Optimal Monetary Policy in an Operational Medium-Sized DSGE Model

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1.Introduction

- Flexible inflation targeting: "Stabilize inflation around the inflation target, with some weight on stabilility of the real economy (output gap)"
- Construct optimal policy projections (OPPs) for Ramses, the Riksbank's open-economy medium-sized DSGE model for forecasting and policy analysis
- The Riksbank Aggregate Model for Studies of the Economy of Sweden (Adolfson, Laséen, Lindé, and Villani) (ALLV)
- OPP: Find instrument-rate path that minimizes quadratic loss function under commitment in a timeless perspective: Alternative to historical empirical or ad hoc instrument rule (Taylor-type rule)

1. Introduction: New

- OPPs in DSGE model of this size
- Estimation requires combination of Klein and AIM algorithms for speed
- Test of whether past policy was optimal or not
- Alternative definitions of the output gap (potential output: trend output, conditional flexprice output, or unconditional flexprice output)
- Commitment in a timeless perspective: Alternative ways of computing initial Lagrange multipliers (past policy: optimal or just systematic)

1. Introduction: Conclusions

- OPPs feasible in Ramses
- Parameter estimates relatively stable
- Past policy not optimal
- Estimated loss-function paramaters: $\lambda_y = 1.1$, $\lambda_{\Delta i} = 0.39$
- Output-gap (potential-output) definition matters
- Initial Lagrange multiplers matter (somewhat)

1. Introduction: Outline

1. Introduction
2. The model
3. Estimation
4. Optimal policy projections
5. Results
6. Conclusions

Appendix

2. The model

- State space form:

$$\begin{bmatrix} X_{t+1} \\ Hx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}$$

- X_t predetermined variables in quarter t ($n_X = 71$),
  x_t forward-looking variables ($n_x = 23$),
  i_t instrument rate, ε_{t+1} i.i.d. shock ($n_\varepsilon = 23$),
  $x_{t+1|t} \equiv \mathrm{E}_t x_{t+1}$
- A, B, C, H estimated with Bayesian methods, considered fixed and known for the optimal projections (certainty equivalence)

2. The model

- Target variables

$$Y_t = D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix},$$

- Period loss function

$$L_t \equiv Y_t' W Y_t = (p_t^c - p_{t-4}^c - \pi^*)^2 + \lambda_y (y_t - \bar{y}_t)^2 + \lambda_{\Delta i}(i_t - i_{t-1})^2,$$

$$Y_t \equiv (p_t^c - p_{t-4}^c - \pi^*, y_t - \bar{y}_t, i_t - i_{t-1})'$$

Flexible inflation targeting: 4-qtr CPIX inflation, alternative definitions of potential output $\bar{y}_t$

- Intertemporal loss function ($0 < \delta < 1$)

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}.$$

2. The model: Optimal policy

- Minimize intertemporaral loss function under commitment in a timeless perspective. Solution:

$$\begin{bmatrix} x_t \\ i_t \end{bmatrix} = \begin{bmatrix} F_x \\ F_i \end{bmatrix} \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix},$$
$$\begin{bmatrix} X_{t+1} \\ \Xi_t \end{bmatrix} = M \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix} + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}.$$

  F_i policy function: depends on A, B, C, H, D, W, δ, but not on $\Sigma_{\varepsilon\varepsilon}$ (certainty equivalence)
  Ξ_{t-1} Lagrange multiplers for equations for forward-looking variables in period $t-1$ ($n_\Xi \equiv n_x = 23$)
- Klein (2000) algorithm returns F_x, F_i, M

2. The model: Simple instrument rule

$$\begin{aligned} i_t &= \rho_R i_{t-1} + (1-\rho_R)\left[\hat{\bar{\pi}}_t^c + r_\pi(\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t^c) + r_y \hat{y}_{t-1} + r_x \hat{\tilde{x}}_{t-1}\right] \\ &\quad + r_{\Delta\pi}(\hat{\pi}_t^c - \hat{\pi}_{t-1}^c) + r_{\Delta y}(\hat{y}_t - \hat{y}_{t-1}) + \varepsilon_{Rt} \end{aligned}$$

- "Implicit" instrument rule

$$i_t = f_X X_t + f_x x_t$$

- Klein algorithm returns F_x, F_i, M

3. Estimation

- Bayesian estimation as in ALLV, following Smets and Wouters (2003)
- Use Swedish data on $n_Z = 15$ macroeconomic variables:

$$Z_t \equiv \begin{array}{l} (\pi_t^d, \Delta \ln(W_t/P_t), \Delta \ln C_t, \Delta \ln I_t, \hat{x}_t, R_t, \hat{H}_t, \ldots \\ \Delta \ln Y_t, \Delta \ln \tilde{X}_t, \Delta \ln \tilde{M}_t, \pi_t^{\mathrm{cpi}}, \pi_t^{\mathrm{def},i}, \Delta \ln Y_t^*, \pi_t^*, R_t^*)'. \end{array}$$

- Foreign output, inflation, and interest rate exogenous
- Sample 1980:1-2007:4
- Fixed exchange rate regime from 1980:1;
  inflation targeting from 1993:1-2007:4;
  unanticipated perceived permanent shift in monetary policy
- Klein algorithm combined with AIM (Anderson-Moore) for speed

3. Estimation

- Simple different instrument rule before and after 1993:1
  Log marginal likelihood $= -2631.56$
- Simple instrument rule before, optimal policy from 1993:1
  Log marginal likelihood $= -2654.45$
- Past policy not optimal
- Model parameters relatively stable, interpreted as structural
- Loss-function parameters estimated with model parameters as in simple instrument rule
  $\lambda_y = 1.1$, $\lambda_{\Delta i} = 0.37$ (baseline)

4. Optimal policy projections

- $y^t \equiv \{y_{t+\tau,t}\}_{\tau=0}^{\infty}$ projection in period t for any variable y_t: mean forecast conditional on information in period t
- Projection model for projections (X^t, x^t, i^t, Y^t) in quarter t is

$$\begin{bmatrix} X_{t+\tau+1,t} \\ Hx_{t+\tau+1,t} \end{bmatrix} = A \begin{bmatrix} X_{t+\tau,t} \\ x_{t+\tau,t} \end{bmatrix} + Bi_{t+\tau,t}, \quad Y_{t+\tau,t} = D \begin{bmatrix} X_{t+\tau,t} \\ x_{t+\tau,t} \\ i_{t+\tau,t} \end{bmatrix}$$

for $\tau \geq 0$.

4. Optimal policy projections

- Optimal projection $(\check{X}^t, \check{x}^t, \check{i}^t, \check{Y}^t)$, minimizes the intertemporal loss function under commitment in a timeless perspective

$$\sum_{\tau=0}^{\infty} \delta^{\tau} L_{t+\tau,t},$$

$$L_{t+\tau,t} = Y_{t+\tau,t}{}'WY_{t+\tau,t}.$$

- $0 < \delta \leq 1$ OK

4. Optimal policy projections

- Solve with Klein or AIM (Anderson-Moore) algorithms: Solution

$$\begin{bmatrix} \check{x}_{t+\tau,t} \\ \check{i}_{t+\tau,t} \end{bmatrix} = F \begin{bmatrix} \check{X}_{t+\tau,t} \\ \Xi_{t+\tau-1,t} \end{bmatrix},$$

$$\begin{bmatrix} \check{X}_{t+\tau+1,t} \\ \Xi_{t+\tau,t} \end{bmatrix} = M \begin{bmatrix} \check{X}_{t+\tau,t} \\ \Xi_{t+\tau-1,t} \end{bmatrix},$$

  for $\tau \geq 0$, where $\check{X}_{t,t} = X_{t|t}$, $\Xi_{t-1,t}$ given
- Decision in quarter t
- Information in quarter t includes data up to $t-1$, $X_{t|t}$ estimated from $X_{t-1|t}$ under the assumption of simple instrument rule in quarter $t-1$

4. OPPs: Initial Lagrange multipliers

- Commitment from scratch in quarter t:

$$\Xi_{t-1,t} = 0$$

- Commitment in timeless perspective:

$$\Xi_{t-1,t} = \Xi_{t-1,t-1},$$

$\Xi_{t-1,t-1}$ determined by OPP in quarter $t-1$

4. OPPs: Initial Lagrange multipliers (1)

- Alternative 1: Assume optimal policy in the past

$$
\begin{aligned}
\Xi_{t-1,t-1} &= M_{\Xi X} X_{t-1|t-1} + M_{\Xi\Xi} \Xi_{t-2,t-2} \\
&= \sum_{\tau=0}^{\infty} (M_{\Xi\Xi})^{\tau} M_{\Xi X} X_{t-1-\tau|t-1-\tau},
\end{aligned}
$$

$$
M \equiv \begin{bmatrix} M_{XX} & M_{X\Xi} \\ M_{\Xi X} & M_{\Xi\Xi} \end{bmatrix}.
$$

Truncate at some $\tau = T$

4. OPPs: Initial Lagrange multipliers (2)

- Alternative 2 (appendix): Assume any systematic policy in the past and find the shadow prices (Lagrange multipliers) $(\xi_t', \Xi_{t-1}')'$ from the first-order conditions:

$$\bar{A}' \begin{bmatrix} \xi_{t+1|t} \\ \Xi_t \end{bmatrix} = \frac{1}{\delta} \bar{H}' \begin{bmatrix} \xi_t \\ \Xi_{t-1} \end{bmatrix} + \bar{W} z_t,$$

where $z_t \equiv (X_t', x_t', i_t')'$

4. OPPs: Initial Lagrange multipliers (2)

- Combine with instrument rule and model equations, find solution:

$$\begin{bmatrix} \xi_t \\ \Xi_t \end{bmatrix} = \bar{B} \begin{bmatrix} \xi_{t-1} \\ \Xi_{t-1} \end{bmatrix} + \sum_{s=0}^{\infty} \bar{F}^s \Phi \bar{W} z_{t+s|t}$$

  The AIM algorithm returns $\bar{B}$, $\bar{F}$, and Φ

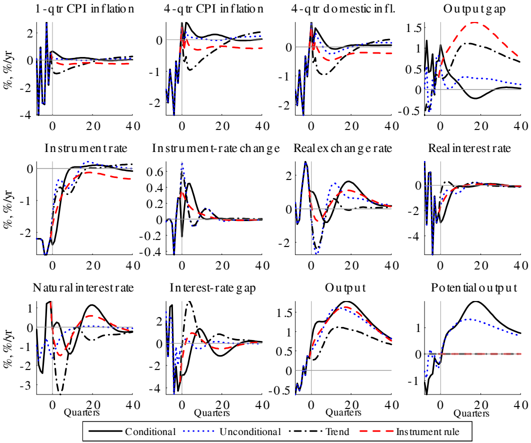
- Thus, for given realizations of $z_{t-\tau}$ and corresponding expectations $\{z_{t-\tau+s|t-\tau}\}_{s=0}^{\infty}$ for $\tau = 0, 1, ..., T$, we can solve for $\{\xi'_{t-\tau}, \Xi'_{t-1-\tau}\}_{\tau=0}^{T}$ and use the resulting Ξ_{t-1} as the initial value for $\Xi_{t-1,t}$

4. OPPs: Alternative output gaps

- Trend output gap (trend output)
- Conditional output gap (flexprice output, prices flexible from this quarter, given capital stock)
- Unconditional output gap (flexprice output, prices flexible from far in the past, different capital stock)
- Corresponding alternative definitions of the real interest-rate gap (neutral real interest rate)

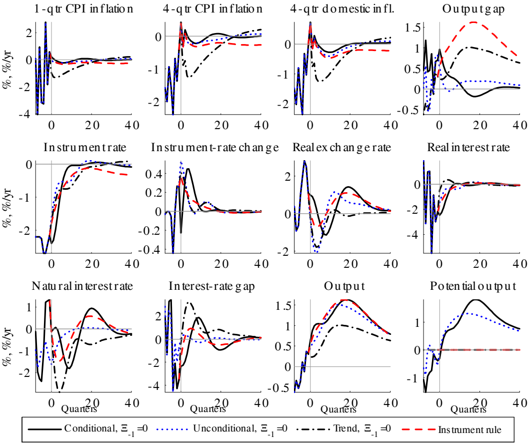
5. Results: Projections in 2006:3

Optimal policy for different output gaps, instrument rule



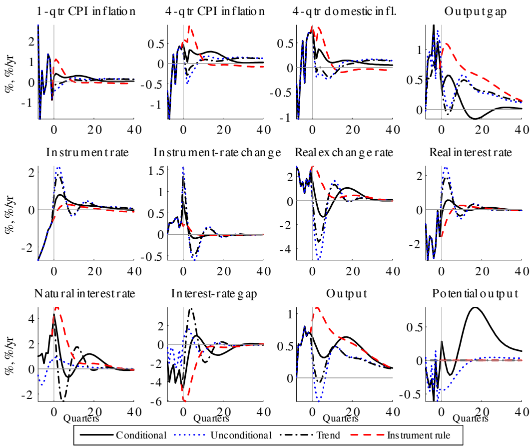
5. Results: Projections in 2006:3

Optimal policy for different output gaps, $\Xi_{t-1,t} = 0$



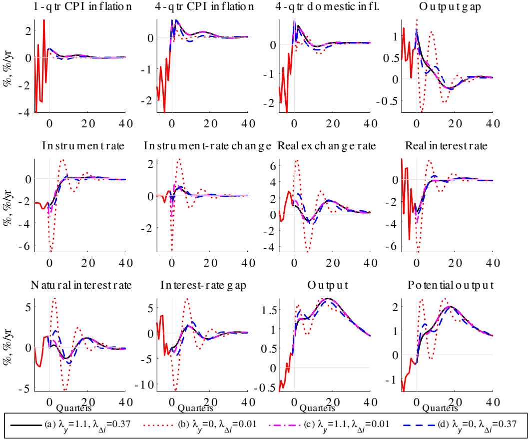
5. Results: Projections in 2007:4

Optimal policy for different output gaps, instrument rule



5. Results: Projections in 2006:3

Optimal policy, different loss functions (cond. output gap)



6. Conclusions

- OPPs feasible in Ramses
- Parameter estimates relatively stable
- Past policy not optimal
- Estimated loss-function paramaters: $\lambda_y = 1.1$, $\lambda_{\Delta i} = 0.39$
- Output-gap (potential-output) definition matters
- Initial Lagrange multipliers matter (somewhat)
- OPPs shown only for two particular quarters (examples)