Optimal Monetary Policy in an Operational Medium-Sized DSGE Model

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1. Introduction

- Flexible inflation targeting: “Stabilize inflation around the inflation target, with some weight on stability of the real economy (output gap)”

- Construct optimal policy projections (OPPs) for Ramses, the Riksbank’s open-economy medium-sized DSGE model for forecasting and policy analysis

- The Riksbank Aggregate Model for Studies of the Economy of Sweden (Adolfson, Laséen, Lindé, and Villani) (ALLV)

- OPP: Find instrument-rate path that minimizes quadratic loss function under commitment in a timeless perspective: Alternative to historical empirical or ad hoc instrument rule (Taylor-type rule)
1. Introduction: New

- OPPs in DSGE model of this size
- Estimation requires combination of Klein and AIM algorithms for speed
- Test of whether past policy was optimal or not
- Alternative definitions of the output gap (potential output: trend output, conditional flexprice output, or unconditional flexprice output)
- Commitment in a timeless perspective: Alternative ways of computing initial Lagrange multipliers (past policy: optimal or just systematic)
1. Introduction: Conclusions

- OPPs feasible in Ramses
- Parameter estimates relatively stable
- Past policy not optimal
- Estimated loss-function parameters: $\lambda_y = 1.1$, $\lambda_{\Delta i} = 0.39$
- Output-gap (potential-output) definition matters
- Initial Lagrange multipliers matter (somewhat)
1. Introduction: Outline

1. Introduction
2. The model
3. Estimation
4. Optimal policy projections
5. Results
6. Conclusions

Appendix
2. The model

- State space form:

\[
\begin{bmatrix}
X_{t+1} \\
Hx_{t+1|t}
\end{bmatrix} = A \begin{bmatrix}
X_t \\
x_t
\end{bmatrix} + B i_t + \begin{bmatrix}
C \\
0
\end{bmatrix} \varepsilon_{t+1}
\]

- \(X_t\) predetermined variables in quarter \(t\) \((n_X = 71)\),
- \(x_t\) forward-looking variables \((n_x = 23)\),
- \(i_t\) instrument rate, \(\varepsilon_{t+1}\) i.i.d. shock \((n_\varepsilon = 23)\),
- \(x_{t+1|t} \equiv E_t x_{t+1}\)

- \(A, B, C, H\) estimated with Bayesian methods, considered fixed and known for the optimal projections (certainty equivalence)
2. The model

- Target variables

\[ Y_t = D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}, \]

- Period loss function

\[ L_t \equiv Y_t' W Y_t = (p_t^c - p_{t-4}^c - \pi^*)^2 + \lambda_y (y_t - \bar{y}_t)^2 + \lambda_{\Delta i} (i_t - i_{t-1})^2, \]

\[ Y_t \equiv (p_t^c - p_{t-4}^c - \pi^*, y_t - \bar{y}_t, i_t - i_{t-1})' \]

Flexible inflation targeting: 4-qtr CPIX inflation, alternative definitions of potential output \( \bar{y}_t \)

- Intertemporal loss function \((0 < \delta < 1)\)

\[ E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}. \]
2. The model: Optimal policy

- Minimize intertemporal loss function under commitment in a timeless perspective. Solution:

\[
\begin{bmatrix}
  x_t \\
  i_t
\end{bmatrix}
= \begin{bmatrix}
  F_x \\
  F_i
\end{bmatrix}
\begin{bmatrix}
  X_t \\
  \Xi_{t-1}
\end{bmatrix},
\]

\[
\begin{bmatrix}
  X_{t+1} \\
  \Xi_t
\end{bmatrix}
= M \begin{bmatrix}
  X_t \\
  \Xi_{t-1}
\end{bmatrix} + \begin{bmatrix}
  C \\
  0
\end{bmatrix} \varepsilon_{t+1}.
\]

- \(F_i\) policy function: depends on \(A, B, C, H, D, W, \delta\), but not on \(\Sigma_{\varepsilon\varepsilon}\) (certainty equivalence)
- \(\Xi_{t-1}\) Lagrange multipliers for equations for forward-looking variables in period \(t - 1\) \((n_{\Xi} \equiv n_x = 23)\)
- Klein (2000) algorithm returns \(F_x, F_i, M\)
2. The model: Simple instrument rule

\[ i_t = \rho_R i_{t-1} + (1 - \rho_R) \left[ \hat{\pi}_t^c + r_\pi (\hat{\pi}_{t-1}^c - \hat{\pi}_t^c) + r_y \hat{y}_{t-1} + r_x \hat{x}_{t-1} \right] \\
+ r_{\Delta\pi} (\hat{\pi}_t^c - \hat{\pi}_{t-1}^c) + r_{\Delta y} (\hat{y}_t - \hat{y}_{t-1}) + \varepsilon_{R_t} \]

- “Implicit” instrument rule

\[ i_t = f_X X_t + f_x x_t \]

- Klein algorithm returns \( F_x, F_i, M \)
3. Estimation

- Bayesian estimation as in ALLV, following Smets and Wouters (2003)
- Use Swedish data on \( n_Z = 15 \) macroeconomic variables:
  \[
  Z_t \equiv (\pi^d_t, \Delta \ln(\frac{W_t}{P_t}), \Delta \ln C_t, \Delta \ln I_t, \hat{x}_t, R_t, \hat{H}_t, ... \\
  \Delta \ln Y_t, \Delta \ln \tilde{X}_t, \Delta \ln \tilde{M}_t, \pi^\text{cpi}_t, \pi^\text{def,i}_t, \Delta \ln Y^*_t, \pi^*_t, R^*_t)'.
  \]
- Foreign output, inflation, and interest rate exogenous
- Fixed exchange rate regime from 1980:1; inflation targeting from 1993:1-2007:4; unanticipated perceived permanent shift in monetary policy
- Klein algorithm combined with AIM (Anderson-Moore) for speed
3. Estimation

- Simple different instrument rule before and after 1993:1
  Log marginal likelihood = −2631.56

- Simple instrument rule before, optimal policy from 1993:1
  Log marginal likelihood = −2654.45

- Past policy not optimal

- Model parameters relatively stable, interpreted as structural

- Loss-function parameters estimated with model parameters as in simple instrument rule
  \( \lambda_y = 1.1, \lambda_{\Delta i} = 0.37 \) (baseline)
4. Optimal policy projections

- \( y^t \equiv \{y_{t+\tau}, t\}_{\tau=0}^{\infty} \) projection in period \( t \) for any variable \( y_t \): mean forecast conditional on information in period \( t \)
- Projection model for projections \((X^t, x^t, i^t, Y^t)\) in quarter \( t \) is

\[
\begin{bmatrix}
X_{t+\tau+1,t} \\
Hx_{t+\tau+1,t}
\end{bmatrix} = A \begin{bmatrix}
X_{t+\tau,t} \\
x_{t+\tau,t}
\end{bmatrix} + Bi_{t+\tau,t}, \quad Y_{t+\tau,t} = D \begin{bmatrix}
X_{t+\tau,t} \\
x_{t+\tau,t} \\
i_{t+\tau,t}
\end{bmatrix}
\]

for \( \tau \geq 0 \).
4. Optimal policy projections

- Optimal projection \((\hat{X}^t, \hat{x}^t, \hat{i}^t, \hat{Y}^t)\), minimizes the intertemporal loss function under commitment in a timeless perspective

\[
\sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau,t},
\]

\[
L_{t+\tau,t} = Y_{t+\tau,t}' W Y_{t+\tau,t}.
\]

- \(0 < \delta \leq 1\) OK
4. Optimal policy projections

- Solve with Klein or AIM (Anderson-Moore) algorithms:
  Solution

\[
\begin{bmatrix}
\tilde{x}_{t+\tau,t} \\
\tilde{y}_{t+\tau,t}
\end{bmatrix} = F \begin{bmatrix}
\tilde{X}_{t+\tau,t} \\
\tilde{E}_{t+\tau-1,t}
\end{bmatrix},
\]

\[
\begin{bmatrix}
\tilde{X}_{t+\tau+1,t} \\
\tilde{E}_{t+\tau,t}
\end{bmatrix} = M \begin{bmatrix}
\tilde{X}_{t+\tau,t} \\
\tilde{E}_{t+\tau-1,t}
\end{bmatrix},
\]

for \( \tau \geq 0 \), where \( \tilde{X}_{t,t} = X_{t|t}, \tilde{E}_{t-1,t} \) given

- Decision in quarter \( t \)

- Information in quarter \( t \) includes data up to \( t - 1 \), \( X_{t|t} \) estimated from \( X_{t-1|t} \) under the assumption of simple instrument rule in quarter \( t - 1 \)
4. OPPs: Initial Lagrange multipliers

- Commitment from scratch in quarter $t$:
  \[ \Xi_{t-1,t} = 0 \]

- Commitment in timeless perspective:
  \[ \Xi_{t-1,t} = \Xi_{t-1,t-1}, \]
  \[ \Xi_{t-1,t-1} \text{ determined by OPP in quarter } t-1 \]
4. OPPs: Initial Lagrange multipliers (1)

Alternative 1: Assume optimal policy in the past

\[
\Xi_{t-1,t-1} = M_{XX} X_{t-1|t-1} + M_{XX} \Xi_{t-2,t-2} \\
= \sum_{\tau=0}^{\infty} (M_{XX})^\tau M_{XX} X_{t-1-\tau|t-1-\tau},
\]

\[
M \equiv \begin{bmatrix} M_{XX} & M_{XE} \\ M_{EX} & M_{EE} \end{bmatrix}.
\]

Truncate at some \( \tau = T \)
Alternative 2 (appendix): Assume any systematic policy in the past and find the shadow prices (Lagrange multipliers) \((\tilde{\zeta}_t', \Xi_{t-1}')\) from the first-order conditions:

\[
\bar{A}' \begin{bmatrix} \tilde{\zeta}_{t+1|t} \\ \Xi_t \end{bmatrix} = \frac{1}{\delta} \bar{H}' \begin{bmatrix} \tilde{\zeta}_t \\ \Xi_{t-1} \end{bmatrix} + \bar{W}z_t,
\]

where \(z_t \equiv (X'_t, x'_t, i'_t)'\)
4. OPPs: Initial Lagrange multipliers (2)

- Combine with instrument rule and model equations, find solution:

\[
\begin{bmatrix}
\xi_t \\
\Xi_t
\end{bmatrix} = \bar{B} \begin{bmatrix}
\xi_{t-1} \\
\Xi_{t-1}
\end{bmatrix} + \sum_{s=0}^{\infty} \bar{F}^s \Phi \bar{W} z_{t+s|t}
\]

The AIM algorithm returns \( \bar{B}, \bar{F}, \) and \( \Phi \)

- Thus, for given realizations of \( z_{t-\tau} \) and corresponding expectations \( \{ z_{t-\tau+s|t-\tau} \}_{s=0}^{\infty} \) for \( \tau = 0, 1, \ldots, T \), we can solve for \( \{ \xi'_t, \Xi'_t \}_{\tau=0}^{T} \) and use the resulting \( \Xi_{t-1} \) as the initial value for \( \Xi_{t-1,t} \)
4. OPPs: Alternative output gaps

- Trend output gap (trend output)
- Conditional output gap (flexprice output, prices flexible from this quarter, given capital stock)
- Unconditional output gap (flexprice output, prices flexible from far in the past, different capital stock)
- Corresponding alternative definitions of the real interest-rate gap (neutral real interest rate)
5. Results: Projections in 2006:3

Optimal policy for different output gaps, instrument rule
5. Results: Projections in 2006:3

Optimal policy for different output gaps, $\Xi_{t-1,t} = 0$
5. Results: Projections in 2007:4

Optimal policy for different output gaps, instrument rule
5. Results: Projections in 2006:3

Optimal policy, different loss functions (cond. output gap)
6. Conclusions

- OPPs feasible in Ramses
- Parameter estimates relatively stable
- Past policy not optimal
- Estimated loss-function parameters: $\lambda_y = 1.1$, $\lambda_{\Delta i} = 0.39$
- Output-gap (potential-output) definition matters
- Initial Lagrange multipliers matter (somewhat)
- OPPs shown only for two particular quarters (examples)