated future expected deviations (from its mean) of the short real interest rate (starting one period ahead) with the expected deviation (from its mean) of the next-period real exchange rate. Alternatively, in view of (2.10), the MCI can be expressed as

$$I_t^{\text{MC}} \approx T r_{t+1|t}^T - \frac{\beta_q}{\beta_\rho} q_{t+1|t},$$

where r_t^T is a real interest rate with a long maturity T. Thus, the MCI combines the expected next-period long real rate with the expected next-period real exchange rate. Obviously, this MCI is rather different from the usual MCI which is a combination of the current short real interest rate and the current real exchange rate. In particular, it is not directly observable and verifiable to external observers.

Also, as any MCI, this MCI is affected by other things than monetary policy. A more precise statement than "I_t^{MC} summarizes the impact of monetary policy on (next-period) aggregate demand" would be "*changes in I_t^{MC} caused by monetary policy* summarize the impact of monetary policy on (next-period) aggregate demand."

In any case, this index is, at most, only relevant as a measure of the monetary policy impact on aggregate demand, not on CPI inflation or domestic inflation. Monetary policy affects CPI inflation and domestic inflation via several channels and with different lags, which cannot be captured by a single index. This is the case in the present model, and by all likelihood even more in the real world.

A separate issue is whether it is feasible for a central bank to have an MCI as an operating target, meaning that the central bank adjusts its instrument so as to keep an MCI on a particular path that is consistent with minimizing its loss function. Within the context of this model, it is demonstrated in appendix F that it is possible to consider an arbitrary MCI, say of the form

$$I_t^{\text{MC}} = r_t - a q_t,$$

for some constant $a > 0$, as an operating target, or formally as a control variable, and that the outcome is the same as if the short nominal interest rate is the control variable. Although the outcome is the same, there is, within the present model, no advantage to, nor any reason for, using an MCI as an operating target.

5 Conclusions

I have presented a relatively simple model of a small open economy, with some microfoundations, and with stylized, reasonably realistic relative lags for the different channels for the transmission

of monetary policy: The direct exchange rate channel to the CPI has the shortest lag (for simplicity set to a zero lag), the aggregate demand channel's effect on the output gap has an intermediate lag (set to one period), and the aggregate demand and expectations channels on domestic inflation have the longest lag (set to two periods).

Within this model, I have examined the properties of strict vs. flexible inflation targeting, and domestic vs. CPI-inflation targeting, especially relating them to the properties of the Taylor rule. This examination shows that flexible inflation targeting, effectively compared to strict inflation targeting, induces less variability in variables other than inflation, by effectively targeting inflation at a longer horizon. Especially, strict CPI-inflation targeting involves using the direct exchange rate channel to stabilize CPI inflation at a short horizon, which induces considerable real exchange rate variability. In contrast, flexible CPI-inflation targeting, compared to both strict CPI-inflation targeting and flexible domestic-inflation targeting, results in considerable stabilization of the real exchange rate. In a situation with weight on stabilization of both inflation and real variables, CPI-inflation targeting appears as an attractive alternative.

The implicit reaction functions arising under domestic-inflation and CPI-inflation targeting differ from the Taylor rule. CPI-inflation targeting deviates conspicuously from the Taylor rule, due to its implicit concern about real exchange rate depreciation. Such concern makes the response to foreign disturbances and variables important, whereas the Taylor rule excludes any direct response to these. Already in a closed economy, the Taylor rule uses only part of the information available; in an open economy it uses an even a smaller part. On the other hand, the Taylor rule does not result in exceptionally large variability in any variable, except possibly the real exchange rate; consequently it appears rather robust.

The model used distinguishes between demand and supply shocks. There are two kinds of supply shocks: cost-push shocks and productivity shocks. The response to a positive demand shock and a negative productivity shock are very similar (except that the response of the real exchange rate is more persistent for the latter). This similarity may appear surprising, given the conventional wisdom that supply shocks cause a conflict between inflation and output stabilization. There are several reasons for the similarity. First, both shocks increase the output gap, and the output gap is the major determinant of domestic inflation. Second, the central bank under flexible inflation targeting, as specified in the loss function I have used, wants to stabilize the variability of the output *gap*, rather than of output itself. For a productivity shock, there is little conflict between stabilizing the output gap and stabilizing output. For a supply

shock, there is a considerable conflict between output-gap stabilization and output stabilization. Then there is little conflict between inflation stabilization and output-gap stabilization, but considerable conflict between inflation stabilization and output stabilization. Thus, since output-gap stabilization rather than output stabilization is one of the goals, there is little difference between positive demand and negative productivity shocks, except that the persistence of the shocks may be quite different. Instead, the conflict between inflation stabilization and output-gap stabilization arises for cost-push supply shocks, rather than for productivity supply shocks.[29]

There are some obvious limitations to the analysis that may indicate suitable directions for future work. *First*, as emphasized above, there is no calibration and/or estimation of the parameters in the current version; the only criterion applied is that they must not be a priori unreasonable. As a consequence, the numerical results are only indicative. *Second*, although some microfoundations are provided for the aggregate supply and demand functions, there is some arbitrariness in the assumptions of partial adjustment and the addition of disturbance terms. At the cost of introducing additional forward-looking variables, it is relatively straightforward to implement the ideas of polynomial costs of adjustment of Pesaran [43] and Tinsley [62]. Also, only sticky prices have been explicitly modelled; with sticky wages, as noted by, for instance, Andersen [1] and Nelson [39], the dynamics can be quite different. *Third*, the model is linear with a quadratic loss function. Nonnegative nominal interest rates is one source of nonlinearity, nonlinear Phillips curves is another. However, any nonlinearity would prevent the use of the convenient and powerful algorithm for the optimal linear regulator with forward-looking variables.[30] *Fourth*, the particular relative lag structure I have used has been imposed on the model; there are obvious alternatives that may be worth pursuing. For instance, the assumption of no lag in the pass-through of exchange rate depreciation to the CPI is certainly an extreme assumption; an alternative is to assume that the direct exchange rate channel has a one period lag, such that import prices are predetermined one period (see footnote 36). Another alternative, at the cost of increased complexity, is to have the lags 1, 2 and 3 for the direct exchange

---

[29] Clarida, Gali and Gertler [13] note that a conflict between inflation stabilization and output-gap stabilization only arises for cost-push shocks but not for demand shocks or productivity shocks. In their framework, the monetary policy response to demand and supply shocks is different: the former are cancelled, and the latter are perfectly accommodated. The reason for this difference from the present model is that they have no lag in the effect of monetary policy, no inertia in aggregate demand, and a random walk in the natural output level. Without a lag in the effect of monetary policy, monetary policy can stabilize both inflation and the output-gap by completely cancelling any demand shock. A permanent shock to the natural output level leads, via the permanent-income hypothesis, to an equal permanent change in aggregate demand, with no effect on the output gap and inflation, and no need for a monetary policy response.

[30] See Fair and Howrey [18] for simulations on optimal monetary policy that take nonnegativity of nominal interest rates into account by adding non-quadratic punishment terms in the loss function.

rate channel to the CPI, the aggregate demand channel to output, and the aggregate demand and expectations channels to domestic inflation, respectively. A third alternative is to impose some partial adjustment of import prices. The theoretically most satisfactory alternative would be to have a separate aggregate supply relation for imported goods, realizing that importers face a pricing decision that is, in principle, similar to that of domestic producers. *Fifth*, the disturbances and the state variables are assumed to be observable to both the central bank and the private sector. In the real world, disturbances and the state variables are not directly observable, and the private sector and the central bank have to solve complicated signal-extraction problems. Some of the implications for inflation targeting of imperfectly observed states and disturbances are examined in Svensson and Woodford [57]. *Sixth*, there is no uncertainty in the model about the central bank's loss function and the inflation target is perfectly credible. Some new results on the consequences of imperfect credibility and less than full transparency of monetary policy are provided by Faust and Svensson [19]. *Seventh*, although it would be very desirable to test the model's predictions empirically, the short periods of inflation targeting in the relevant countries probably imply that several additional years of data are necessary for any serious empirical testing.

Finally, inflation targeting has been modelled as the minimization of a loss function over inflation deviations from the target and output deviations from the natural rate. There is considerable agreement in the literature that inflation targeting involves such a loss function. Furthermore, the explicit inflation target and the high degree of transparency and accountability ensure, better than for any other monetary policy regime, that monetary policy is systematic, rational and goal-directed, and therefore can be well described as optimizing. Indeed, the explicit inflation target and the high degree of transparency can be interpreted as a commitment mechanism, through which the central bank commits itself to minimizing a loss function, since deviations from optimizing behavior can more easily be spotted by central-bank watchers.[31]

The operating procedure of an inflation-targeting central bank, inflation-forecast targeting, discussed for instance in Freedman [22], Haldane [26], Mayes and Riches [33] and Svensson [52], serves to ensure such optimizing behavior. The result of such behavior can be approximated by

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[31] When representing inflation targeting as the minimization of a quadratic loss function over inflation and the output gap, I have sometimes encountered the objection that then inflation targeting is "no different from other monetary policy." I believe the appropriate response to that objection is that "other" monetary policy need not be well-represented by the minimization of a given loss function, precisely because it does not normally include any mechanisms that prevent policy from being unsystematic and shifting, for instance due to shifting or inconsistent preferences of the central bank. For instance, who knows what different loss functions different FOMC members and chairmen may have, and how those objectives are aggregated at FOMC meetings? The discussion about the so called opportunistic approach to disinflation, cf. Orphanides and Wilcox [40] and Rudebusch [47], is arguably evidence of the uncertainty about FOMC objectives.

an optimal reaction function of the form (2.25). However, this does not mean that the bank actually follows a prescribed explicit instrument rule that tells the bank how to respond to current information about the economy. Instead, the bank's staff produces alternative forecasts of inflation and other relevant variables, conditional upon different instrument paths and the current information about the economy. The bank's governor, monetary policy committee, or governing board then selects the particular instrument path and corresponding forecasts of the relevant goal variables that is considered to best fulfill the bank's objectives. If this selection is rational, the result is as if the bank is minimizing a loss function subject to its model and information about the economy. Similarly, the selection can be described as the solution of the first-order conditions of the optimization problem. These first-order conditions are a system of equations for the conditional forecasts of the goal variables, and can be seen as a generalized "intermediate-targeting rule" (cf. Rudebusch and Svensson [48]).

Thus, the operating procedure under inflation targeting can be seen as the implementation of an intermediate-targeting rule involving conditional forecasts of the goal variables. The analytical details of this implementation are discussed in a simple backward-looking model in Svensson [52] and [53] and (in greater detail, with an empirical model of the U.S. economy, including simple approximations to the optimal targeting rule and comparisons with various explicit instrument rules) in Rudebusch and Svensson [48]. The construction of conditional forecasts is relatively straightforward in a backward-looking model, but it presents some new problems in a forward-looking model, for instance, the present model. Furthermore, monetary policy cannot, so far, rely on models alone; there must always be room for judgemental adjustments and the use of extraneous information. The combination of extraneous information with a formal model, and consistent judgemental adjustments to the model, presents another very relevant problem for practical monetary policy. These and related problems are further discussed in Svensson [56].

A The aggregate demand equation

Consider a representative domestic consumer with an additively separable CES utility function of aggregate real consumption with intertemporal elasticity of substitution σ.[32] Intertemporal optimization will imply the first-order condition

$$c_t = c_{t+1|t} - \sigma(i_t - \pi^c_{t+1|t}), \tag{A.1}$$

where c_t denotes (log) aggregate real domestic consumption (possibly a deviation from a constant trend), and $i_t - \pi^c_{t+1|t}$ is the real CPI interest rate's deviation from a long-run mean real interest

---

[32] See appendix C for details on the utility function.

rate (which equals the rate of time preference when there is no trend in consumption).[33]

Let aggregate consumption be a CES function of consumption of domestic goods and foreign goods, with an elasticity of substitution θ. Then (log) domestic demand for domestically produced goods, c_t^h, is given by

$$
\begin{aligned}
c_t^h &= c_t - \theta(p_t - p_t^c) \\
&= c_t + \theta\omega q_t.
\end{aligned}
\tag{A.2}
$$

Substitution of (A.2) and (2.5) into the first-order condition (A.1) results in

$$
\begin{aligned}
c_t^h &= c_{t+1|t}^h - \theta\omega(q_{t+1|t} - q_t) - \sigma(i_t - \pi_{t+1|t}) + \sigma\omega(q_{t+1|t} - q_t) \\
&= c_{t+1|t}^h - \sigma(i_t - \pi_{t+1|t}) + (\sigma - \theta)\omega(q_{t+1|t} - q_t).
\end{aligned}
$$

Let us assume that there is a steady state for c_t^h.[34] Then $\lim_{\tau\to\infty} c_{t+\tau|t}^h = 0$. Let us also assume that the infinite sums below converge. It follows that

$$
\begin{aligned}
c_t^h &= -\sigma\sum_{\tau=0}^{\infty}(i_{t+\tau|t} - \pi_{t+\tau+1|t}) + (\sigma - \theta)\omega\sum_{\tau=0}^{\infty}(q_{t+\tau+1|t} - q_{t+\tau|t}) \\
&= -\sigma\sum_{\tau=0}^{\infty}(i_{t+\tau|t} - \pi_{t+\tau+1|t}) + (\sigma - \theta)\omega(\lim_{\tau\to\infty} q_{t+\tau|t} - q_t).
\end{aligned}
$$

Assume $\lim_{\tau\to\infty} q_{t+\tau|t} = 0$, then

$$
\begin{aligned}
c_t^h &= -\sigma\sum_{\tau=0}^{\infty}(i_{t+\tau|t} - \pi_{t+\tau+1|t}) - (\sigma - \theta)\omega q_t \\
&= -\sigma\rho_t - (\sigma - \theta)\omega q_t,
\end{aligned}
$$

where

$$
\rho_t \equiv \sum_{\tau=0}^{\infty}(i_{t+\tau|t} - \pi_{t+\tau+1|t}).
$$

Let the (log) foreign demand for home goods, c_t^{*h}, be

$$
\begin{aligned}
c_t^{*h} &= c_t^* + \theta^*\omega^* q_t \\
&= \bar{\beta}_y^* y_t^* + \theta^*\omega^* q_t,
\end{aligned}
$$

where c_t^* is (log) foreign real consumption, θ^* and ω^* are the foreign atemporal elasticity of substitution and the share of domestically produced goods in foreign consumption, respectively, $\bar{\beta}_y^*$ is the income elasticity of foreign real consumption, and y_t^* is (log) foreign output. Let y_t^d denote the (total log) aggregate demand for domestically produced goods, and let κ be the share of domestic aggregate demand in the total aggregate demand. Then we have

$$
\begin{aligned}
y_t^d &= \kappa c_t^h + (1 - \kappa)c_t^{*h} \\
&= -\kappa\sigma\rho_t - \kappa(\sigma - \theta)\omega q_t + (1 - \kappa)\bar{\beta}_y^* y_t^* + (1 - \kappa)\theta^*\omega^* q_t \\
&= -\tilde{\beta}_\rho\rho_t + \tilde{\beta}_y^* y_t^{*\cdot} + \tilde{\beta}_q q_t,
\end{aligned}
$$

---

[33] See Kerr and King [28], McCallum and Nelson [36], Woodford [66] and Rotemberg and Woodford [46] for similar derivations of aggregate demand for one-good closed economies.

[34] This assumption presumes that net foreign assets are stationary. Thus I avoid the well-known problem that a small open economy with infinitely-lived consumers that can borrow at an exogenous world interest rate normally has nonstationary net foreign assets (and consumption). Such nonstationarity generally violates the assumption of an exogenous world interest rate, since the economy may become arbitrary large. Overlapping generations, as in the Blanchard-Yaari model, [8], restores stationarity.

where

$$\begin{aligned}
\tilde{\beta}_\rho &\equiv \kappa\sigma \\
\tilde{\beta}_y^* &\equiv (1-\kappa)\bar{\beta}_y^* \\
\tilde{\beta}_q &\equiv (1-\kappa)\theta^*\omega^* - \kappa(\sigma-\theta)\omega.
\end{aligned}$$

Now assume that real consumption and aggregate demand are predetermined one period. If the derivation above is repeated under this assumption, we get

$$y_{t+1|t}^d = -\tilde{\beta}_\rho \rho_{t+1|t} + \tilde{\beta}_y^* y_{t+1|1}^* + \tilde{\beta}_q q_{t+1|t}. \tag{A.3}$$

Next, let us assume that, due to costs of adjustment, habit formation, or some other mechanism, aggregate demand and output adjust only partially, such that aggregate demand, y_{t+1}^d, is a convex combination of lagged aggregate demand, y_t^d, and the right side of (A.3),

$$\begin{aligned}
y_{t+1}^d &= \beta_y y_t^d + (1-\beta_y)(-\tilde{\beta}_\rho \rho_{t+1|t} + \tilde{\beta}_y^* y_{t+1|1}^* + \tilde{\beta}_q q_{t+1|t}) + \eta_{t+1}^d \\
&= \beta_y y_t^d - \beta_\rho \rho_{t+1|t} + \beta_y^* y_{t+1|t}^* + \beta_q q_{t+1|t} + \eta_{t+1}^d, \tag{A.4}
\end{aligned}$$

where a serially uncorrelated zero-mean demand shock, η_{t+1}^d, has been added, and where

$$\begin{aligned}
\beta_\rho &\equiv (1-\beta_y)\tilde{\beta}_\rho \\
\beta_y^* &\equiv (1-\beta_y)\tilde{\beta}_y^* \\
\beta_q &\equiv (1-\beta_y)\tilde{\beta}_q.
\end{aligned}$$

In terms of the output gap, (2.2), we get

$$\begin{aligned}
y_{t+1} &\equiv y_{t+1}^d - y_{t+1}^n \\
&= \beta_y y_t^d - \beta_\rho \rho_{t+1|t} + \beta_y^* y_{t+1|t}^* + \beta_q q_{t+1|t} + \eta_{t+1}^d - y_{t+1}^n \\
&= \beta_y y_t - \beta_\rho \rho_{t+1|t} + \beta_y^* y_{t+1|t}^* + \beta_q q_{t+1|t} - (\gamma_y^n - \beta_y)y_t^n + \eta_{t+1}^d - \eta_{t+1}^n, \tag{A.5}
\end{aligned}$$

which is (2.7).[35]

_____

[35] Suppose the inertia is in terms of the output gap instead of aggregate demand. First, express (A.3) in terms of the output gap,

$$y_{t+1|t} \equiv y_{t+1|t}^d - y_{t+1|t}^n = -\tilde{\beta}_r \rho_{t+1|t} + \tilde{\beta}_y^* y_{t+1|1}^* + \tilde{\beta}_q q_{t+1|t} - y_{t+1|t}^n. \tag{A.6}$$

Then, assume that the intertia is in terms of the output gap, such that $y_{t+1|t}$ is a convex combination of the lagged output gap, y_t, and the right side of (A.6),

$$\begin{aligned}
y_{t+1|t} &= \beta_y y_t + (1-\beta_y)(-\tilde{\beta}_r \rho_{t+1|t} + \tilde{\beta}_y^* y_{t+1|1}^* + \tilde{\beta}_q q_{t+1|t} - y_{t+1|t}^n) \\
&= \beta_y y_t - \beta_r \rho_{t+1|t} + \beta_y^* y_{t+1|t}^* + \beta_q q_{t+1|t} - \beta_n y_{t+1|t}^n,
\end{aligned}$$

where $\beta_n \equiv 1 - \beta_y$.

Finally, add a temporary demand disturbance η_{t+1}^d such that

$$y_{t+1} = \beta_y y_t - \beta_r \rho_{t+1|t} + \beta_y^* y_{t+1|t}^* + \beta_q q_{t+1|t} - \beta_n \gamma_y^n y_t^n + \eta_{t+1}^d - \beta_n \eta_{t+1}^n. \tag{A.7}$$

B The infinite-horizon market discount factor

Let $d_{t,T}$ denote the (log of the deviation from the mean of) the (real market) T-horizon discount factor (for domestically produced goods) between period t and period $T > t$, the present value (measured in domestic goods) in period t of one unit of domestic goods in period T. Under the expectations hypothesis the discount factor will fulfill

$$d_{t,T} = -\sum_{\tau=0}^{T-1} r_{t+\tau|t}.$$

Let the infinite-horizon discount factor d_t be given by

$$d_t \equiv \lim_{T\to\infty} d_{t,T} = -\sum_{\tau=0}^{\infty} r_{t+\tau|t}, \tag{B.1}$$

assuming that the limit exists. Then, from (2.8) it follows that

$$\rho_t \equiv -d_t, \tag{B.2}$$

the sum of accumulated current and future expected real interest rates, ρ_t, is simply the negative of the infinite-horizon discount factor, d_t.

Given this, we can further clarify the relation to the real exchange rate. Let d_t^* denote the foreign infinite-horizon discount factor, which, under the expectations hypothesis, will fulfill

$$d_t^* = -\sum_{\tau=0}^{\infty} r_{t+\tau|t}^*. \tag{B.3}$$

It follows from (2.16) and (B.1)–(B.3) that

$$q_t = d_t - d_t^* + \tilde{\varphi}_t. \tag{B.4}$$

Due to the assumptions about the foreign interest rate and foreign inflation, by (2.17) d_t^* fulfills

$$d_t^* = -i_t^* - \frac{(f_\pi^* - 1)\gamma_\pi^*}{1 - \gamma_\pi^*}\pi_t^* - \frac{f_y^*\gamma_y^*}{1 - \gamma_y^*}y_t^*, \tag{B.5}$$

and $\tilde{\varphi}_t$ is an (accumulated) infinite-horizon foreign-exchange risk premium given by

$$\tilde{\varphi}_t \equiv \sum_{\tau=0}^{\infty} \varphi_{t+\tau|t} = \frac{1}{1 - \gamma_\varphi}\varphi_t. \tag{B.6}$$

Clearly, the model and analysis could be expressed in terms of the infinite-horizon discount factor, d_t, instead of the sum of current and future expected real interest rates, ρ_t.

C The aggregate supply equation

The derivation of the aggregate supply function is an open-economy variant of that of Woodford [66] and Rotemberg-Woodford [46]. Let the economy have a continuum (of measure one) of consumers/producers indexed by j, $0 \leq j \leq 1$, where each consumer/producer has the same intertemporal utility function

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^{\tau} U(C_{t+\tau}^h, C_{t+\tau}^f),$$

where C_t^h and C_t^f denote consumption of domestic and foreign goods respectively (in equilibrium all consumers/producers will have the same consumption) and δ is the discount factor ($0 < \delta < 1$). Furthermore, the utility function fulfills

$$U(C_t^h, C_t^f) = \frac{C(C_t^h, C_t^f)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}},$$

where $\sigma > 0$ is the intertemporal elasticity of substitution and $C(C_t^h, C_t^f)$ is a CES subutility function (with an elasticity of substitution $\theta > 0$) that defines aggregate real consumption (see appendix A). Suppose consumption decisions are made one period in advance, that is, consumption is predetermined one period. The first-order condition with respect to C_{t+1}^h, predetermined in period t, is

$$\mathrm{E}_t U_h(C_{t+1}^h, C_{t+1}^f) = \mathrm{E}_t[\Lambda_{t+1} P_{t+1}] \equiv \mathrm{E}_t \tilde{\Lambda}_{t+1}.$$

Here, Λ_t is the marginal utility of nominal income in period t, and $\tilde{\Lambda}_t$ is the marginal utility of domestic goods.

Suppose there is a continuum of differentiated domestic goods, such that domestic good of type j ($0 \le j \le 1$) is produced by consumer/producer j with a composite input, with price W_t. The cost of producing the quantity Y_t^j is then $W_t V(Y_t^j)/A_t$, where the input requirement function $V(Y_t^j)/A_t$ is the same for all goods j and A_t is an exogenous economy-wide productivity parameter. Suppose there exists a Dixit-Stiglitz aggregate of domestic goods with elasticity of substitution $\vartheta > 1$, such that the demand for domestic goods can be written

$$Y_t^j = (C_t^h + C_t^{*h})\left(\frac{P_t^j}{P_t}\right)^{-\vartheta} \equiv Y_t^d \left(\frac{P_t^j}{P_t}\right)^{-\vartheta},$$

where C_t^h and C_t^{*h} are aggregate domestic and foreign consumption of domestic goods, $Y_t^d \equiv C_t^h + C_t^*$ is the total aggregate demand for domestic goods, P_t^j is the nominal price for domestic good j, and P_t is the Dixit-Stiglitz price index for domestic goods.

Suppose, as in Calvo [11], that in the beginning of any period a consumer/producer is free to set a new price with probability $1 - \alpha$, but must keep the same price as the previous period with probability α. Suppose consumer/producer j is free to set a new price in period t. It follows that the decision problem can be written

$$\max_{\tilde{P}_t} \mathrm{E}_t \left\{ \sum_{\tau=0}^{\infty} \alpha^{\tau} \delta^{\tau} \tilde{\Lambda}_{t+\tau} \left[\frac{\tilde{P}_t}{P_{t+\tau}} Y_{t+\tau}^d \left(\frac{\tilde{P}_t}{P_{t+\tau}}\right)^{-\vartheta} - \frac{W_{t+\tau}}{P_{t+\tau}} \frac{V\left(Y_{t+\tau}^d \left(\frac{\tilde{P}_t}{P_{t+\tau}}\right)^{-\vartheta}\right)}{A_t} \right] \right\},$$

where $\tilde{P}_t$ denotes the new price chosen in period t. The first-order condition is

$$\mathrm{E}_t \left\{ \sum_{\tau=0}^{\infty} \alpha^{\tau} \delta^{\tau} \tilde{\Lambda}_{t+\tau} \left[\frac{\tilde{P}_t}{P_{t+\tau}} - \zeta \frac{W_{t+\tau}}{P_{t+\tau}} \frac{V'\left(Y_{t+\tau}^d \left(\frac{\tilde{P}_t}{P_{t+\tau}}\right)^{-\vartheta}\right)}{A_t} \right]_t Y_{t+\tau}^d \left(\frac{\tilde{P}_t}{P_{t+\tau}}\right)^{-\vartheta} \right\} = 0,$$

where $\zeta \equiv \frac{\vartheta}{\vartheta-1} > 1$. The first-order condition can be written

$$
E_t \left\{ \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \tilde{\Lambda}_{t+\tau} \left[\frac{X_t}{\prod_{s=1}^{\tau} \Pi_{t+s}} - \zeta \frac{W_{t+\tau}}{P_{t+\tau}} \frac{V'\left(Y_{t+\tau}^d \left(\frac{X_t}{\prod_{s=1}^{\tau} \Pi_{t+s}} \right)^{-\vartheta} \right)}{A_t} \right] Y_{t+\tau}^d \left(\frac{X_t}{\prod_{s=1}^{\tau} \Pi_{t+s}} \right)^{-\vartheta} \right\} = 0,
$$

where

$$
X_t \equiv \frac{\tilde{P}_t}{P_t}
$$

$$
\Pi_t \equiv \frac{P_t}{P_{t-1}}.
$$

In equilibrium each consumer/producer that chooses a new price in period t will choose the same new price, $\tilde{P}_t$, and the same level of output. Then the (aggregate) price of domestic goods will obey

$$
P_t = \left[\alpha P_{t-1}^{1-\vartheta} + (1-\alpha)\tilde{P}_t^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}.
$$

We note that

$$
\Pi_t = \left[\alpha + (1-\alpha)\Pi_t^{1-\vartheta} X_t^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}
$$

$$
\left[1 - (1-\alpha)X_t^{1-\vartheta} \right] \Pi_t^{1-\vartheta} = \alpha
$$

$$
\Pi_t = \alpha^{\frac{1}{1-\vartheta}} \left[1 - (1-\alpha)X_t^{1-\vartheta} \right]^{\frac{1}{\vartheta-1}}.
$$

Next, we do a loglinear approximation. We allow bounded fluctuations in $(Y_t^d, \Pi_t, X_t, \tilde{\Lambda}_t, A_t, \frac{W_t}{P_t})$ around a steady state $(C^h, 1, 1, \tilde{\Lambda}, 1, 1)$, and we let $x \equiv \text{d}\ln X$, etc. Then we have

$$
\begin{aligned}
v' &= \tilde{\omega} y^j \\
w_t &= (1-\gamma)p_t + \gamma p_t^f \qquad\qquad\qquad\qquad\qquad\qquad (\text{C.1})\\
y_{t+\tau}^d &= \kappa c_{t+\tau}^h + (1-\kappa)c_{t+\tau}^{*h} \\
\pi_t &= \frac{1}{\vartheta-1} \frac{-(1-\alpha)}{1-(1-\alpha)}(1-\vartheta)x_t = \frac{1-\alpha}{\alpha}x_t, \qquad\qquad (\text{C.2})
\end{aligned}
$$

where $\tilde{\omega} > 0$ is the elasticity of V' with respect to Y_t^j, $0 \le \gamma \le 1$ is the share of foreign goods in the composite input, p_t^f is the (log) domestic-currency price of foreign goods, and $0 < \kappa < 1$ is the share of domestic demand in the aggregate demand for domestic goods. Furthermore, we let the log of the productivity parameter, a_t, fulfill

$$
a_t \equiv \tilde{\omega} y_t^n,
$$

in which case we can identify y_t^n as the log of the natural output level, as we shall see below.

A log-linearization of the first-order condition results in

$$
\begin{aligned}
0 &= E_t \left\{ \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[x_t - \sum_{s=1}^{\tau} \pi_{t+s} - w_{t+\tau} + p_{t+\tau} - \tilde{\omega}\left(y_{t+\tau}^d - \vartheta(x_t - \sum_{s=1}^{\tau} \pi_{t+s}) \right) + \tilde{\omega} y_{t+\tau}^n \right] \right\} \\
&\equiv E_t \left\{ \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[(1+\tilde{\omega}\vartheta)(x_t - \sum_{s=1}^{\tau} \pi_{t+s}) - z_{t+\tau} \right] \right\}, \qquad\qquad (\text{C.3})
\end{aligned}
$$

where z_t fulfills

$$z_t \equiv \tilde{\omega}y_t + w_t - p_t = \tilde{\omega}y_t + \gamma q_t,$$

where we recall (2.2), (C.1) and (2.6).[36]

We note that we can change the summation order in (C.3) as follows:

$$\sum_{\tau=0}^{\infty} \alpha^{\tau}\delta^{\tau} \sum_{s=1}^{\tau} \pi_{t+s} = \sum_{s=1}^{\infty} \pi_{t+s} \sum_{\tau=s}^{\infty} \alpha^{\tau}\delta^{\tau} = \sum_{s=1}^{\infty} \pi_{t+s} \frac{\alpha^s \delta^s}{1-\alpha\delta} = \frac{1}{1-\alpha\delta} \sum_{\tau=1}^{\infty} \alpha^{\tau}\delta^{\tau}\pi_{t+\tau};$$

and write

$$\mathrm{E}_t\left\{ \frac{1+\tilde{\omega}\vartheta}{1-\alpha\delta}x_t - \frac{1+\tilde{\omega}\vartheta}{1-\alpha\delta} \sum_{\tau=1}^{\infty} \alpha^{\tau}\delta^{\tau}\pi_{t+\tau} - \sum_{\tau=0}^{\infty} \alpha^{\tau}\delta^{\tau}z_{t+\tau} \right\} = 0$$

$$x_t = \mathrm{E}_t\left\{ \sum_{\tau=1}^{\infty} \alpha^{\tau}\delta^{\tau}\pi_{t+\tau} + \frac{1-\alpha\delta}{1+\tilde{\omega}\vartheta} \sum_{\tau=0}^{\infty} \alpha^{\tau}\delta^{\tau}z_{t+\tau} \right\}$$

$$= \mathrm{E}_t\left\{ \alpha\delta\pi_{t+1} + \frac{1-\alpha\delta}{1+\tilde{\omega}\vartheta}z_t \right\} + \alpha\delta\mathrm{E}_t x_{t+1}.$$

Using (C.2) we get

$$\frac{\alpha}{1-\alpha}\pi_t = \mathrm{E}_t\left\{ \alpha\delta\pi_{t+1} + \frac{1-\alpha\delta}{1+\tilde{\omega}\vartheta}z_t \right\} + \alpha\delta\frac{\alpha}{1-\alpha}\mathrm{E}_t\pi_{t+1},$$

and finally

$$\pi_t = \delta\mathrm{E}_t\pi_{t+1} + \xi z_t,$$

where

$$\xi \equiv \frac{(1-\alpha)(1-\alpha\delta)}{\alpha(1+\tilde{\omega}\vartheta)}.$$

So far, this has followed [66] and [46], except that the open economy aspects have been added. Now, assume that inertia and/or adjustment costs results in a simple partial adjustment,

$$\pi_t = \alpha_{\pi}\pi_{t-1} + (1-\alpha_{\pi})(\pi_{t+1|t} + \xi z_t),$$

where $0 \leq \alpha_{\pi} < 1$. Furthermore, let π_t be predetermined two periods, and approximate δ by unity, so as to ensure the natural-rate hypothesis,

$$\pi_{t+2|t} = \alpha_{\pi}\pi_{t+1|t} + (1-\alpha_{\pi})(\delta\pi_{t+3|t} + \xi z_{t+2|t})$$

$$= \alpha_{\pi}\pi_{t+1|t} + (1-\alpha_{\pi})(\pi_{t+3|t} + \xi\tilde{\omega}y_{t+2|t} + \xi\gamma q_{t+2|t}).$$

Finally, write the aggregate supply curve as

$$\pi_{t+2} = \alpha_{\pi}\pi_{t+1} + (1-\alpha_{\pi})\pi_{t+3|t} + \alpha_y y_{t+2|t} + \alpha_y\beta_y(y_{t+1} - y_{t+1|t}) + \alpha_q q_{t+2|t} + \varepsilon_{t+2},$$

---

[36] This is when p_t^f, the domestic-currency price of foreign goods, is not predetermined. When p_t^f is predetermined one period, we have

$$p_t^f = p_{t|t-1}^* + s_{t|t-1} = q_{t|t-1} + p_{t|t-1}$$

$$p_t^f - p_t = q_{t|t-1} - (p_t - p_{t|t-1}) = q_{t|t-1} - (\pi_t - \pi_{t|t-1}).$$

where

$$\alpha_y \equiv (1 - \alpha_\pi)\xi\tilde{\omega}$$
$$\alpha_q \equiv (1 - \alpha_\pi)\xi\gamma.$$

Here, an error term ε_{t+2}, a cost-push shock, has been added. Also, to the term $\alpha_\pi \pi_{t+1|t} + \alpha_y y_{t+2|t}$ I have added $\alpha_\pi(\pi_{t+1} - \pi_{t+1|t}) + \alpha_y\beta_y(y_{t+1} - y_{t+1|t})$, in order to allow an effect on $\pi_{t+2|t+1}$ and π_{t+2} of the shocks ε_{t+1}, η_{t+1}^d and η_{t+1}^n (cf. the aggregate demand function (2.7)).

D State-space form

First, write

$$\pi_{t+1} = \pi_{t+1|t} + \varepsilon_{t+1} \tag{D.1}$$
$$\pi_{t+2|t+1} = \pi_{t+2|t} + \alpha_\pi \varepsilon_{t+1} + \alpha_y \beta_y (\eta_{t+1}^d - \eta_{t+1}^n) \tag{D.2}$$
$$\rho_{t+1|t} = \rho_t - i_t + \pi_{t+1|t}. \tag{D.3}$$

Next, in order to write the model on state-space form, take the expectation in period t of (2.1), and leave only $\pi_{t+3|t}$ on the left side,

$$(1 - \alpha_\pi)\pi_{t+3|t} = \pi_{t+2|t} - \alpha_\pi \pi_{t+1|t} - \alpha_y y_{t+2|t} - \alpha_q q_{t+2|t}.$$

Lead (2.7) one period, take the expectation in period t, and substitute for $y_{t+2|t}$. Similarly, lead (D.3) one period, take the expectation in period t, and substitute for $\rho_{t+2|t}$. Finally, lead (2.11) one period, take the expectation in period t, and substitute for $q_{t+2|t} - q_{t+1|t}$. This gives

$$
\begin{aligned}
(1 - \alpha_\pi)\pi_{t+3|t} = {} & \pi_{t+2|t} - \alpha_\pi \pi_{t+1|t} \\
& - \alpha_y[\beta_y y_{t+1|t} - \beta_\rho \rho_{t+1|t} + \beta_y^* y_{t+2|t}^* + \beta_q(q_{t+1|t} - i_{t+1|t}^* + \pi_{t+2|t}^* - \varphi_{t+1|t})] \\
& - \alpha_y(\beta_\rho + \beta_q)(i_{t+1|t} - \pi_{t+2|t}) - \alpha_y[-(\gamma_y^n - \beta_y)]y_{t+1|t}^n \\
& - \alpha_q(q_{t+1|t} + i_{t+1|t} - \pi_{t+2|t} - i_{t+1|t}^* + \pi_{t+2|t}^* - \varphi_{t+1|t}) \\
= {} & -\alpha_\pi \pi_{t+1|t} + [1 + \alpha_y(\beta_\rho + \beta_q) + \alpha_q]\pi_{t+2|t} - \alpha_y\beta_y y_{t+1|t} + \alpha_y\beta_\rho \rho_{t+1|t} \\
& - \alpha_y\beta_y^* y_{t+2|t}^* - (\alpha_y\beta_q + \alpha_q)q_{t+1|t} + (\alpha_y\beta_q + \alpha_q)(i_{t+1|t}^* - \pi_{t+2|t}^* + \varphi_{t+1|t}) \\
& + \alpha_y(\gamma_y^n - \beta_y)y_{t+1|t}^n - [\alpha_y(\beta_\rho + \beta_q) + \alpha_q]i_{t+1|t}. \tag{D.4}
\end{aligned}
$$

We can now collect equations (D.1), (2.7), (2.12)–(2.15) and (D.2) for the predetermined variables, and add the trivial equations for the lagged variable q_{t-1} and i_{t-1}. Similarly, we can collect equations (2.11), (D.3) and (D.4) for the forward-looking variables. This results in (2.22) with the matrices

$$
A = \begin{bmatrix}
e_{10} \\
\beta_y e_2 - \beta_\rho A_{n_1+2\cdot} + \beta_y^* \gamma_y^* e_4 + \beta_q A_{n_1+1\cdot} - (\gamma_y^n - \beta_y)e_7 \\
\gamma_\pi^* e_3 \\
\gamma_y^* e_4 \\
f_\pi^* \gamma_\pi^* e_3 + f_y^* \gamma_y^* e_4 \\
\gamma_\varphi e_6 \\
\gamma_y^n e_7 \\
e_{n_1+1} \\
e_0 \\
e_n \\
e_{n_1+1} - e_{10} + A_{3\cdot} - e_5 - e_6 \\
e_{10} + e_{n_1+2} \\
A_{n\cdot}
\end{bmatrix},
$$

36

where

$$A_{n\cdot} = \frac{1}{1-\alpha_\pi}\left\{ \begin{array}{l} -\alpha_\pi e_{10} + [1+\alpha_y(\beta_\rho+\beta_q)+\alpha_q]e_n - \alpha_y\beta_y A_{2\cdot} + \alpha_y\beta_\rho A_{n_1+2\cdot} - \alpha_y\beta_y^*\gamma_y^* A_{4\cdot} \\ -(\alpha_y\beta_q+\alpha_q)A_{n_1+1\cdot} + (\alpha_y\beta_q+\alpha_q)(A_{5\cdot}-\gamma_\pi^* A_{3\cdot}+A_{6\cdot}) + \alpha_y(\gamma_y^n-\beta_y)A_{7\cdot} \end{array} \right\},$$

$$B = \begin{bmatrix} 0 \\ \beta_\rho+\beta_q \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ -1 \\ \frac{-1}{1-\alpha_\pi}[\alpha_y(1+\beta_y)(\beta_\rho+\beta_q)+\alpha_q] \end{bmatrix}, \quad B^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{-1}{1-\alpha_\pi}[\alpha_y(\beta_\rho+\beta_q)+\alpha_q] \end{bmatrix},$$

where e_j, $j=0,...,n$, denotes a $1\times n$ row vector, for $j=0$ with all elements equal to zero, for $j\neq 0$ with element j equal to unity and all other elements equal to zero, and $A_{j\cdot}$ denotes row j of the matrix A.

The $n_3 \times n$ matrix C_Z and the $n_3 \times 1$ column vector C_i in (2.23) are given by

$$C_Z = \begin{bmatrix} e_1+\omega(e_{n_1+1}-e_8) \\ e_1 \\ e_2 \\ e_0 \\ -e_9 \end{bmatrix}, \quad C_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

E The linear regulator problem with forward-looking variables and expected future controls

Consider the decision problem to choose i_t in period t to minimize (2.21) (with $0<\delta<1$) under discretion, that is, subject to (2.22)–(2.24) and

$$i_{t+1} = f_{t+1}X_{t+1} \tag{E.1}$$
$$x_{t+1} = H_{t+1}X_{t+1}, \tag{E.2}$$

where f_{t+1} and H_{t+1} are determined by the decision problem in period $t+1$. First, combine (E.1) with (2.22) to eliminate $i_{t+1|t}$ in (2.22). This results in the new system

$$\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = \tilde{A}_t \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \tilde{B}_t i_t + \begin{bmatrix} v_{t+1} \\ 0 \end{bmatrix}, \tag{E.3}$$

where

$$\tilde{A}_t \equiv \left(I - B^1[f_{t+1}\ 0]\right)^{-1} A \tag{E.4}$$
$$\tilde{B}_t \equiv \left(I - B^1[f_{t+1}\ 0]\right)^{-1} B. \tag{E.5}$$

Then the algorithm presented in Oudiz and Sachs [41], further discussed in Backus and Driffill [2] and Currie and Levin [14], and for instance applied in Svensson [50], can be applied directly.[37] More precisely, suppose the optimal value of the problem in period $t + 1$ is given by $X'_{t+1}V_{t+1}X_{t+1} + w_{t+1}$, where V_{t+1} is a positive semidefinite matrix and w_{t+1} is a scalar. Then the optimal value of the problem in period t is associated with the positive semidefinite matrix V_t and the scalar w_t and fulfills

$$X'_t V_t X + w_t = \min_{i_t} \left\{ L_t + \delta \mathrm{E}_t[X'_{t+1}V_{t+1}X_t + w_{t+1}] \right\}.$$

Rewrite the period loss function (2.24) as

$$L_t = Z'_t Q Z_t + 2Z'_t U i_t + i'_t R i_t,$$

where we, for generality, formally allow the control variable i_t to be a column vector, and where

$$Q \equiv C'_Z K C_Z, \ U \equiv C'_Z K C_i, \ R \equiv C'_i K C_i.$$

Decompose $\tilde{A}_t$, $\tilde{B}_t$, Q and U according to $(X'_t, x'_t)'$ into

$$\tilde{A}_t = \begin{bmatrix} \tilde{A}_{t11} & \tilde{A}_{t12} \\ \tilde{A}_{t21} & \tilde{A}_{t22} \end{bmatrix}, \ \tilde{B}_t = \begin{bmatrix} \tilde{B}_{t1} \\ \tilde{B}_{t2} \end{bmatrix}, \ Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}, \ U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}.$$

Then, given $(f_{t+1}, H_{t+1}, V_{t+1})$, construct (f_t, H_t, V_t) according to (E.4), (E.5) and

$$\begin{aligned}
D_t &= (\tilde{A}_{t22} - H_{t+1}\tilde{A}_{t12})^{-1}(H_{t+1}\tilde{A}_{t11} - \tilde{A}_{t21}) \\
G_t &= (\tilde{A}_{t22} - H_{t+1}\tilde{A}_{t12})^{-1}(H_{t+1}\tilde{B}_{t1} - \tilde{B}_{t2}) \\
A^*_t &= \tilde{A}_{t11} + \tilde{A}_{t12}D_t \\
B^*_t &= \tilde{B}_{t1} + \tilde{A}_{t12}G_t \\
Q^*_t &= Q_{11} + Q_{12}D_t + D'_t Q_{21} + D'_t Q_{22}D_t \\
U^*_t &= Q_{12}G_t + D'_t Q_{22}G_t + U_1 + D'_t U_2 \\
R^*_t &= R + G'_t Q_{22}G_t + G'_t U_2 + U'_2 G_t \\
f_t &= -(R^*_t + \delta B^{*'}_t V_{t+1} B^*_t)^{-1}(U^{*'}_t + \delta B^{*'}_t V_{t+1} A^*_t) \\
H_t &= D_t + G_t f_t \\
V_t &= Q^*_t + U^*_t f_t + f'_t U^{*'}_t + f'_t R^*_t f_t + \delta (A^*_t + B^*_t f_t)' V_{t+1} (A^*_t + B^*_t f_t).
\end{aligned}$$

The solution to the decision problem is a fix point (f, H, V) of the mapping defined by the above algorithm. It is obtained as the limit of (f_t, H_t, V_t) when $t \to -\infty$.

The equation for w_t is $w_t = \delta[\mathrm{trace}(V_{t+1}\Sigma) + w_{t+1}]$, where Σ is the covariance matrix of v_t. The limit of w_t is $\frac{\delta}{1-\delta}\mathrm{trace}(V\Sigma)$. Consequently, the optimal value, J_t, of the problem is given by

$$J_t = X'_t V X_t + \frac{\delta}{1-\delta}\mathrm{trace}(V\Sigma). \tag{E.6}$$

When $\delta \to 1$, the sum in (2.21) and J_t in (E.6) become unbounded. However, the first component of J_t, $X'_t V X_t$, which corresponds to the deterministic optimization problem when all shocks are zero, remains bounded for $\delta = 1$ (because the deterministic terms in (2.21) approach zero quickly enough), and the decision problem is actually well-defined also for that case. For $\delta \to 1$, (2.21) is dominated by the infinite sum of terms that approach the unconditional mean of the period loss function, $\mathrm{E}[L_t]$. Then, the scaled loss function $(1-\delta)\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}$ approaches $\mathrm{E}[L_t]$. Furthermore, from (E.6) we see that $(1 - \delta)\mathrm{E}[J_t]$ approaches $\mathrm{trace}(V\Sigma)$. It follows that the optimization problem, thus interpreted, is well-defined for $\delta = 1$, with the loss function $\mathrm{E}[L_t]$, (2.20), and that the optimal value of the problem then fulfills $\mathrm{E}[L_t] = \mathrm{trace}(V\Sigma)$.

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[37] See Söderlind [49] for a detailed presentation.

F   An arbitrary MCI as a control variable

Suppose we somewhat arbitrarily define an MCI as

$$
\begin{aligned}
I_t^{\mathrm{MC}} &\equiv r_t - a q_t \\
&= i_t - \pi_{t+1|t} - a q_t,
\end{aligned}
$$

where $a > 0$ is a given constant, for instance $a = 0.4$. Can such an MCI be considered a control variable in the model used here? Recall that in the model, we have

$$
\begin{aligned}
\pi_{t+1|t} &\equiv e_{10} Z_t \\
q_t &\equiv e_{n_1+1} Z_t.
\end{aligned}
$$

Hence we can write

$$
\begin{aligned}
i_t &= I_t^{\mathrm{MC}} + \pi_{t+1|t} + a q_t \\
&\equiv I_t^{\mathrm{MC}} + (e_{10} + e_{n_1+1}) Z_t. \tag{F.1}
\end{aligned}
$$

Then we can rewrite the model (2.22) and (2.23) as

$$
\begin{aligned}
\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} &= A Z_t + B i_t + B^1 i_{t+1|t} + \begin{bmatrix} v_{t+1} \\ 0 \end{bmatrix} \\
&= A Z_t + B[I_t^{\mathrm{MC}} + (e_{10} + e_{n_1+1}) Z_t] + B^1[+(e_{10} + e_{n_1+1}) Z_{t+1|t}] + \begin{bmatrix} v_{t+1} \\ 0 \end{bmatrix} \\
&\equiv \bar{A} Z_t + \bar{B} I_t^{\mathrm{MC}} + \bar{B}^1 I_{t+1|t}^{\mathrm{MC}} + \begin{bmatrix} v_{t+1} \\ 0 \end{bmatrix} \tag{F.2} \\
Y_t &= C_Z Z_t + C_i[I_t^{\mathrm{MC}} + (e_{10} + e_{n_1+1}) Z_t] \\
&\equiv \bar{C}_Z Z_t + C_i I_t^{\mathrm{MC}}, \tag{F.3}
\end{aligned}
$$

where the matrices $\bar{A}, \bar{B}, \bar{B}^1$ and $\bar{C}_Z$ fulfill

$$
\begin{aligned}
\bar{A} &= \left[I - B^1(e_{10} + e_{n_1+1})\right]^{-1}\left[A + B(e_{10} + e_{n_1+1})\right] \\
\bar{B} &= \left[I - B^1(e_{10} + e_{n_1+1})\right]^{-1} B \\
\bar{B}^1 &= \left[I - B^1(e_{10} + e_{n_1+1})\right]^{-1} B^1 \\
\bar{C}_Z &= C_Z + (e_{10} + e_{n_1+1}).
\end{aligned}
$$

Thus, formally we can rewrite the model as in (F.2) and (F.3), where I_t^{MC} is considered the control variable. The optimal solution will be given by a row vector $\bar{f}$,

$$
I_t^{\mathrm{MC}} = \bar{f} X_t.
$$

However, from (F.1), (2.28) and (3.4), we directly realize that the optimal solutions $\bar{f}$ for I_t^{MC} and f for i_t are related as

$$
\bar{f} \equiv f - (\bar{e}_{10} + H_{1\cdot}),
$$

where $\bar{e}_{10}$ is a 1×10 row vector with element 10 equal to 1 and all other elements equal to zero.

However, this only shows that the same model can formally have several alternative control variables. The present model does not provide any support for a particular MCI, other than possibly the one in (4.1).

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Figure 3.1: Strict domestic-inflation targeting. Impulse responses
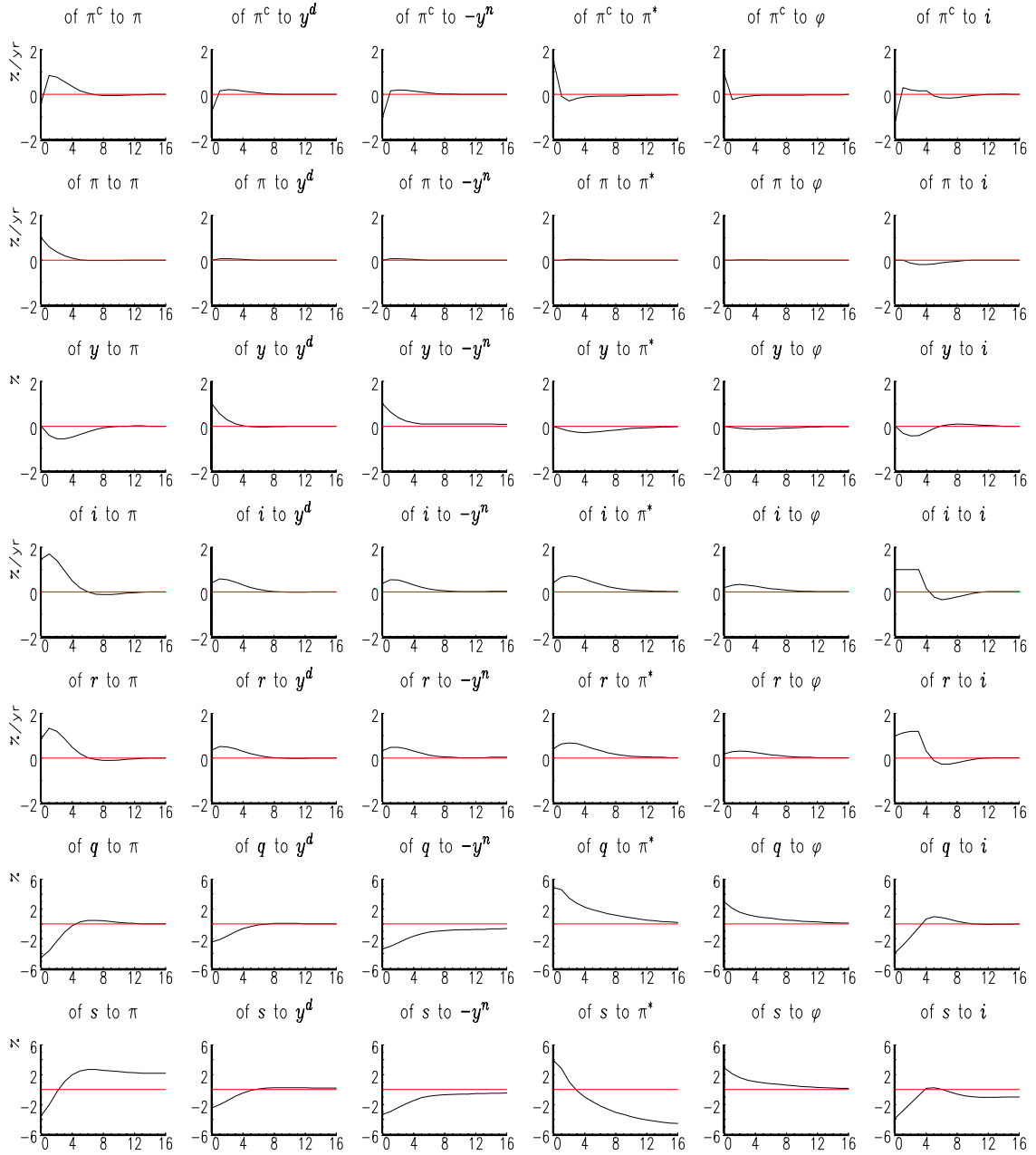
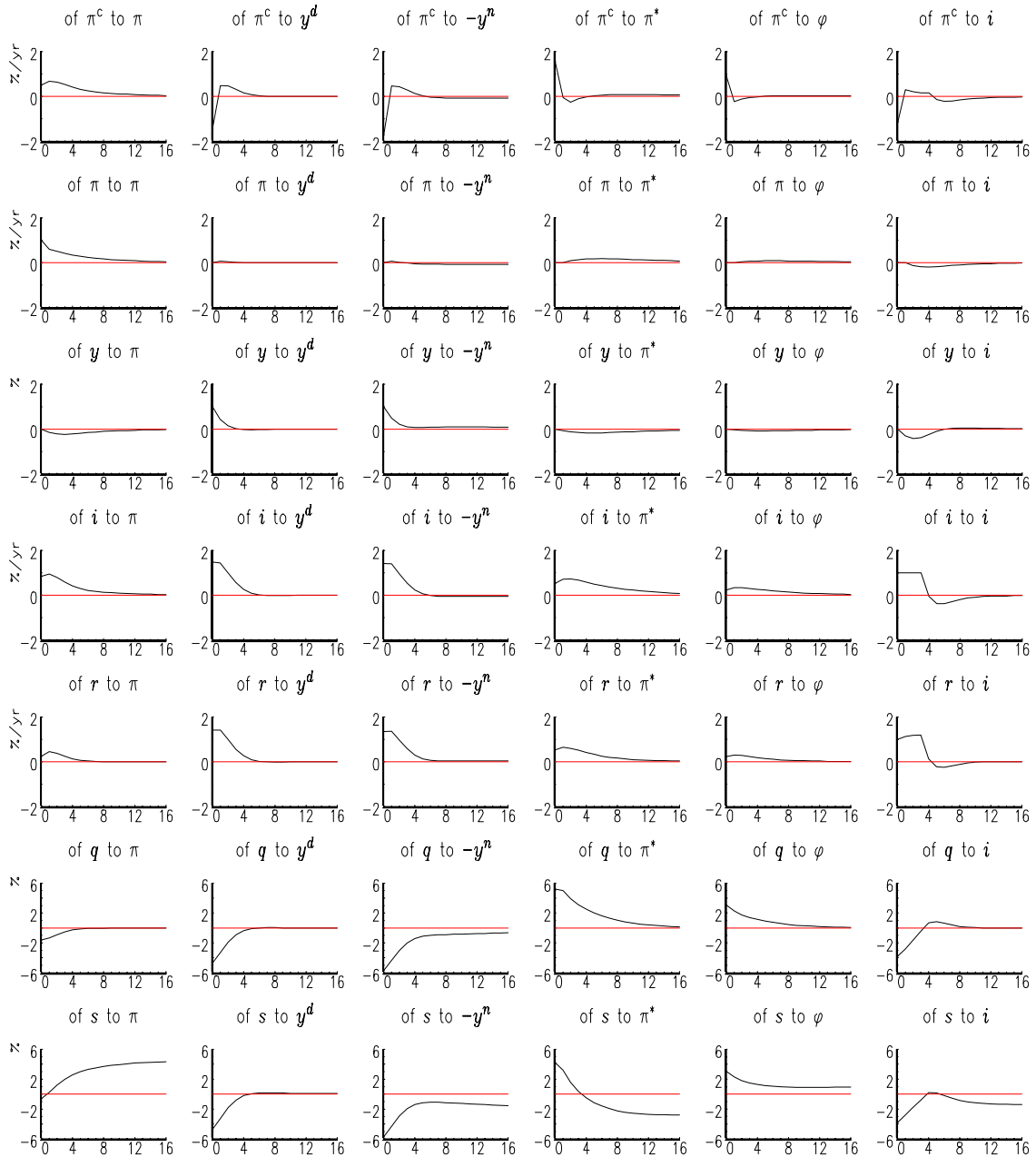Figure 3.2: Flexible domestic-inflation targeting. Impulse responses

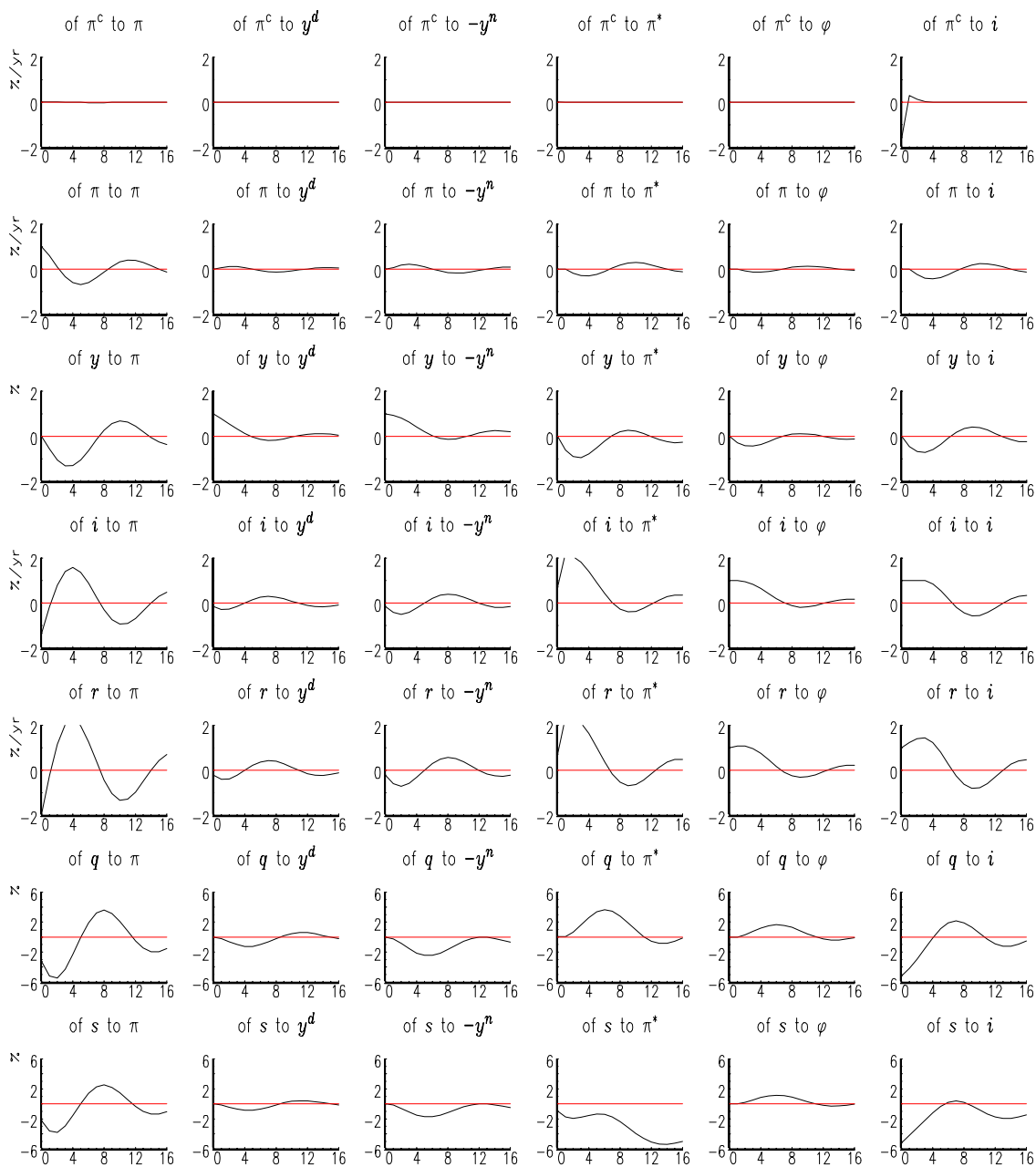Figure 3.3: Strict CPI-inflation targeting. Impulse responses

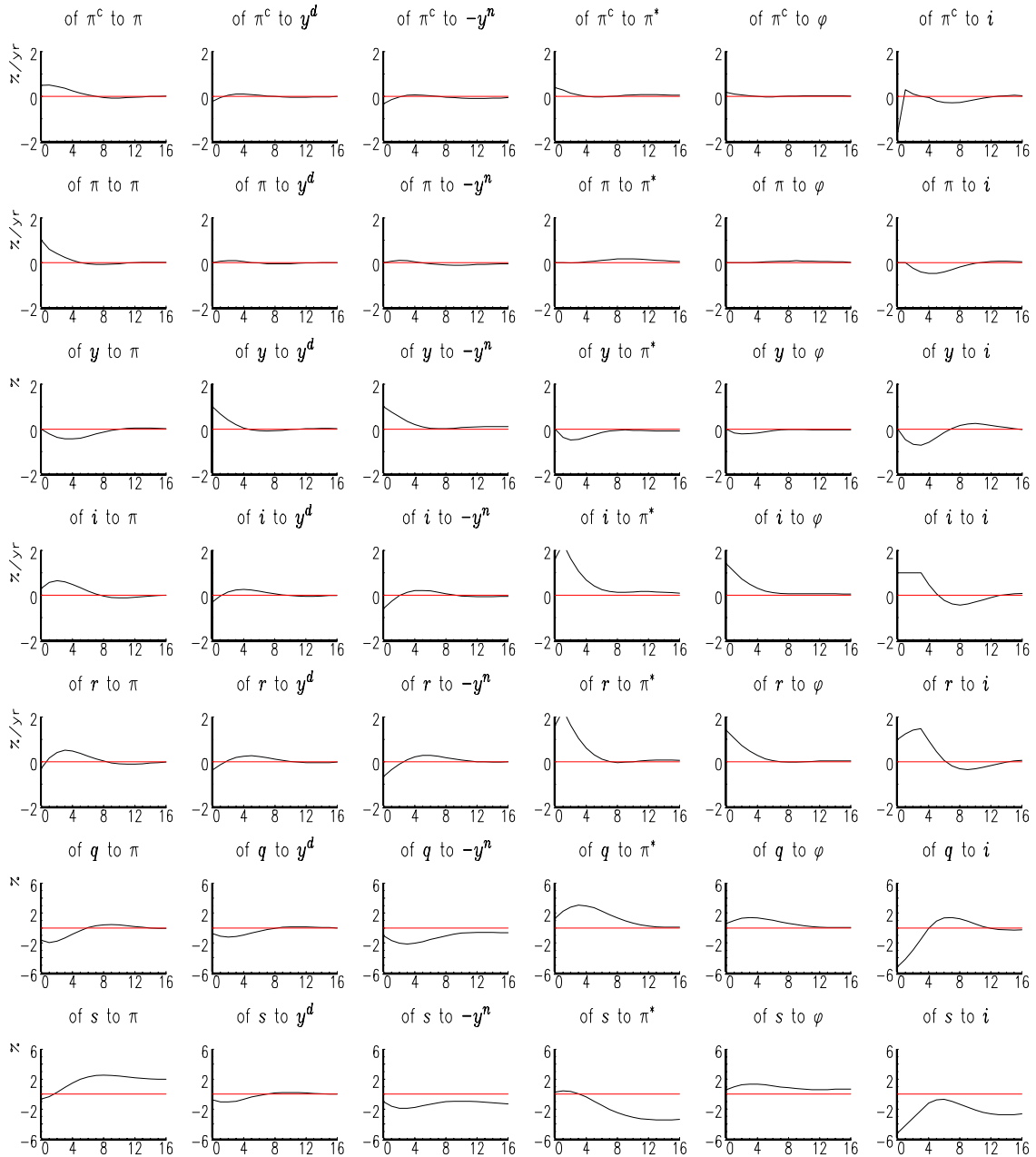Figure 3.4: Flexible CPI-inflation targeting. Impulse responses

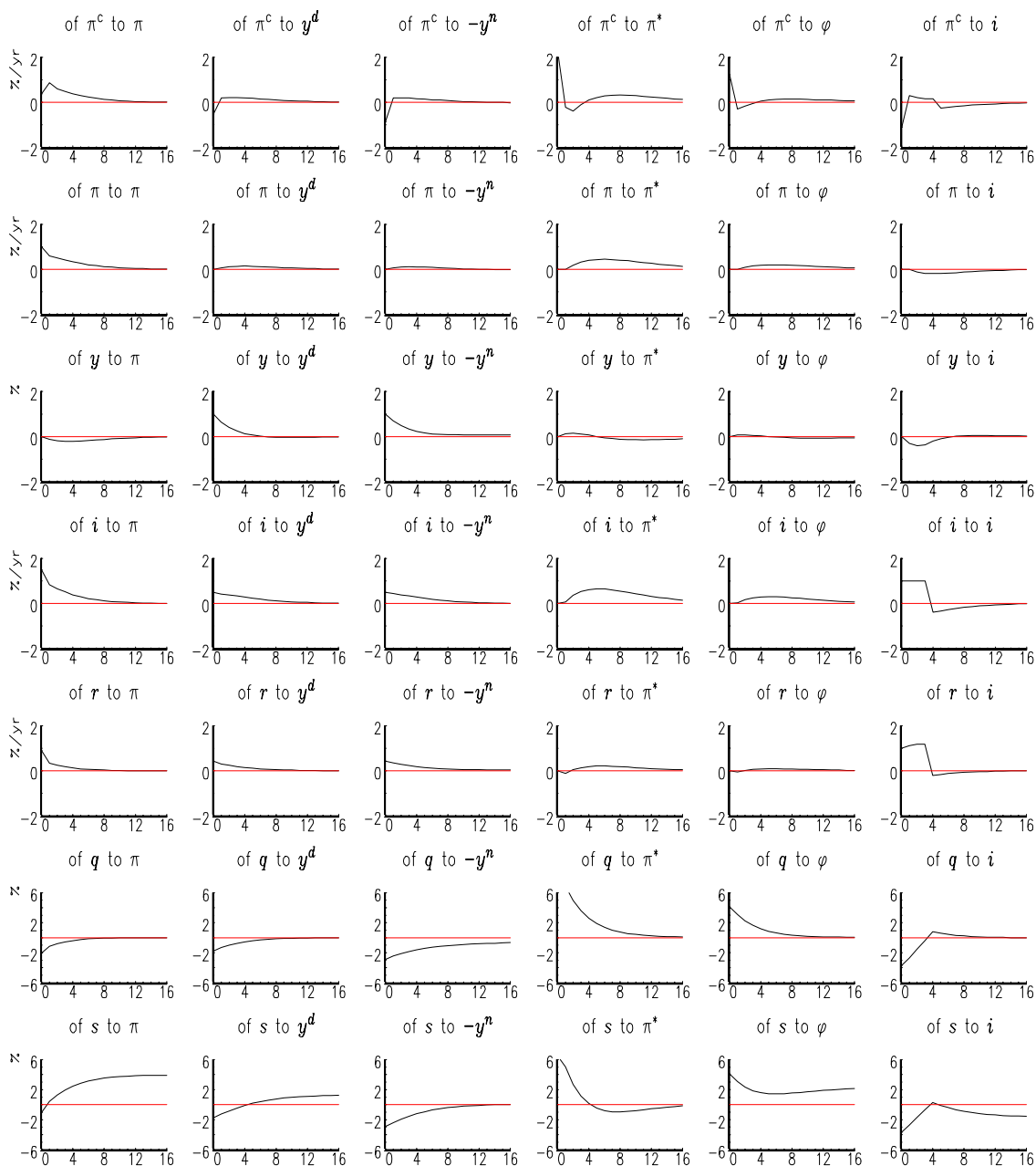Figure 3.5: Taylor rule for domestic inflation. Impulse responses

Figure 3.6: Taylor rule for CPI inflation. Impulse responses