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Inflation Targeting: Some Extensions

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Abstract

Previous analysis of the implementation of inflation targeting is extended to monetary policy responses to different shocks, consequences of model uncertainty, effects of interest rate smoothing and stabilization, a comparison with nominal GDP targeting, and implications of forward-looking behavior. Model uncertainty, output stabilization, and interest rate stabilization or smoothing all call for a more gradual adjustment of the conditional inflation forecast toward the inflation target. The conditional inflation forecast is the natural intermediate target during inflation targeting.

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1 Introduction

Explicit inflation targeting has received considerable attention during the last few years. Monetary policy in New Zealand, Canada, UK, Sweden, Finland, Australia and Spain has explicit inflation targets. Two recent conference volumes, Leiderman and Svensson (1995) and Haldane (1995), and an increasing number of research papers deal with different aspects of inflation targeting. At a recent symposium (Federal Reserve Bank of Kansas City (1996)), the four major papers (by Stanley Fischer, Charles Freedman, Mervyn King and John Taylor) recommended explicit inflation targeting as the best way to achieve and maintain low and stable inflation.[1]

In Svensson (1997a), I examined both the implementation and the monitoring of inflation targeting. In a simple closed-economy model, I showed that inflation targeting implies that the central bank's conditional inflation forecast for a horizon corresponding to the control lag becomes an intermediate target (in line with explicit statements in King (1994) and Bowen (1995)).[2] Under what we can call *strict inflation targeting*, with low and stable inflation being the only goal for monetary policy (a zero weight on output stabilization), this implies that the central bank should adjust its instrument such that the conditional inflation forecast for the control lag equals the inflation target. Under what we may call *flexible inflation targeting* (with a positive weight on output stabilization), the conditional inflation forecast should instead be adjusted gradually towards the inflation target. I also argued that inflation targeting allows efficient monitoring of monetary policy by the public, especially if the central bank makes the conditional inflation forecast an explicit intermediate target, and publishes and allows public scrutiny of its inflation forecast, including models, analyses and judgements. Then the conditional inflation forecast becomes an ideal intermediate target, in that it is the current variable most correlated with the goal, is easier to control than the goal, is easier to observe than the goal, and by implying extremely transparent principles for monetary policy is most conducive to public understanding of monetary policy. I also showed that inflation targeting is more efficient, in the sense of bringing lower inflation variability, than money growth or exchange rate targeting.

In the present paper I extend the analysis of the implementation of inflation targeting to

[1] See Svensson (1997a), for instance, for further references to the literature on inflation targeting.

[2] I would like to argue that this is also in line with statements in Mayes and Riches (1996, p. 7), and certainly with the practice at the Reserve Bank of New Zealand: "The current operational framework employed by the Reserve Bank is based directly on forecasts of inflation. No intermediate targets are set. To determine if monetary conditions should be tighter or looser than at present, the current inflation forecast is compared with the specified objective. If inflation is forecast to be outside the target band within the forecast horizon (the next two to eight quarters) then some change of policy settings is required." In spite of the second sentence in this quotation, I interpret it as implying that the inflation forecast is used as an intermediate target. Then, the second sentence might be interpreted as "No intermediate targets [for variables other than the inflation forecast] are set."

the monetary policy response to different shocks (section 3), to the consequences of model uncertainty (section 4), to the effects of interest rate smoothing and stabilization (section 5), to a comparison with nominal GDP targeting (section 6), and to the implications of forward-looking behavior (section 7). Section 2 restates the result that inflation targeting implies that the conditional inflation forecast becomes an intermediate target, and shows how that intermediate target is affected by a positive weight on output gap stabilization. This section goes beyond Svensson (1997a) in incorporating a stochastic "natural rate" level of output and exogenous variables. Section 8 concludes. Appendices A to E contain some technical details.

2 Inflation forecast targeting

This section shows that conditional inflation targeting implies that the conditional inflation forecast for a horizon corresponding to the control lag becomes an intermediate target. Although the result can be demonstrated in a much more elaborate model with a more explicit role for agents' expectations, a much simpler model is sufficient.[3] The model nevertheless has some structural similarity to more elaborate models used by certain central banks. Section 7 discusses some issues that arise with a more forward-looking model.

The important aspects of the model are that the monetary authority has imperfect control over inflation, that inflation and the output gap react with a lag to changes in the monetary policy instrument, that inflation reacts with a longer lag than the output gap, and that a stochastic persistent "natural (rate)" level of output and some exogenous variables (like oil prices) also play a role. Consider the following model with an acceleration Phillips curve and an aggregate demand equation,

$$\pi_{t+1} = \pi_t + \alpha_y y_t + \varepsilon_{t+1} \quad (2.1)$$

$$y_{t+1} = \tilde{\beta}_y y_t + \beta_x x_t - \beta_r (i_t - \pi_{t+1|t}) + \eta_{t+1} \quad (2.2)$$

$$x_{t+1} = \gamma x_t + \theta_{t+1}, \quad (2.3)$$

where $\pi_t = p_t - p_{t-1}$ is the inflation (rate) in year t, p_t is the (log) price level, y_t is the output gap (the log of the ratio of output to the natural output level), x_t is an exogenous variable, i_t is the monetary policy instrument or operating target (for instance, a short repo rate or the

[3] For instance, it is not necessary to assume the systematic discretionary inflation bias (due to 'time-consistency' problems) emphasized in the modern 'principal-agent' approach to central banking (for instance in the work by Barro and Gordon (1983), Rogoff (1985), Cukierman (1992), Walsh (1995), Persson and Tabellini (1993) and Svensson (1997b)) and disputed in the 'traditional' approach (for instance in McCallum (1995) and Romer and Romer (1996)); see Tabellini (1995) for discussion of these approaches.

federal funds rate), $\pi_{t+1|t}$ denotes $\mathrm{E}_t\pi_{t+1}$ (the inflation in year $t+1$ expected in year t), and ε_t, η_t and θ_t are i.i.d. shocks in year t that are not known in year $t-1$. The coefficients α_y, $\tilde{\beta}_y$ and β_r are assumed to be positive; γ fulfills $0 \leq \gamma \leq 1$.

In this annual discrete-time model, the instrument i_t can be interpreted as a short interest rate that is held constant by the monetary authority from one year to the next. Then i_t can be interpreted as a one-year interest rate controlled by the monetary authority, and $i_t - \pi_{t+1|t}$ as a real one-year interest rate.

The change in inflation is increasing in the lagged output gap. The output gap is serially correlated and decreasing in the lagged real interest rate, $i_t - \pi_{t+1|t}$. The real interest rate affects the output gap with a one-year lag, and hence inflation with a two-year lag, the control lag for inflation in the model. That the instrument affects inflation with a longer lag than it affects the output gap is consistent with results from a number of VAR-studies. The average output gap, $\mathrm{E}[y_t]$, is zero, and the average real interest rate, $\mathrm{E}[i_t - \pi_{t+1|t}]$, is normalized to zero. As clarified in appendix A, the exogenous variable x_t can be interpreted (when $\gamma > 0$) as a persistent disturbance to the natural level of output (in which case η_{t+1} is the difference between a temporary demand shock and a shock to the natural output level), or a persistent disturbance to aggregate demand.[4]

Inflation expectations $\pi_{t+1|t}$ in year t are by (2.1) predetermined and fulfill

$$\pi_{t+1|t} = \pi_t + \alpha_y y_t. \tag{2.4}$$

Using (2.4) in (2.2) results in the reduced form aggregate demand equation

$$y_{t+1} = \beta_y y_t + \beta_x x_t - \beta_r(i_t - \pi_t) + \eta_{t+1}, \tag{2.5}$$

where

$$\beta_y = \tilde{\beta}_y + \alpha_y \beta_r,$$

and $i_t - \pi_t$ may be called a "pseudo-real" repo rate. Thus, the model can be represented by (2.1), (2.5) and (2.3).[5]

[4] A more elaborate model would include a long real interest rate in the aggregate demand function and link the long nominal rate to the repo rate via the expectations hypothesis, for instance as in Fuhrer and Moore (1995).
Strictly speaking, cf. McCallum (1989), the model violates the natural-rate hypothesis (of no long-run effect on output or employment of any monetary policy), in that a steady increasing inflation rate permanently increases the output gap. Such policies will never be optimal with the loss functions to be used in this paper. If such policies would be attempted, the presumption is that the model would break down.

[5] In this form, the model is the same as the one in Taylor (1994), except for the lag in the aggregate demand equation and the exogenous variable (the explicit natural output level in appendix A).

Interpret inflation targeting as monetary policy being conducted by a monetary authority with a long-run inflation target π^* (say 2 percent per year) but with no long-run output gap target (other than the long-run average, zero). Furthermore, in the short-run, the monetary authority wants to reduce inflation fluctuations around the long-run inflation target, and output gap fluctuations around zero.[6] This can be formalized as the monetary authority's intertemporal loss function being

$$\mathrm{E}_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\pi_\tau, y_\tau), \tag{2.6}$$

where E_t denotes expectations conditional upon information available in year t, the discount factor δ fulfills $0 < \delta < 1$, and the period loss function $L(\pi_\tau, y_\tau)$ is

$$L(\pi_\tau, y_t) = \frac{1}{2}\left[(\pi_\tau - \pi^*)^2 + \lambda y_t^2\right], \tag{2.7}$$

where $\lambda \geq 0$ is the weight on output gap stabilization. That is, the monetary authority wishes to minimize the expected sum of discounted squared future deviations of inflation and output from the inflation target and the natural output level, respectively.[7]

In appendix B, it is shown that the first-order condition for minimizing (2.6) over the repo rate can be written

$$\pi_{t+2|t}(i_t) = \pi^* + c(\lambda)\left(\pi_{t+1|t} - \pi^*\right). \tag{2.8}$$

Here $\pi_{t+2|t}(i_t)$ denotes the "two-year conditional inflation forecast", $\mathrm{E}\left[\pi_{t+2}|i_t; \pi_t, y_t, x_t\right]$, the forecast for annual inflation from year $t+1$ to year $t+2$, conditional upon a given instrument level i_t, and conditional upon the predetermined state variables in year t (π_t, y_t and x_t). It is given by

$$\pi_{t+2|t}(i_t) \equiv \pi_t + a_y y_t + a_x x_t - a_r (i_t - \pi_t), \tag{2.9}$$

where

$$a_y = \alpha_y(1+\beta_y),\ a_x = \alpha_y\beta_x \text{ and } a_r = \alpha_y\beta_r. \tag{2.10}$$

The one-year inflation forecast, $\pi_{t+1|t}$, is predetermined and given by (2.4). The coefficient $c(\lambda)$ is a function of the relative weight λ given by

$$c(\lambda) \equiv \frac{\lambda}{\lambda + \delta\alpha_y^2 k(\lambda)} \tag{2.11}$$

[6] Cf. Fischer (1996), King (1996) and Svensson (1997b) on whether inflation targeting also involves implicit goals for output or employment.

[7] Since the central bank does not have perfect control over inflation it is not meaningful to minimize the *realized* squared deviations, only the *expected* squared deviations (conditional upon the information available when the repo rate is set).

and fulfills $0 \leq c(\lambda) < 1$, and the coefficient $k(\lambda)$ is another function of λ given by

$$k(\lambda) \equiv \frac{1}{2}\left(1 - \frac{\lambda(1-\delta)}{\delta\alpha_y^2} + \sqrt{\left(1 + \frac{\lambda(1-\delta)}{\delta\alpha_y^2}\right)^2 + \frac{4\lambda}{\alpha_y^2}}\right) \geq 1. \tag{2.12}$$

Under *strict inflation targeting*, when the weight on output gap stabilization is zero ($\lambda = 0$) and only inflation enters in the loss function, the coefficients fulfill $c(0) = 0$ and $k(0) = 1$. Then the first-order condition simplifies to

$$\pi_{t+2|t}(i_t) = \pi^*. \tag{2.13}$$

The monetary authority should adjust its instrument such that the two-year conditional inflation forecast always equals the inflation target.

Under *flexible inflation targeting*, when there is a positive weight on output gap stabilization ($\lambda > 0$) and both inflation and the output gap enter the loss function, the interpretation of the first-order condition (2.8) is still intuitive. The monetary authority should adjust the instrument such that the deviation of the two-year conditional inflation forecast from the long-run inflation target is a fraction $c(\lambda)$ of the deviation of the pre-determined one-year inflation forecast from the inflation target. Instead of always adjusting the two-year conditional inflation forecast all the way to the long-run inflation target, the monetary authority should adjust the two-year conditional inflation forecast gradually towards the long-run inflation target. The intuition is that this reduces output gap fluctuations, which is apparent from (2.1). The higher the weight on output gap stabilization, the slower the adjustment of the conditional inflation forecast towards the long-run inflation target (the larger the coefficient $c(\lambda)$, see appendix B). The right-hand side of (2.8) can hence be interpreted as a variable short-run target for the two-year inflation forecast.

In general, (2.8) and its variant (2.13) imply that the two-year conditional inflation forecast, the conditional inflation forecast corresponding to the control lag, can be interpreted as an explicit intermediate target. As in Svensson (1997a), I call (2.8) and its variant (2.13) a(n) (intermediate-)*target rule*, a rule that specifies the intermediate-target variable and how its target level is determined. The monetary authority then adjusts the repo rate so as to always fulfill the target rule. If the two-year conditional inflation forecast exceeds (falls short of) the right-hand sides of (2.8) or (2.13), the repo rate should be increased (decreased). This results in an endogenous reaction function, an *instrument rule*, expressing the instrument as a function of current information.[8]

Thus, substitution of the forecasts (2.4) and (2.9) into (2.8) leads to the optimal instrument rule

$$i_t = \pi_t + f_\pi(\lambda)(\pi_t - \pi^*) + f_y(\lambda) y_t + f_x x_t, \tag{2.14}$$

where

$$f_\pi(\lambda) = \frac{1 - c(\lambda)}{\alpha_y \beta_r}, \; f_y(\lambda) = \frac{\beta_y + 1 - c(\lambda)}{\beta_r} \text{ and } f_x = \frac{\beta_x}{\beta_r}. \tag{2.15}$$

The instrument rule (2.14) is of the same form as the Taylor rule (1993, 1996), except that it also depends on the exogenous variable. The pseudo-real repo rate $i_t - \pi_t$ is increasing in the excess of current inflation over the inflation target and in the current output gap. The instrument depends on current variables, not because current variables are targeted (they are predetermined) but because current variables predict future variables. Even if the weight on output gap stabilization is zero, so that only future inflation is targeted, the instrument will depend on all current variables that help predict future inflation.[9]

Note that the instrument rule can also be written as a function of the predetermined one-year inflation expectations, $\pi_{t+1|t}$, rather than in terms of current inflation,

$$i_t = \pi_{t+1|t} + f_\pi(\lambda)(\pi_{t+1|t} - \pi^*) + \tilde{f}_y y_t + f_x x_t,$$

where

$$\tilde{f}_y = \frac{\tilde{\beta}_y}{\beta_r}.$$

Indeed, by leading (2.1) and (2.2) one period and taking expectations,

$$\begin{aligned} \pi_{t+2|t} &= \pi_{t+1|t} + \alpha_y y_{t+1|t} \\ y_{t+1|t} &= \tilde{\beta}_y y_t + \beta_x x_t - \beta_r (i_t - \pi_{t+1|t}), \end{aligned}$$

we realize that we can consistently regard $\pi_{t+1|t}$, y_t and x_t as the relevant state variables, rather than π_t, y_t and x_t.

[8] Bryant, Hooper and Mann (1993) use the terminology "exact targeting" ("full instrument adjustment") and "inexact targeting" ("partial instrument adjustment"). Under the former, the instrument is adjusted to make the intermediate-target variable exactly equal to its desired value. Under the latter, the instrument is only adjusted to partially reduce the deviation of the intermediate-target variable from its desired value.

Strict inflation targeting is then an example of exact targeting, since the instrument is adjusted to make the intermediate target $\pi_{t+2|t}(i_t)$ exactly equal to its desired value, π^*. Flexible inflation targeting can be seen as either exact targeting or inexact targeting, depending upon whether the "desired value" is identified with the *short*-run target level, $\pi^* + c(\lambda)(\pi_{t+1|t} - \pi^*)$, or the *long*-run target level, π^*.

[9] See Broadbent (1996) for an insightful discussion of Taylor rules in relation to inflation targeting. See also the comment by Svensson (1996) on Taylor (1996).

Actual inflation in year $t+2$ will unavoidably deviate from the inflation target and the two-year conditional inflation forecast by a forecast error,

$$\pi_{t+2} - \pi_{t+2|t} = \varepsilon_{t+1} + \alpha_y \eta_{t+1} + \varepsilon_{t+2}, \tag{2.16}$$

due to disturbances that occur within the control lag, after the monetary authority has set the instrument. Here $\pi_{t+2|t}$ denotes the two-year inflation forecast (2.9) conditional upon the instrument rule (2.14),

$$\pi_{t+2|t} \equiv \pi_{t+2|t} \left[\pi_t + f_\pi (\pi_t - \pi^*) + f_y y_t + f_x x_t \right].$$

From (2.8) and (2.4) the two-year inflation forecast will follow

$$\pi_{t+2|t} - \pi^* = c(\lambda) (\pi_t - \pi^*) + c(\lambda) \alpha_y y_t.$$

From (2.5), (2.14) and (2.15), the output gap will follow

$$\begin{aligned} y_{t+1} &= \beta_y y_t + \beta_x x_t - \beta_r \left[f_\pi (\pi_t - \pi^*) + f_y y_t + f_x x_t \right] + \eta_{t+1} \\ &= -\frac{1 - c(\lambda)}{\alpha_y} (\pi_t - \pi^*) - [1 - c(\lambda)] \, y_t + \eta_{t+1}. \end{aligned}$$

To generalize from this model, inflation targeting implies that the conditional inflation forecast for a horizon corresponding to the control lag (two years in the model) becomes an intermediate target. Under strict inflation targeting (no weight on output gap stabilization), the instrument should be set so as to make the conditional inflation forecast equal to the inflation target. Under flexible inflation targeting (some weight on output gap stabilization), the instrument should be set so as to make the two-year conditional inflation forecast approach the long-run inflation target gradually. This behavior results in the optimal reaction function of the monetary authority. Since the conditional inflation forecast depends on all relevant information, the instrument will be a function of all relevant information.

The monetary authority's conditional inflation forecast must, in practice, combine both formal and informal components, for instance with judgemental adjustments of more formal structural forecasts. Forecasts will hardly ever be purely mechanical. This view is strengthened by the results of Cecchetti (1995), who has examined mechanical reduced-form inflation forecasts for the United States, with rather negative results. Forecast errors are sizeable, and there are frequent structural shifts in the forecast equations. However, forecast errors for one-year inflation rates, for instance for the one-to-two-year inflation rate emphasized in the model used here, are smaller than for one-quarter inflation rates. As emphasized by Kohn (1995), more structural modeling and use of extramodel information and judgment by forecasters are likely to produce forecasts with acceptable precision. In addition, forecasting inflation may be easier in a situation when the monetary authority actively pursues inflation targeting and the public expects the monetary authority to pursue inflation targeting so that inflation expectations are stabilized.[10] This is illustrated in section 7 which deals with forward-looking behavior.

3 Response to shocks

How should monetary policy react to shocks?[11] The conventional wisdom is that monetary policy should neutralize aggregate demand shocks, since these move inflation and the output gap in the same direction. With regard to supply shocks, the conventional wisdom is that the response depends on the weight on output gap stabilization. With a positive weight, it is optimal to partially accommodate supply shocks, since they affect inflation and the output gap in opposite directions. With a zero weight, the supply shock effect on inflation is neutralized, even though this enhances the effect on the output gap.

When lags are taken into account, the conventional wisdom must be modified. First, the monetary authority cannot affect the first-round effects on inflation and the output gap of supply and demand shocks, due to the lags. It can only mitigate the second-round effects. Second, the reaction to temporary demand and supply shocks appears more symmetric. Third, the reaction to both shocks differs with the weight on output gap stabilization. Under strict inflation targeting (with a zero weight on output gap stabilization), the two-year conditional inflation forecast is brought in line with the long-run inflation target, regardless of how the shocks have affected the one-year inflation forecast. Hence, shocks are not allowed to let the two-year conditional inflation forecast deviate from the long-run target. Under flexible inflation targeting (with a positive weight on output gap stabilization), the two-year conditional inflation forecast is adjusted less in response to the shocks. The effect of these shocks on future inflation is only gradually eliminated.

A general, and operational, way to determine the appropriate response to the shocks is to "filter the shocks through the conditional inflation forecast, and then take appropriate action."

[10] For instance, the Reserve Bank of New Zealand has been able to keep the underlying inflation rate in New Zealand within a 1.3 percentage point range since 1991 (Reserve Bank of New Zealand (1996)).

[11] See Freedman (1996) for a more detailed discussion of the optimal response to shocks under inflation targeting, including the response to different shifts in inflation expectations. Such shifts can be examined in the forward-looking model in section 7.

More specifically, the effects of the shocks on the one-year and two-year inflation forecasts are assessed, and then the instrument is adjusted so that the first-order condition (2.8) still holds.

In order to see this, consider shocks in year t. By (2.4) these shocks will change the one-year inflation forecast by

$$\begin{aligned}\pi_{t+1|t} - \pi_{t+1|t-1} &= (\pi_t - \pi_{t|t-1}) + \alpha_y(y_t - y_{t|t-1}) \\ &= \varepsilon_t + \alpha_y(\tilde{\eta}_t - \xi_t), \qquad (3.1)\end{aligned}$$

where I use the more elaborate model in appendix A in which the shock to the output gap,

$$\eta_t = \tilde{\eta}_t - \xi_t,$$

consists of the difference between a temporary demand shock, $\tilde{\eta}_t$, and a shock to the natural output level, ξ_t. By the analog of (2.9) in appendix A, (A.9), the shocks will change the two-year conditional inflation forecast by

$$\begin{aligned}\pi_{t+2|t} - \pi_{t+2|t-1} &= \Big[(1+a_r)(\pi_t - \pi_{t|t-1}) + a_y(y_t - y_{t|t-1}) + a_z(z_t - z_{t|t-1}) + a_n(y_t^n - y_{t|t-1}^n)\Big] \\ &\quad - a_r(i_t - i_{t|t-1}) \\ &= [(1+a_r)\varepsilon_t + a_y(\tilde{\eta}_t - \xi_t) + a_z\zeta_t + a_n\xi_t] - a_r(i_t - i_{t|t-1}), \qquad (3.2)\end{aligned}$$

where z_t is a persistent demand disturbance, ζ_t is a shock to this demand disturbance, y_t^n is (the log of) the natural output level, and the coefficients a_r, a_y, a_z and a_n are given by (A.10)–(A.13). The term within brackets in (3.2) is the change in the two-year conditional inflation forecast due to the shocks, and the other term is the change due to the change in the instrument, $i_t - i_{t|t-1}$.

The changes in the one-year and two-year inflation forecasts must obey the first-order condition (2.8), which implies that they must fulfill

$$\pi_{t+2|t} - \pi_{t+2|t-1} = c(\lambda)\left(\pi_{t+1|t} - \pi_{t+1|t-1}\right). \qquad (3.3)$$

Thus, (3.1)–(3.3) determine the required change in the instrument.

Solving for the instrument change results in

$$\begin{aligned}i_t - i_{t|t-1} &= \frac{[(1+a_r)\varepsilon_t + a_y(\tilde{\eta}_t - \xi_t) + a_z\zeta + a_n\xi_t] - c(\lambda)\left[\varepsilon_t + \alpha_y(\tilde{\eta}_t - \xi_t)\right]}{a_r} \\ &= \frac{[1+\alpha_y\beta_r - c(\lambda)]\,\varepsilon_t + \alpha_y[1+\beta_y - c(\lambda)]\tilde{\eta}_t - \alpha_y\left[1+\alpha_y\beta_r + \gamma_n - c(\lambda)\right]\xi_t + \alpha_y\beta_z\zeta_t}{\alpha_y\beta_r},\end{aligned}$$

(3.4)

where I have used (A.5)–(A.6) and (A.10)–(A.13), and γ_n $(0 \leq \gamma_n \leq 1)$ is the degree of persistence of the natural output level (for $\gamma_n = 1$ the natural output level is a random walk). The numerator in (3.4) is the change in the two-year conditional inflation forecast caused by the shocks, less the fraction $c(\lambda)$ of the change in the one-year inflation forecast due to the shock. The denominator is the policy multiplier of the instrument for the two-year conditional inflation forecast.

We see that the response to the shocks vary with the relative weight on output gap stabilization, λ, via the effect on the coefficient $c(\lambda)$. A positive inflation shock, ε_t, and a positive temporary demand shock, $\tilde{\eta}_t$, both motivate an increase in the instrument. Those increases are smaller with a higher weight on output stabilization, since $c(\lambda)$ is increasing in λ. A positive shock to the natural output level, ξ_t, motivates a fall in the instrument. The fall is larger for more persistence, γ_n, and a lower relative weight on output gap stabilization, λ. A shock to the persistent demand disturbance, ζ_t, leads to an increase in the instrument, independent of the weight on output gap stabilization.

The response coefficients for the shocks in (3.4) are of course the same coefficients as in the instrument rule for the more elaborate model in appendix A, (A.14).

4 Model uncertainty

In this section, I consider model uncertainty, in the form of uncertainty about the coefficients in the model (2.1)–(2.3). Let me simplify the model somewhat by disregarding the exogenous variable ($\beta_x = 0$). Restate the model as

$$\pi_{t+1} = \pi_t + \alpha_{yt} y_t + \varepsilon_{t+1} \tag{4.1}$$

$$y_{t+1} = \tilde{\beta}_{yt} y_t - \beta_{rt} \left(i_t - \pi_{t+1|t} \right) + \eta_{t+1}, \tag{4.2}$$

where the coefficients α_y, $\tilde{\beta}_y$ and β_r have been dated according to the year they refer to. For simplicity, consider only the case of strict inflation targeting ($\lambda = 0$). Then the problem to minimize (2.6) simplifies to the period-by-period problem (see Svensson (1997a) for details)

$$\min_{i_t} \delta^2 \mathrm{E}_t \left[\frac{1}{2} \left(\pi_{t+2} - \pi^* \right)^2 \right]$$

subject to

$$\pi_{t+2} = \pi_{t+2|t} \left(i_t \right) + \varepsilon_{t+1} + \alpha_{y,t+1} \eta_{t+1} + \varepsilon_{t+2},$$

where

$$\begin{aligned}
\pi_{t+2|t}(i_t) &= \pi_{t+1|t} + \alpha_{y,t+1} y_{t+1|t} \\
&= \pi_{t+1|t} + \tilde{a}_{y,t+1} y_t - a_{r,t+1}\left(i_t - \pi_{t+1|t}\right) \\
\pi_{t+1|t} &= \pi_t + \alpha_{yt} y_t, \qquad (4.3)
\end{aligned}$$

and where I use the notation

$$\tilde{a}_{y,t+1} = \alpha_{y,t+1}\tilde{\beta}_{yt} \text{ and } a_{r,t+1} = \alpha_{y,t+1}\beta_{rt}$$

and observe that $\pi_{t+1|t}$ is predetermined.

Assume first that the coefficients α_{yt}, $\tilde{\beta}_{yt}$ and β_{rt} remain constant. Then the first-order condition for this problem is the target rule

$$\pi_{t+2|t}(i_t) = \pi^*, \qquad (4.5)$$

as we saw in section 2.

Now, following the classic analysis of Brainard (1967) (the relevance of which has recently been emphasized by Blinder (1995), see also Chow (1975, chapt. 10)), consider the alternative problem when there is model uncertainty in the form of uncertainty in year t, when the instrument is chosen, about the coefficient $\tilde{a}_{y,t+1}$ and the policy multiplier $a_{r,t+1}$, resulting from uncertainty about the coefficients α_{yt}, $\tilde{\beta}_{yt}$ and β_{rt}. More precisely, let the α_{yt} be known at t, and let

$$\begin{aligned}
\alpha_{y,t+1} &= \alpha_y + \nu_{\alpha y,t+1} \\
\tilde{\beta}_{yt} &= \tilde{\beta}_y + \nu_{\beta yt} \\
\beta_{rt} &= \beta_r + \nu_{\beta rt}
\end{aligned}$$

where $\nu_{\alpha y,t+1}$, $\nu_{\beta yt}$ and $\nu_{\beta rt}$ are i.i.d. stochastic disturbances with zero means and given variances/covariances. The realizations of these disturbances become known in year $t+1$. For simplicity, assume that $\nu_{\alpha y,t+1}$ is uncorrelated with $\nu_{\beta yt}$ and $\nu_{\beta rt}$. Then we can write

$$\begin{aligned}
\tilde{a}_{y,t+1} &= \tilde{a}_y + \nu_{y,t+1} \\
a_{r,t+1} &= a_r + \nu_{r,t+1},
\end{aligned}$$

where $\nu_{y,t+1}$ and $\nu_{r,t+1}$ are zero mean i.i.d. disturbances, and

$$\tilde{a}_y = \alpha_y\tilde{\beta}_y,\; a_r = \alpha_y\beta_r. \qquad (4.6)$$

Thus, in year t, the parameters in the current Phillips curve are known, but not those of next year's Phillips curve, and not those of the current aggregate demand equation. These are instead known in year $t+1$. That is, we assume that all uncertainty relevant for the policy decision in year t is resolved in year $t+1$. In particular, there is a new realization of the stochastic disturbance terms each year, with unchanged variances and covariances. Therefore, there is nothing that can be learned to reduce the uncertainty, and there is no point in experimenting in order to learn more about the stochastic properties of the model. The fact that there is no role for experimentation and learning simplifies the analysis considerably.[12]

Under these assumptions, the constraint in year t can be written

$$\pi_{t+2} = \pi_{t+1|t} + (\tilde{a}_y + \nu_{y,t+1})\, y_t - (a_r + \nu_{r,t+1}) \left(i_t - \pi_{t+1|t}\right) + \varepsilon_{t+1} + \alpha_{y,t+1}\eta_{t+1} + \varepsilon_{t+2}, \quad (4.7)$$

where the one-year inflation forecast, $\pi_{t+1|t}$, remains predetermined and given by (4.3). Let $\nu_{y,t+1}$ and $\nu_{r,t+1}$ have variances and covariance σ_y^2, σ_r^2 and σ_{yr}, respectively.[13] Furthermore, let the covariance of $\nu_{r,t+1}$ with $\varphi_{t+1} \equiv \varepsilon_{t+1} + \alpha_y\eta_{t+1}$ be $\sigma_{\varphi r}$. It follows that the two-year conditional inflation forecast is given by

$$\pi_{t+2|t}(i_t) \equiv \pi_{t+1|t} + \tilde{a}_y y_t - a_r \left(i_t - \pi_{t+1|t}\right). \quad (4.8)$$

With the constraint (4.7), the first-order condition is

$$\begin{aligned}
0 &= \delta^2 \mathrm{E}_t \left[(\pi_{t+2} - \pi^*) \frac{\partial \pi_{t+2}}{\partial i_t} \right] \\
&= -\delta^2 \mathrm{E}_t \Big\{ \Big[\pi_{t+1|t} + (\tilde{a}_y + \nu_{y,t+1})\, y_t - (a_r + \nu_{r,t+1})(i_t - \pi_{t+1|t}) \\
&\qquad + \varphi_{t+1} + \varepsilon_{t+2} - \pi^* \Big] \Big\} (a_r + \nu_{r,t+1}) \\
&= -\delta^2 \left(\pi_{t+2|t}(i_t) - \pi^*\right) a_r - \delta^2 \sigma_{yr} y_t + \delta^2 \sigma_r^2 \left(i_t - \pi_{t+1|t}\right) - \delta^2 \sigma_{\varphi r}.
\end{aligned}$$

We can rewrite the first-order condition as

$$\pi_{t+2|t}(i_t) - \pi^* = -\frac{\sigma_{yr}}{a_r} y_t + \frac{\sigma_r^2}{a_r}\left(i_t - \pi_{t+1|t}\right) - \frac{\sigma_{\varphi r}}{a_r}. \quad (4.9)$$

It is clear that with multiplier uncertainty, the variances and covariances of the multiplier will affect the solution and make it deviate from (4.5). The standard certainty-equivalence in the linear-quadratic model breaks down.

[12] On learning and experimenting, see, for instance, Prescott (1972), Chow (1975, chapt. 11), Bertocchi and Spagat (1993) and Balvers and Cosimano (1994).

[13] If there is uncertainty in $\alpha_{y,t+1}$ (or $\tilde{\beta}_{yt}$) alone with variance $\sigma_{\alpha y}^2$ (or $\sigma_{\beta y}^2$), we have $\sigma_r^2 = \beta_r^2 \sigma_{\alpha y}^2$ (or $\sigma_r^2 = 0$) and $\sigma_{yr} = \tilde{\beta}_y \beta_r \sigma_{\alpha y}^2$ (or $\sigma_{yr} = 0$). If there is uncertainty in β_{rt} alone, with variance $\sigma_{\beta r}^2$, we have $\sigma_r^2 = \alpha_y^2 \sigma_{\beta r}^2$ and $\sigma_{yr} = 0$.

We can discuss the optimal policy either in terms of target rules or instrument rules. Let us first look at instrument rules. Using (4.8) in (4.9), we can solve for the optimal instrument rule,

$$i_t = \pi_{t+1|t} + \frac{1}{(1+v_r)\,a_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{\tilde{a}_y + \sigma_{yr}/a_r}{(1+v_r)\,a_r} y_t + \frac{\sigma_{\varphi r}/a_r}{(1+v_r)\,a_r}, \tag{4.10}$$

where

$$v_r = \frac{\sigma_r^2}{a_r^2}$$

is the coefficient of variation of the policy multiplier a_r.

In order to interpret the instrument rule (4.10), consider the special case of "independent multiplier uncertainty", when $\sigma_r^2 > 0$, but ν_r is not correlated with ν_y or φ, that is, $\sigma_{yr} = \sigma_{\varphi r} = 0$. This is the case when there is uncertainty in β_{rt} alone, and when β_{rt} is uncorrelated with φ_{t+1}. Then (4.10) simplifies to

$$i_t = \pi_{t+1|t} + \frac{1}{(1+v_r)\,a_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{\tilde{a}_y}{(1+v_r)\,a_r} y_t. \tag{4.11}$$

We see that more uncertainty (a higher coefficient of variation v_r) leads to a more "conservative" and less activist policy, in the sense of reducing the magnitude of the response coefficients.

In order to interpret the policy further, consider two extreme cases. First, consider the case with no (policy) multiplier uncertainty ($\sigma_r^2 = 0$), as in section 2. Then $v_r = 0$, and the instrument rule is

$$i_t = i_t^0 \equiv \pi_{t+1|t} + \frac{1}{a_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{\tilde{a}_y}{a_r} y_t, \tag{4.12}$$

which I call the "no-multiplier-uncertainty" policy.

Next, consider the other extreme, with infinite uncertainty ($\sigma_r^2 \to \infty$). The model and its policy are of course meaningless with unbounded uncertainty, so this case only serves as a hypothetical reference point. It follows from (4.11) that the optimal policy is then to set the interest equal to expected inflation, so as to make the real interest rate equal to zero,

$$i_t = i^\infty \equiv \pi_{t+1|t}. \tag{4.13}$$

I call this the "infinite-multiplier-uncertainty" policy. Intuitively, with large uncertainty in the coefficient β_{rt} in (4.2), it is best to choose the instrument so that the real interest rate is close to zero, in order to limit the variability of inflation. For infinite uncertainty, when the real interest rate is held constant at zero, inflation becomes non-stationary.[14]

[14] The appropriate response when uncertainty becomes very large is of course dependent on the precise model and nature of the uncertainty. From (4.1) and (4.2) it is apparent that if the uncertainty is in α_{yt} or in $\tilde{\beta}_{yt}$ rather than in β_{rt}, the appropriate response with infinite uncertainty is to set the instrument such that $y_{t+1|t} = 0$, rather than $i_t - \pi_{t+1|t} = 0$.

The instrument rule (4.11) can now be written as a convex combination of the no-multiplier-uncertainty instrument rule and the infinite-multiplier-uncertainty instrument rule,

$$i_t = \frac{1}{1+v_r} i_t^0 + \frac{v_r}{1+v_r} i_t^{\infty}. \tag{4.14}$$

Thus, the monetary authority is more conservative with independent multiplier uncertainty than without any multiplier uncertainty, in the sense that its policy is an average of the policy without uncertainty and the policy for infinite uncertainty (which makes the real interest rate equal to its long-run average).

Next, we shall look at this in terms of target rules. The two-year conditional inflation forecast that corresponds to the no-multiplier-uncertainty policy and the infinite-multiplier-uncertainty policy is $\pi_{t+2|t}\left(i_t^0\right) = \pi^*$ and

$$\pi_{t+2|t}\left(i_t^{\infty}\right) = \pi_{t+2|t}^{\infty} \equiv \pi_{t+1|t} + \tilde{a}_y y_t,$$

respectively. Since the two-year conditional inflation forecast is linear in the instrument, it follows that it will be a convex combination of the long-run inflation target and the infinite-multiplier-uncertainty two-year conditional inflation forecast with the same weight as in (4.14),

$$\begin{aligned}
\pi_{t+2|t}\left(i_t\right) &= \frac{1}{1+v_r}\pi^* + \frac{v_r}{1+v_r}\pi_{t+2|t}^{\infty} \\
&= \pi^* + \frac{v_r}{1+v_r}\left(\pi_{t+2|t}^{\infty} - \pi^*\right) \\
&= \pi^* + \frac{v_r}{1+v_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{v_r \tilde{a}_y}{1+v_r} y_t.
\end{aligned}$$

Thus, the two-year conditional inflation forecast deviates from the inflation target by a fraction $\frac{v_r}{1+v_r}$ of the deviation of the infinite-multiplier-uncertainty two-year inflation forecast from the inflation target. Equivalently, the two-year conditional inflation forecast deviates from the inflation target by the sum of the same fraction of the deviation of the *one*-year inflation forecast from the inflation target and a term proportional to the output gap. In the case of flexible inflation targeting, the two-year conditional inflation forecast is only gradually adjusted towards the inflation target.

In the general case, when multiplier uncertainty is not independent, the policy (4.10) involves a constant, $\frac{\sigma_{\varphi r}/a_r}{(1+v_r)a_r}$. The coefficient of y_t is also modified, and affected by the covariance σ_{yr}. The constant will make average inflation deviate from the long-run inflation target. The long-run average follows directly from (4.9) and fulfills

$$\mathrm{E}\left[\pi_t\right] = \pi^* - \frac{\sigma_{\varphi r}}{a_r}, \tag{4.15}$$

where I have used that $\mathrm{E}[y_t] = 0$ and $\mathrm{E}[i_t] = \mathrm{E}[\pi_t]$. Thus, the average inflation deviates from the inflation target, the bias being positive or negative depending on the sign of the covariance between the policy multiplier and the disturbance to inflation, $\sigma_{\varphi r}$. The two-year conditional inflation forecast will be

$$\begin{aligned}\pi_{t+2|t}(i_t) &= \pi^* + \frac{v_r}{1+v_r}\left(\pi^{\infty}_{t+2|t} - \pi^*\right) - \frac{\sigma_{yr}/a_r}{1+v_r} y_t - \frac{\sigma_{\varphi r}/a_r}{1+v_r} \\ &= \pi^* + \frac{v_r}{1+v_r}\left(\pi_{t+1|t} - \pi^*\right) + \frac{v_r \tilde{a}_y - \sigma_{yr}/a_r}{1+v_r} y_t - \frac{\sigma_{\varphi r}/a_r}{1+v_r}.\end{aligned}$$

The two-year conditional inflation forecast is mean-reverting and gradually adjusted towards (4.15).

In summary, model uncertainty in the form of policy-multiplier uncertainty motivates deviations from the long-run inflation target. Under strict inflation targeting, without any multiplier uncertainty, the two-year conditional inflation forecast should always equal the long-run inflation target. With independent policy-multiplier uncertainty, the optimal policy is a convex combination of the no-multiplier-uncertainty policy and the infinite-multiplier-uncertainty policy, which results in the two-year conditional inflation forecast being gradually adjusted towards the long-run inflation target. When policy-multiplier uncertainty is not independent, there may be a bias in average inflation, and the response of the two-year conditional inflation forecast to the output gap is modified.

5 Interest rate stabilization and smoothing

How is inflation targeting affected by attempts to stabilize and/or smooth the instrument?[15] Modify the period loss function to

$$L(\pi_t, y_t, i_t, i_t - i_{t-1}) = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda y_t^2 + \mu\,(i_t - \pi_t)^2 + \nu\,(i_t - i_{t-1})^2\right]. \tag{5.1}$$

This allows for a weight $\mu \geq 0$ on stabilizing the pseudo-real rate, $i_t - \pi_t$, as well as a weight $\nu \geq 0$ on smoothing the instrument (stabilizing the first-difference of the instrument). Alternatives to stabilizing the pseudo-real rate are, of course, to stabilize the real interest rate, $i_t - \pi_{t+1|t}$, or the nominal interest rate i_t itself. Since other variables than inflation enters the loss function, this is another case of flexible inflation targeting.

Minimizing the intertemporal loss function (2.6) with the period loss function (2.7) replaced by (5.1) generally seems to require a numerical solution of the standard linear-quadratic optimal

[15] See Goodhart (1996) for a recent discussion of interest rate smoothing.

control problem, see appendix C. In particular, when $\nu > 0$, the lagged instrument enters as a state variable, which together with inflation and the output gap brings the number of state variables to three (excluding the exogenous variable).

In order to gain some insight into the effects of interest rate stabilization and smoothing, without having to resort to numerical analysis, let me make a few simplifications. First, the weight on output stabilization is set to zero. Second, each period the monetary authority solves the simple problem

$$\min_{i_t} \frac{1}{2}\left[\delta^2\left(\pi_{t+2}-\pi^*\right)^2+\mu\left(i_t-\pi_t\right)^2+\nu(i_t-i_{t-1})^2\right] \tag{5.2}$$

subject to

$$\pi_{t+2}=\pi_t+a_y y_t-a_r(i_t-\pi_t)+\varepsilon_{t+1}+\alpha_y\eta_{t+1}+\varepsilon_{t+2},$$

where I use (2.9), (2.10) and (2.16), and for simplicity disregard the exogenous variable ($a_x = \beta_x = 0$).

The monetary authority is assumed to minimize the loss function in (5.2) each period, taking last year's interest rate as given, but disregarding that today's instrument setting will affect next year's loss function. When $\lambda = \mu = \nu = 0$, this problem is equivalent to the general intertemporal problem, as demonstrated in Svensson (1997a). When either μ or ν differs from zero, this is no longer true. Nevertheless, the simple case of (5.2) helps to understand the general consequences of instrument stabilization and smoothing.

The first-order condition is

$$\delta^2\left(\pi_{t+2|t}\left(i_t\right)-\pi^*\right)(-a_r)+\mu(i_t-\pi_t)+\nu(i_t-i_{t-1})=0. \tag{5.3}$$

We can write the first-order condition as

$$\pi_{t+2|t}\left(i_t\right)=\pi^*+\frac{\mu}{\delta^2 a_r}\left(i_t-\pi_t\right)+\frac{\nu}{\delta^2 a_r}(i_t-i_{t-1}),$$

and observe that when μ or ν differ from zero, the two-year conditional inflation forecast will generally deviate from the inflation target.

We can solve for the instrument rule and get

$$i_t=\frac{\mu+\delta^2 a_r^2}{\mu+\nu+\delta^2 a_r^2}\pi_t+\frac{\nu}{\mu+\nu+\delta^2 a_r^2}i_{t-1}+\frac{\delta^2 a_r}{\mu+\nu+\delta^2 a_r^2}(\pi_t-\pi^*)+\frac{\delta^2 a_r a_y}{\mu+\nu+\delta^2 a_r^2}y_t. \tag{5.4}$$

With a zero weight on instrument smoothing ($\nu = 0$), the instrument rule does not depend on the lagged interest rate and is given by

$$i_t=\pi_t+\frac{\delta^2 a_r}{\mu+\delta^2 a_r^2}(\pi_t-\pi^*)+\frac{\delta^2 a_r a_y}{\mu+\delta^2 a_r^2}y_t.$$

Hence, the effect of a positive weight on (pseudo-real) interest rate stabilization is simply to reduce the coefficients of $\pi_t - \pi^*$ and y_t.

With a zero weight on interest rate stabilization ($\mu = 0$), the instrument rule depends on the lagged interest rate and becomes

$$i_t = \frac{\delta^2 a_r^2}{\nu + \delta^2 a_r^2}\pi_t + \frac{\nu}{\nu + \delta^2 a_r^2} i_{t-1} + \frac{\delta^2 a_r}{\nu + \delta^2 a_r^2}(\pi_t - \pi^*) + \frac{\delta^2 a_r a_y}{\nu + \delta^2 a_r^2} y_t.$$

Note that the instrument rule is not simply a rule for the first-difference of the instrument.[16]

In this simple case, the instrument rule has an interesting interpretation. Let i_t^π denote the instrument rule under strict inflation targeting, when $\mu = \nu = 0$. It is given by

$$i_t^\pi = \pi_t + \frac{1}{a_r}(\pi_t - \pi^*) + \frac{a_y}{a_r} y_t. \tag{5.5}$$

Furthermore, let i_t^i and $i_t^{\Delta i}$ denote the instrument rules under strict pseudo-real interest rate stabilization ($\mu \to \infty$, $\nu = 0$) and strict interest rate smoothing ($\nu \to \infty$, $\mu = 0$), respectively. They are given by $i_t^i = \pi_t$ and $i_t^{\Delta i} = i_{t-1}$. Then the optimal instrument rule can be written as a convex combination of the three rules,

$$i_t = \frac{\delta^2 a_r^2}{\mu + \nu + \delta^2 a_r^2} i_t^\pi + \frac{\mu}{\mu + \nu + \delta^2 a_r^2} i_t^i + \frac{\nu}{\mu + \nu + \delta^2 a_r^2} i_t^{\Delta i}. \tag{5.6}$$

Let $\pi_{t+2|t}^j$, $j = \pi,\ i,\ \Delta i$, denote the two-year conditional inflation forecast that corresponds to each strict rule. They are given by

$$\begin{aligned}
\pi_{t+2|t}^\pi &= \pi^* \\
\pi_{t+2|t}^i &= \pi_t + a_y y_t = \pi_t + \alpha_y y_t + \tilde{a}_y y_t = \pi_{t+1|t} + \tilde{a}_y y_t. \\
\pi_{t+2|t}^{\Delta i} &= \pi_{t+1|t} - a_r(i_{t-1} - \pi_t),
\end{aligned}$$

where I use that by (2.10) and (4.6) $a_y = \alpha_y + \tilde{a}_y$. It follows that the two-year conditional inflation forecast is the same convex combination of these three forecasts,

$$\begin{aligned}
\pi_{t+2|t}(i_t) &= \frac{\delta^2 a_r^2}{\mu + \nu + \delta^2 a_r^2}\pi_{t+2|t}^\pi + \frac{\mu}{\mu + \nu + \delta^2 a_r^2}\pi_{t+2|t}^i + \frac{\nu}{\mu + \nu + \delta^2 a_r^2}\pi_{t+2|t}^{\Delta i} \qquad (5.7)\\
&= \pi^* + \frac{\mu}{\mu + \nu + \delta^2 a_r^2}\left(\pi_{t+2|t}^i - \pi^*\right) + \frac{\nu}{\mu + \nu + \delta^2 a_r^2}\left(\pi_{t+2|t}^{\Delta i} - \pi^*\right) \\
&= \pi^* + \frac{\mu + \nu}{\mu + \nu + \delta^2 a_r^2}\left(\pi_{t+1|t} - \pi^*\right) + \frac{\mu + \nu}{\mu + \nu + \delta^2 a_r^2}\tilde{a}_y y_t - \frac{\nu}{\mu + \nu + \delta^2 a_r^2} a_r(i_{t-1} - \pi_t). \qquad (5.8)
\end{aligned}$$

[16] In the monetary policy literature, it is quite common to consider only restricted classes of rules. Typical restrictions are that the instrument, or the change in the instrument, is a linear function of the deviation of a target variable from a target level only (cf. Bryant, Hooper and Mann (1993)). The above illustrates that such restricted rules are generally not optimal.

Equations (5.7)–(5.8) can be interpreted as equivalent forms of a target rule for the two-year conditional inflation forecast, implying that the two-year conditional inflation forecast is gradually adjusted to the inflation target.

Generally, concerns about interest stabilization and smoothening leads to a less active policy. The two-year conditional inflation forecast, as for flexible inflation targeting, is adjusted gradually towards the inflation target. Numerical analysis of the general intertemporal problem confirms this insight.

As far as I can see, the result that an instrument rule can be written as a convex combination of strict instrument rules does not necessarily hold in the general intertemporal problem. In some special cases the result holds, but the weights are more complicated to determine.[17]

6 Nominal GDP targeting

Nominal GDP targeting is recommended by several researchers, for instance, Bean (1983), McCallum (1989), Hall and Mankiw (1994) and Feldstein and Stock (1994). Nominal GDP targeting is easily examined in the current framework.

Let me define g_t and Y_t as the (log) nominal GDP (gap) growth and nominal GDP (gap) level, respectively. That is,

$$\begin{aligned} g_t &= \pi_t + y_t - y_{t-1} \\ Y_t &= p_t + y_t. \end{aligned}$$

Then nominal GDP growth targeting with a nominal GDP target growth rate g^* can be interpreted as having the period loss function

$$L(\pi_t, y_t, y_{t-1}) = \frac{1}{2}\left(\pi_t + y_t - y_{t-1} - g^*\right)^2. \tag{6.1}$$

Similarly, nominal GDP level targeting with a nominal GDP target level Y^* can be interpreted has having the period loss function

$$L(p_t, y_t) = \frac{1}{2}\left(p_t + y_t - Y^*\right)^2. \tag{6.2}$$

Ball (1996) has demonstrated a somewhat surprising result. Both nominal GDP growth and level targeting lead to instability of inflation and the output gap in the present model . This section will restate and discuss Ball's result.

[17] Broadbent (1996) observes for simple loss functions that the instrument rule can be written as a convex combination of pure instrument rules.

Disregard, for simplicity the exogenous variable, and let the model be given by (2.1) and (2.5), the latter with $\beta_x = 0$. Consider first nominal GDP growth targeting, so the monetary authority's problem is to minimize (2.6) where the period loss function (2.6) is replaced by (6.1). We realize that a first-order condition for this problem is

$$\pi_{t+1|t} + y_{t+1|t} - y_t = g^*. \tag{6.3}$$

In year t the monetary authority controls $y_{t+1|t}$ whereas $\pi_{t+1|t}$ and y_t are predetermined. The monetary authority can hence achieve the desired nominal GDP growth rate in year $t+1$ in expectation.

Ball shows that (6.3) implies that y_t and π_t become unstable, in spite of the nominal GDP growth target being achieved in expectation. In order to see this, rewrite (6.3) as

$$y_{t+1|t} = -\left(\pi_{t+1|t} - g^*\right) + y_t = -\left(\pi_t - g^*\right) + \left(1 - \alpha_y\right) y_t.$$

We then realize that the dynamics of π_t and y_t are given by the system

$$\begin{bmatrix} \pi_{t+1} - g^* \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \alpha_y \\ -1 & 1 - \alpha_y \end{bmatrix} \begin{bmatrix} \pi_t - g^* \\ y_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \end{bmatrix}.$$

The eigenvalues are the roots μ of the characteristic equation

$$0 = \begin{vmatrix} \mu - 1 & -\alpha_y \\ 1 & \mu - 1 + \alpha_y \end{vmatrix} = \mu^2 - \left(2 - \alpha_y\right)\mu + 1. \tag{6.4}$$

Appendix D shows that for $\alpha_y \leq 4$ both roots are complex and on the unit circle, whereas for $\alpha_y > 4$ one root is outside and one root inside the unit circle. Hence, in both cases both π_t and y_t are unstable. In contrast, nominal GDP growth g_t fulfills

$$g_t = \pi_t + \left(y_t - y_{t-1}\right) = g^* + \varepsilon_t + \eta_t$$

and is stationary.

What is the intuition for this result? First, nominal GDP growth targeting implies that there is a *constant* marginal rate of substitution (equal to unity) between inflation and output growth in the period loss function, since only the sum of these matter. Hence, in contrast to the period loss function (2.7), there is no loss associated with divergent inflation and output growth, as long as the sum remains stationary.

Second, the realistic property of this model that the control lags for output and inflation are different, creates problems for nominal GDP growth targeting. Consider an initial situation when current inflation and forecasts of future inflation rates are all g^*, and the current output gap and forecasts of future output gaps are all zero, so forecasts of future nominal GDP growth rates are all on target. Then, suppose there is a positive shock, $\varepsilon_t > 0$, to current inflation. This will increase the inflation forecast $\pi_{t+1|t}$ by the same magnitude. The appropriate monetary policy response is now to increase the interest rate i_t, in order to reduce the output gap forecast $y_{t+1|t}$ by the same magnitude. This way the nominal GDP growth forecast remains on target.

Now consider next year, $t+1$, and the outlook for nominal GDP growth in year $t+2$. Suppose, for simplicity, that both disturbances ε_{t+1} and η_{t+1} are zero in year $t+1$. The inflation forecast $\pi_{t+2|t+1}$ is given by

$$\pi_{t+2|t+1} = \pi_{t+1} + \alpha_y y_{t+1}.$$

Because of the shock to inflation in year t, inflation in year $t+1$ is ε_t. Output in year $t+1$ has fallen to $-\varepsilon_t$, due to the increase in the interest rate in year t. Assume, realistically, that $\alpha_y < 1$ (the instability results also holds for $\alpha_y \geq 1$). Then the inflation forecast for year $t+2$ is

$$\pi_{t+2|t+1} = (1 - \alpha_y)\,\varepsilon_t > 0.$$

The forecast for nominal GDP growth in year $t+2$ is

$$g_{t+2|t+1} = \pi_{t+2|t+1} + y_{t+2|t+1} - y_{t+1},$$

where $\pi_{t+2|t+1}$ is up, and y_{t+1} is down. Hence, for unchanged $y_{t+2|t+1}$, $g_{t+2|t}$ is double up. Then, $y_{t+2|t+1}$ must be brought down even further than y_{t+1}, in order to keep $g_{t+2|t+1}$ in line with g^*, which requires another increase in the interest rate in year $t+1$. Clearly, inflation and the output gap are onto divergent paths.

The different control lags for inflation and the output gap result in the instability of inflation and output. Then a modified definition of staggered nominal GDP growth can restore stability. Consider the following definition of staggered nominal GDP growth,

$$\tilde{g}_{t+1} \equiv \pi_{t+1} + y_t - y_{t-1},$$

where output gap growth is lagged one year. Consider stabilizing this staggered nominal GDP growth around a target growth rate g^*, that is, with the period loss function

$$L(\pi_{t+1}, y_t - y_{t-1}) = \frac{1}{2}(\pi_{t+1} + y_t - y_{t-1} - g^*)^2. \quad (6.5)$$

The first-order condition is

$$\pi_{t+2|t} + y_{t+1|t} - y_t = g^*. \quad (6.6)$$

The staggered nominal GDP growth will fulfill

$$\tilde{g}_{t+2} = g^* + \varepsilon_{t+1} + (1 + \alpha_y)\, \eta_{t+1} + \varepsilon_{t+2}.$$

Appendix D shows that this case results in stability. Intuitively, this is because the instrument i_t affects both $\pi_{t+2|t}$ and $y_{t+1|t}$. Clearly, the staggered nominal GDP growth is a far-fetched construction, though.

Ball shows that π_t and y_t are unstable also for nominal GDP *level* targeting (see appendix D for details).

A frequently mentioned rationale for nominal GDP targeting is that the monetary authority allegedly controls only nominal GDP growth, but not the decomposition of nominal GDP growth into inflation and real GDP growth. It is often claimed that little is understood about the determinants of that decomposition. Given such lack of understanding, it is considered safer for the monetary authority to achieve a certain nominal GDP growth rate, rather than attempting to control inflation and/or output separately. Interestingly, the present model is very different. Here, the transmission mechanism of monetary policy is via aggregate demand to inflation, with a longer control lag for inflation. Hence, in this model, the knowledge about the separate effects of the instrument on aggregate demand and inflation is substantial, in particular the different lags of those effects, and the nominal aggregate demand does not play any role in the transmission of monetary policy by itself. For further discussion of the role of money in this model, including a comparison between money growth targeting and inflation targeting, see Svensson (1997a). If a money demand equation is added, it is easy to generate the high long-run correlation of inflation and money growth, without implying that nominal aggregate demand plays a crucial role in the transmission mechanism of monetary policy in business cycle frequencies.

Regardless of how robust Ball's instability result may be, the present model does not provide any support for nominal GDP targeting.

7 Forward-looking behavior

The model used so far is very simple and in particular does not incorporate any explicit forward-looking behavior in the Phillips curve and aggregate demand equation (other then a trivial inflation-expectations term in the real interest rate). Let me therefore consider a simple forward-looking alternative.

Roberts (1995) has demonstrated that several different forward-looking "New Keynesian" models of Phillips curves boil down to a Phillips curve of the form

$$\pi_t = \pi_{t+1|t} + \alpha_y y_t + \varepsilon_t, \tag{7.1}$$

where $\alpha_y > 0$.[18] Woodford (1996) and McCallum and Nelson (1997) use a forward-looking aggregate demand curve consistent with intertemporal optimization that, expressed in terms of the output gap y_t, can be written

$$y_t = y_{t+1|t} - \beta_r \left(i_t - \pi_{t+1|t} \right) + \eta_t, \tag{7.2}$$

where $\beta_r > 0$. (Appendix E shows that this assumes that the natural output level is a random walk.) The disturbances ε_t and η_t are i.i.d. with zero means.

Furthermore, assume that costs of adjustment, overlapping contracts, or some other mechanism leads to $\pi_{t+1|t}$ in the Phillips curve being replaced by $(1 - \alpha_\pi)\pi_{t+1|t} + \alpha_\pi \pi_{t-1}$, and $y_{t+1|t}$ in the aggregate demand equation being replaced by $(1 - \beta_y)y_{t+1|t} + \beta_y y_{t-1}$, where $0 < \alpha_\pi < 1$ and $0 < \beta_y < 1$,

$$\begin{aligned} \pi_t &= (1 - \alpha_\pi)\pi_{t+1|t} + \alpha_\pi \pi_{t-1} + \alpha_y y_t + \varepsilon_t \\ y_t &= (1 - \beta_y)y_{t+1|t} + \beta_y y_{t-1} - \beta_r \left(i_t - \pi_{t+1|t} \right) + \eta_t. \end{aligned}$$

I would like to maintain the assumption that both inflation and output are pre-determined two and one periods, respectively. Bernanke and Woodford (1996) let inflation be predetermined one period and model this as the left-hand side of (7.1) depending on expectations one period earlier of the right-hand side. Here I take expectations two periods earlier of the right-hand side of (7.1), and one period earlier for the right-hand side of (7.2). Assuming a current disturbance in each equation, and leading them one period, gives

$$\pi_{t+1} = (1 - \alpha_\pi)\pi_{t+2|t-1} + \alpha_\pi \pi_{t|t-1} + \alpha_y y_{t+1|t-1} + \varepsilon_{t+1} \tag{7.3}$$

$$y_{t+1} = (1 - \beta_y)y_{t+2|t} + \beta_y y_t - \beta_r \left(i_{t+1|t} - \pi_{t+2|t} \right) + \eta_{t+1}. \tag{7.4}$$

Finally, I approximate the term $\alpha_\pi \pi_{t|t-1} + \alpha_y y_{t+1|t-1}$ in the Phillips curve by $\alpha_\pi \pi_t + \alpha_y y_t$ (appendix E discusses what is involved in this approximation). The final result is then

$$\pi_{t+1} = (1 - \alpha_\pi)\pi_{t+2|t-1} + \alpha_\pi \pi_t + \alpha_y y_t + \varepsilon_{t+1} \tag{7.5}$$

$$y_{t+1} = (1 - \beta_y)y_{t+2|t} + \beta_y y_t - \beta_r(i_{t+1|t} - \pi_{t+2|t}) + \eta_{t+1}. \tag{7.6}$$

[18] See also Kiley (1996) and Nelson (1997) for recent discussions of Phillips curves.

Note that (7.6) can also be derived from a utility function that is not additively separable in consumption over time. The accelerationist Phillips curve (2.1) simply sets $\alpha_\pi = 1$ (or $\pi_{t+2|t-1} = \pi_t$). The simple aggregate demand function (2.2) replaces $y_{t+2|t}$ by zero, and $i_{t+1|t} - \pi_{t+2|t}$ by $i_t - \pi_{t+1|t}$.

Solving the model with the intertemporal loss function (2.6) with the period loss function (2.7) generally requires a numerical solution. Appendix E shows how the model can be written in state-space form as a linear stochastic regulator problem with forward looking variables, which problem is solved in Backus and Driffill (1986) and Currie and Levine (1993) and applied in Svensson (1994). With forward-looking expectations, the optimal solutions under discretion and commitment are different. Under the more realistic discretion solution, the forward looking variables are linear functions of the state variables, as is the optimal solution. The state variables for (7.5) and (7.6) are π_t, $\pi_{t+1|t}$ and y_t. However, by leading (7.5) one period and taking expectations at t we get

$$\pi_{t+2|t} = (1 - \alpha_\pi)\pi_{t+3|t} + \alpha_\pi \pi_{t+1|t} + \alpha_y y_{t+1|t}, \tag{7.7}$$

and we realize that only the state variables $\pi_{t+1|t}$ and y_t are relevant for the optimal policy. Thus the optimal instrument rule under discretion will be of the form

$$i_{t+1|t} = f_0 + f_\pi \pi_{t+1|t} + f_y y_t.$$

We note the intricate property that the expected future interest rate $i_{t+1|t}$ rather than the current interest rate is the control variable.

7.1 Strict inflation targeting

Consider the case of strict inflation targeting ($\lambda = 0$). It is then clear from (2.7) that we would like to make

$$\pi_{t+\tau|t} = \pi^*, \tag{7.8}$$

for $\tau = 2, 3, ...$, if possible. Using (7.8) in (7.7) results in

$$y_{t+1|t} = -\frac{\alpha_\pi}{\alpha_y}\left(\pi_{t+1|t} - \pi^*\right). \tag{7.9}$$

We then realize that

$$y_{t+\tau|t} = 0 \tag{7.10}$$

for $\tau = 2, 3, ...$, because of (7.8). Using (7.8)–(7.10) in (7.6) gives

$$i_{t+1|t} = \pi^* + \frac{\alpha_\pi}{\beta_r \alpha_y} \left(\pi_{t+1|t} - \pi^* \right) + \frac{\beta_y}{\beta_r} y_t,$$

which is the optimal instrument rule.

Under these circumstances the equilibrium will be

$$\begin{aligned} \pi_{t+1|t} &= \pi^* + \alpha_\pi \varepsilon_t + \alpha_y \eta_t \\ \pi_{t+1} &= \pi_{t+1|t} + \varepsilon_{t+1} \\ y_{t+1} &= -\frac{\alpha_\pi}{\alpha_y} \left(\pi_{t+1|t} - \pi^* \right) + \eta_{t+1}. \end{aligned}$$

In this case, the Phillips curve and aggregate demand are given by

$$\pi_{t+1} = (1 - \alpha_\pi) \pi^* + \alpha_\pi \pi_t + \alpha_y y_t + \varepsilon_{t+1} \quad (7.11)$$

$$y_{t+1} = \beta_y y_t - \beta_r (i_{t+1|t} - \pi^*) + \eta_{t+1}. \quad (7.12)$$

since $\pi_{t+2|t-1} = \pi^*$ and $y_{t+2|t} = 0$, rather than (2.1)–(2.2). The term $(1 - \alpha_\pi)\pi^*$ in (7.12) represents a favorable "credibility effect".

Note that when $\alpha_\pi = \beta_y = 0$, as in (7.1) and (7.2), the optimal instrument rule is trivially $i_{t+1|t} = \pi^*$, and the equilibrium is

$$\begin{aligned} \pi_{t+1|t} &= \pi^* + \alpha_y \eta_t \\ \pi_{t+1} &= \pi_{t+1|t} + \varepsilon_{t+1} \\ y_{t+1} &= \eta_{t+1}. \end{aligned}$$

8 Conclusions

Inflation targeting makes the conditional inflation forecast (conditional upon the current state of the economy and the current instrument setting) an intermediate target. Thus, inflation targeting can be described as a target rule, a rule that specifies the intermediate-target variable and how its target level is determined. Implementation of this target rule then leads to an implicit endogenous instrument rule. Inflation targeting can be interpreted as a commitment to a target rule, where the monetary authority has discretion in selecting the appropriate instrument rule that achieves the target rule.

The present paper has examined inflation targeting with regard to the appropriate monetary policy response to different shocks, the consequences of model uncertainty, the effects of interest rate smoothing and stabilization, a comparison with nominal GDP targeting, and the implications of forward-looking behavior of the private sector. The analysis distinguishes between strict inflation targeting, when nothing but inflation enters the monetary authority's loss function, and flexible inflation targeting, when the monetary authority is also concerned about the stability of the output gap or the instrument.

Under strict inflation targeting, the target rule is very simple. The instrument should be adjusted such that the conditional inflation forecast for a horizon corresponding to the control lag always equals the inflation target. Any shock causing a deviation between the conditional inflation forecast and the inflation target should then be met by an instrument adjustment that eliminates the deviation.

Under flexible inflation targeting, the target rule is not quite as simple, but very intuitive. The instrument should be adjusted such that the conditional inflation forecast gradually approaches the long-run inflation target. For instance, when there is some weight on output stabilization in the monetary authority's loss function, the two-year conditional inflation forecast's deviation from the long-run inflation target should be a given proportion of the predetermined one-year inflation forecast's deviation from the same target, when there is some weight on instrument stabilization or smoothing, the conditional forecast should also be gradually adjusted towards the long-run inflation target. As a consequence, there is a more gradual response to shocks. The intuition for this result is, of course, that a more gradual adjustment requires less output and instrument variability.

Interestingly, a gradual adjustment of the conditional inflation forecast towards the long-run inflation target is also the appropriate policy under model uncertainty. Here, the intuition is that uncertainty about the policy multiplier requires a more muted instrument response, in order to reduce the part of the variability in inflation that is caused by the variability of the policy multiplier.

Thus, both flexible inflation targeting and model uncertainty lead to a gradual adjustment of the conditional inflation forecast toward the long-run inflation target. This also means that they may have consequences that are observation equivalent; observations of gradual adjustment to the long-run inflation target by actual monetary authorities do not directly reveal the precise reasons for this. In this context, it is interesting to note that the 0–2 percent per year range for the Reserve Bank for New Zealand was recently increased to 0–3 percent per year, in the modification of the Policy Target Agreement in December 1996. In the debate in New Zealand, some observers have suggested that the original target range requires an excessive degree of activism on the part of the Reserve Bank, and that a slightly wider band would be sensible (Brash (1997)).

The examination of nominal GDP targeting has restated Ball's (1996) result about resulting possible instability of inflation and output growth. In such models, where a great deal is known about the separate effects of the instrument on inflation and the output gap, and where these effects have different lags, there is no support for nominal GDP targeting. A loss function where the marginal rate of substitution between inflation and output growth is constant and independent of inflation and output deviations generally seems problematic. The loss function associated with flexible inflation targeting, where stability of inflation and the output gap enter separately, seems more advantageous and intuitive.

The examination of a model with forward-looking behavior by the private sector showed how a similar control lag structure as in the simple model can be constructed. It is apparent that forward-looking behavior makes some of the coefficients of the simple model depend on the parameters in the loss function, which generally, aside from the case of strict inflation targeting, requires numerical solutions. For given parameters in the loss function, the coefficients are given, and the results of the simple model apply.

Many inflation-targeting issues remain and seem suitable for future research. The model used here is annual, and it remains to apply these ideas in a quarterly, more empirical framework. Rudebusch and Svensson (1997) compare different inflation targeting rules and explicit instrument rules, for instance the Taylor rule, in an empirical quarterly model for the United States.

Inflation-targeting with imperfectly observed shocks results in a signal-extraction problem for the monetary authority. Imperfect identification of shocks may be a separate reason for a gradual adjustment of the conditional inflation forecast toward the long-run inflation target. This remains to be examined.

The real world inflation-targeting regimes are all very open economies. In an open economy, there is also a direct exchange rate channel for the transmission of monetary policy, with by all accounts a shorter lag than the aggregate demand channel emphasized in the present paper. In an open economy there is also a choice between targeting only inflation in domestic prices (the GDP deflator) or a consumer price index where imports enter. These and other issues in open-economy inflation targeting are examined in Svensson (1998).

A The natural output level and the output gap

Consider the model

$$\begin{aligned}\pi_{t+1} &= \pi_t + \alpha_y(y_t^d - y_t^n) + \varepsilon_{t+1} && \text{(A.1)}\\ y_{t+1}^d &= \tilde{\beta}_y y_t^d + \beta_z z_t - \beta_r(i_t - \pi_{t+1|t}) + \tilde{\eta}_{t+1} && \text{(A.2)}\\ z_{t+1} &= \gamma_z z_t + \zeta_{t+1} && \text{(A.3)}\\ y_{t+1}^n &= \gamma_n y_t^n + \xi_{t+1}, && \text{(A.4)}\end{aligned}$$

where y_t^d is (log) aggregate demand, y_t^n is the natural output level, z_t is a persistent aggregate demand disturbance, $0 \leq \gamma_z < 1$, $0 \leq \gamma_n \leq 1$, and ε_t, $\tilde{\eta}_t$, ζ_t and ξ_t are i.i.d. disturbances. Subtract y_{t+1}^n from (A.2),

$$\begin{aligned}y_{t+1}^d - y_{t+1}^n &= \tilde{\beta}_y\left(y_t^d - y_t^n\right) + \beta_z z_t - \beta_r(i_t - \pi_{t+1|t}) + \left(\tilde{\beta}_y y_t^n - y_{t+1}^n\right) + \tilde{\eta}_{t+1}\\ &= \tilde{\beta}_y\left(y_t^d - y_t^n\right) + \beta_z z_t - \beta_r(i_t - \pi_{t+1|t}) + \left(\tilde{\beta}_y - \gamma_n\right) y_t^n + \tilde{\eta}_{t+1} - \xi_{t+1},\end{aligned}$$

and introduce the output gap,

$$y_t = y_t^d - y_t^n.$$

Then the model can be written

$$\begin{aligned}\pi_{t+1} &= \pi_t + \alpha_y y_t + \varepsilon_{t+1}\\ y_{t+1} &= \tilde{\beta}_y y_t + \beta_z z_t + \left(\tilde{\beta}_y - \gamma_n\right) y_t^n - \beta_r(i_t - \pi_{t+1|t}) + \tilde{\eta}_{t+1} - \xi_{t+1}\\ &= \beta_y y_t + \beta_z z_t + \beta_n y_t^n - \beta_r(i_t - \pi_t) + \eta_{t+1},\end{aligned}$$

where

$$\begin{aligned}\beta_y &= \tilde{\beta}_y + \alpha_y \beta_r && \text{(A.5)}\\ \beta_n &= \tilde{\beta}_y - \gamma_n && \text{(A.6)}\\ \eta_{t+1} &= \tilde{\eta}_{t+1} - \xi_{t+1}. && \text{(A.7)}\end{aligned}$$

The one-year and two-year inflation forecasts are

$$\begin{aligned}\pi_{t+1|t} &= \pi_t + \alpha_y y_t && \text{(A.8)}\\ \pi_{t+2|t}(i_t) &= \pi_t + a_y y_t + a_z z_t + a_n y_t^n - a_r(i_t - \pi_t), && \text{(A.9)}\end{aligned}$$

where

$$a_y = \alpha_y(1+\beta_y) \tag{A.10}$$

$$a_z = \alpha_y\beta_z \tag{A.11}$$

$$a_n = \alpha_y\beta_n \tag{A.12}$$

$$a_r = \alpha_y\beta_r. \tag{A.13}$$

With the period loss function (2.7) the optimal policy rule can be written on the forms

$$i_t = \pi_t + f_\pi(\lambda)(\pi_t - \pi^*) + f_y(\lambda)y_t + f_z z_t + f_n y_t^n, \tag{A.14}$$

where $f_\pi(\lambda)$ and $f_y(\lambda)$ are given by (2.15), $f_z = \frac{\beta_z}{\beta_r}$ and $f_n = \frac{\beta_n}{\beta_r}$.

In (2.3), x_t represents either the persistent demand disturbance z_t or the natural rate y_t^n (or both, if it is interpreted as a vector and γ as a diagonal matrix).

B Inflation targeting with output gap stabilization and exogenous variables

B.1 One-year control lag for inflation

In order to derive the first-order condition (2.8), it is practical to first study the simpler problem

$$V(\pi_t) = \min_{y_t} \left\{ \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda y_t^2 \right] + \delta \mathrm{E}_t V(\pi_{t+1}) \right\} \tag{B.1}$$

subject to

$$\pi_{t+1} = \pi_t + \alpha_y y_t + \varepsilon_{t+1}, \tag{B.2}$$

where the output gap y_t is regarded as a control variable and the indirect loss function $V(\pi_t)$ remains to be determined.

The indirect loss function $V(\pi_t)$ will be quadratic,

$$V(\pi_t, x_t) = k_0 + \frac{1}{2}k\left(\pi_t - \pi^*\right)^2, \tag{B.3}$$

where the coefficients k_0 and k remain to be determined (I will only need k). The first-order condition is

$$\lambda y_t + \delta \mathrm{E}_t V_\pi(\pi_{t+1})\alpha_y = \lambda y_t + \delta\alpha_y k \left(\pi_{t+1|t} - \pi^*\right) = 0,$$

where I have used (B.3). This can be written

$$\pi_{t+1|t} - \pi^* = -\frac{\lambda}{\delta\alpha_y k} y_t. \tag{B.4}$$

The decision rule for the output gap fulfills

$$\begin{aligned} y_t &= -\frac{\delta\alpha_y k}{\lambda}\left(\pi_{t+1|t}-\pi^*\right) \\ &= -\frac{\delta\alpha_y k}{\lambda}\left(\pi_t-\pi^*\right)-\frac{\delta\alpha_y^2 k}{\lambda}y_t \\ &= -\frac{\delta\alpha_y k}{\lambda+\delta\alpha_y^2 k}\left(\pi_t-\pi^*\right), \end{aligned}$$

where I have used (2.4).

Then the equilibrium inflation forecast fulfills

$$\begin{aligned} \pi_{t+1|t} &= \pi_t+\alpha_y y_t \\ &= \pi^*+\frac{\lambda}{\lambda+\delta\alpha_y^2 k}\left(\pi_t-\pi^*\right). \end{aligned} \tag{B.5}$$

In order to identify k, I exploit the envelope theorem for (B.1) and (B.3) and use (B.5), which gives

$$\begin{aligned} V_\pi(\pi_t) &\equiv k\left(\pi_t-\pi^*\right) \\ &\equiv \left(\pi_t-\pi^*\right)+\delta k\left(\pi_{t+1|t}-\pi^*\right) \\ &= \left(1+\frac{\delta\lambda k}{\lambda+\delta\alpha_y^2 k}\right)\left(\pi_t-\pi^*\right). \end{aligned}$$

Identification of the coefficient for $\pi_t-\pi^*$ gives

$$k = 1+\frac{\delta\lambda k}{\lambda+\delta\alpha_y^2 k}.$$

The right-hand side is equal to unity for $k=0$ and increases towards $1+\frac{\lambda}{\alpha_y^2}$ for $k\to\infty$. We realize that there is a unique positive solution that fulfills $k\geq 1$. It can be solved analytically from

$$k^2-\left(1-\frac{\lambda(1-\delta)}{\delta\alpha_y^2}\right)k-\frac{\lambda}{\delta\alpha_y^2}=0$$

and is given by

$$k=k(\lambda)\equiv\frac{1}{2}\left(1-\frac{\lambda(1-\delta)}{\delta\alpha_y^2}+\sqrt{\left(1+\frac{\lambda(1-\delta)}{\delta\alpha_y^2}\right)^2+\frac{4\lambda}{\alpha_y^2}}\right)\geq 1. \tag{B.6}$$

B.2 Two-year control lag for inflation

After these preliminaries, consider the problem

$$\min_{i_t} \mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L\left(\pi_{t+\tau}, y_{t+\tau}\right)$$

subject to

$$
\begin{aligned}
L(\pi_t, y_t) &= \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda y_t^2\right] \\
\pi_{t+1} &= \pi_t + \alpha_y y_t + \varepsilon_{t+1} \\
y_{t+1} &= \beta_y y_t + \beta_x x_t - \beta_r (i_t - \pi_t) + \eta_{t+1}.
\end{aligned}
$$

We realize that this can be formulated as

$$
V(\pi_{t+1|t}) = \min_{y_{t+1|t}} \left\{ \frac{1}{2}\left[\left(\pi_{t+1|t} - \pi^*\right)^2 + \lambda y_{t+1|t}^2\right] + \delta \mathrm{E}_t V(\pi_{t+2|t+1}) \right\}
$$

subject to

$$
\begin{aligned}
\pi_{t+2|t+1} &= \pi_{t+1} + \alpha_y y_{t+1} \\
&= \pi_{t+1|t} + \alpha_y y_{t+1|t} + \left(\varepsilon_{t+1} + \alpha_y \eta_{t+1}\right),
\end{aligned}
$$

where $y_{t+1|t}$ is regarded as the control, and where the optimal repo rate can be inferred from

$$
i_t - \pi_t = -\frac{1}{\beta_r} y_{t+1|t} + \frac{\beta_y}{\beta_r} y_t + \frac{\beta_x}{\beta_r} x_t. \tag{B.7}
$$

We realize that this problem is analogous to the problem (B.1) subject to (B.2), where $\pi_{t+1|t}$, $y_{t+1|t}$ and $\varepsilon_{t+1} + \alpha_y \eta_{t+1}$ replace π_t, y_t and ε_{t+1}. Thus, in analogy with (B.4), the first-order condition can be written

$$
\pi_{t+2|t} - \pi^* = -\frac{\lambda}{\delta \alpha_y k(\lambda)} y_{t+1|t}, \tag{B.8}
$$

where $k(\lambda)$ will obey (B.6).

Since by (2.1) we have

$$
y_{t+1|t} = \frac{1}{\alpha_y}\left(\pi_{t+2|t} - \pi_{t+1|t}\right),
$$

we can eliminate $y_{t+1|t}$ from (B.8) and get

$$
\pi_{t+2|t} - \pi^* = \frac{\lambda}{\lambda + \delta \alpha_y^2 k(\lambda)}\left(\pi_{t+1|t} - \pi^*\right),
$$

that is,

$$
\pi_{t+2|t} = \pi^* + c(\lambda)\left(\pi_{t+1|t} - \pi^*\right) \tag{B.9}
$$

where $c(\lambda)$ is given by

$$
0 \leq c(\lambda) \equiv \frac{\lambda}{\lambda + \delta \alpha_y^2 k(\lambda)} < 1. \tag{B.10}
$$

Since by (B.8)

$$
y_{t+1|t} = -\frac{\delta \alpha_y k(\lambda)}{\lambda}\left(\pi_{t+2|t} - \pi^*\right), \tag{B.11}
$$

by (B.7) the reaction function will fulfill

$$\begin{aligned} i_t - \pi_t &= -\frac{1}{\beta_r} y_{t+1|t} + \frac{\beta_y}{\beta_r} y_t + \frac{\beta_x}{\beta_r} x_t \\ &= \frac{\delta \alpha_y k(\lambda)}{\lambda \beta_r} \left[\pi_t - \pi^* + \alpha_y \left(1 + \beta_y\right) y_t + \alpha_y \beta_x x_t - \alpha_y \beta_r \left(i_t - \pi_t\right) \right] + \frac{\beta_y}{\beta_r} y_t + \frac{\beta_x}{\beta_r} x_t, \end{aligned}$$

where I have used

$$\pi_{t+2|t} = \pi_t + \alpha_y (1 + \beta_y) y_t + \alpha_y \beta_x x_t - \alpha_y \beta_r \left(i_t - \pi_t\right).$$

We get

$$\begin{aligned} \left(1 + \frac{\delta \alpha_y^2 k(\lambda)}{\lambda}\right) (i_t - \pi_t) &= \frac{\delta \alpha_y k(\lambda)}{\lambda \beta_r} \left[\pi_t - \pi^* + \alpha_y \left(1 + \beta_y\right) y_t + \alpha_y \beta_x x_t \right] + \frac{\beta_y}{\beta_r} y_t + \frac{\beta_x}{\beta_r} x_t \\ &= \frac{\delta \alpha_y k(\lambda)}{\lambda \beta_r} (\pi_t - \pi^*) + \frac{1}{\lambda \beta_r} \left[\delta \alpha_y^2 \left(1 + \beta_y\right) k(\lambda) + \lambda \beta_y \right] y_t \\ &\quad + \frac{\beta_x (\lambda + \delta \alpha_y^2 k(\lambda))}{\lambda \beta_r} x_t \end{aligned}$$

and

$$\begin{aligned} i_t - \pi_t &= \frac{\delta \alpha_y k(\lambda)}{\beta_r \left(\lambda + \delta \alpha_y^2 k(\lambda)\right)} (\pi_t - \pi^*) + \frac{1}{\beta_r \left(\lambda + \delta \alpha_y^2 k(\lambda)\right)} \left[\delta \alpha_y^2 \left(1 + \beta_y\right) k(\lambda) + \lambda \beta_y \right] y_t + \frac{\beta_x}{\beta_r} x_t \\ &= f_\pi(\lambda) (\pi_t - \pi^*) + f_y(\lambda) y_t + f_x x_t, \end{aligned} \tag{B.12}$$

where

$$f_\pi(\lambda) = \frac{1 - c(\lambda)}{\alpha_y \beta_r}, \quad f_y(\lambda) = \frac{\beta_y + 1 - c(\lambda)}{\beta_r} \text{ and } f_x = \frac{\beta_x}{\beta_r}.$$

It is shown in Svensson (1997) that the coefficients $\frac{\lambda}{\delta \alpha_y k(\lambda)}$ in (B.4) and $c(\lambda)$ in (B.10) are (i) increasing in λ and (ii) decreasing in α_y. The coefficient $c(\lambda)$ increases monotonically from 0 to 1 when λ goes from 0 to infinity.

C Interest rate stabilization and smoothing

Introduce $X_t = (\pi_t, y_t, i_{t-1})'$ and $v_t = (\varepsilon_t, \eta_t, 0)'$ (the exogenous variable is disregarded, $\beta_x = 0$). Then the model (2.1)–(2.2) can be written

$$X_{t+1} = A X_t + B i_t + v_{t+1},$$

where

$$A = \begin{bmatrix} 1 & \alpha_y & 0 \\ \beta_r & \beta_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ -\beta_r \\ 1 \end{bmatrix}.$$

Let for simplicity $\pi^* = 0$ (interpret π_t and i_t as inflation and instrument deviations from π^*). Then the period loss function (5.1) can be written

$$L(X_t, i_t) = X_t' Q X_t + 2X_t' U i_t + R i_t^2,$$

where

$$Q = \frac{1}{2}\begin{bmatrix} 1+\mu & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \nu \end{bmatrix}, \; U = \frac{1}{2}\begin{bmatrix} -\mu \\ 0 \\ -\nu \end{bmatrix}, \; R = \frac{\mu+\nu}{2}.$$

This is the standard stochastic linear optimal regulator problem (see, for instance, Chow (1975) and Sargent (1987)). The optimal instrument rule is of the form

$$i_t = f X_t,$$

where the 3×1 row vector f is given by

$$f = -\left(R + \delta B' V B\right)^{-1}\left(\delta B' V A + U'\right)$$

and the 3×3 matrix V is the solution to the matrix Riccati equation

$$V = Q + Uf + f'U' + f'Rf + \delta\,(A + Bf)'\, V(A + Bf).$$

D Nominal GDP targeting

D.1 Nominal GDP growth targeting

The roots to (6.4) fulfill

$$\mu = \frac{2 - \alpha_y \pm \sqrt{\alpha_y\,(\alpha_y - 4)}}{2}.$$

Consider first the case $\alpha_y \leq 4$. Then the roots are complex and given by

$$\mu_{1,2} = \frac{2 - \alpha_y \pm j\sqrt{\alpha_y\,(4 - \alpha_y)}}{2} \tag{D.1}$$

where j denotes $\sqrt{-1}$ and

$$\left\|\mu_{1,2}\right\| = \sqrt{\frac{(2-\alpha_y)^2 + \alpha_y\,(4-\alpha_y)}{4}} = 1.$$

The roots are on the unit circle.

For the case $\alpha_y > 4$, the roots are real and fulfill

$$\mu_{1,2} = \frac{2-\alpha_y \pm \sqrt{(2-\alpha_y)^2 - 4}}{2}. \tag{D.2}$$

Since

$$\mu_1 \mu_2 = 1,$$

we realize that one root is inside the unit circle, the other is outside,

$$\begin{aligned} \mu_1 &= \frac{2-\alpha_y - \sqrt{(2-\alpha_y)^2 - 4}}{2} < 1 \\ \mu_2 &= \frac{2-\alpha_y + \sqrt{(2-\alpha_y)^2 - 4}}{2} > 1. \end{aligned}$$

In both cases the system is unstable.

D.2 Staggered nominal GDP growth targeting

The first-order condition (6.6) can be written

$$y_{t+1|t} = -\frac{1}{1+\alpha_y}\left(\pi_{t+1|t} - g^*\right) + \frac{1}{1+\alpha_y} y_t = -\frac{1}{1+\alpha_y}\left(\pi_t - g^*\right) + \frac{1-\alpha_y}{1+\alpha_y} y_t.$$

We realize that π_t and y_t will follow

$$\begin{bmatrix} \pi_{t+1} - g^* \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \alpha_y \\ -\frac{1}{1+\alpha_y} & \frac{1-\alpha_y}{1+\alpha_y} \end{bmatrix} \begin{bmatrix} \pi_t - g^* \\ y_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \end{bmatrix}.$$

The eigenvalues are the roots μ of the characteristic equation

$$0 = \begin{vmatrix} \mu - 1 & -\alpha_y \\ \frac{1}{1+\alpha_y} & \mu - \frac{1-\alpha_y}{1+\alpha_y} \end{vmatrix} = \mu^2 - \frac{2}{1+\alpha_y}\mu + \frac{1}{1+\alpha_y}.$$

The roots are complex and inside the unit circle,

$$\begin{aligned} \mu_{1,2} &= \frac{1 \pm j\sqrt{\alpha_y}}{1+\alpha_y} \\ \left\|\mu_{1,2}\right\| &= \frac{\sqrt{1+\alpha_y}}{1+\alpha_y} < 1. \end{aligned}$$

Hence the system is stable.

D.3 Nominal GDP level targeting

For nominal GDP (gap) level targeting, the first-order condition is

$$p_{t+1|t} + y_{t+1|t} = Y^*.$$

Rewrite this as

$$y_{t+1|t} = -p_{t+1|t} + Y^* = -\pi_{t+1|t} - (p_t - Y^*) = -\pi_t - \alpha_y y_t - (p_t - Y^*).$$

We realize that π_t, y_t and p_t follow

$$\begin{bmatrix} \pi_{t+1} \\ y_{t+1} \\ p_{t+1} - Y^* \end{bmatrix} = \begin{bmatrix} 1 & \alpha_y & 0 \\ -1 & -\alpha_y & -1 \\ 1 & \alpha_y & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ p_t - Y^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \\ \varepsilon_{t+1} \end{bmatrix}.$$

The eigenvalues are the roots μ of the characteristic equation

$$0 = \begin{vmatrix} \mu - 1 & -\alpha_y & 0 \\ 1 & \mu + \alpha_y & 1 \\ -1 & -\alpha_y & \mu - 1 \end{vmatrix} = \mu \left[(\mu - 1)(\mu - 1 + \alpha_y) + \alpha_y \right].$$

One root is zero. The other two are the roots of the equation

$$\mu^2 - (2 - \alpha_y)\mu + 1 = 0,$$

which are the same roots as in (D.1) or (D.2) above. Thus the system is unstable, although the nominal GDP level is stationary and fulfills

$$Y_{t+1} = p_{t+1} + y_{t+1} = Y^* + \varepsilon_{t+1} + \eta_{t+1}.$$

E Forward-looking behavior

E.1 Output gap and natural output rate

Let y_t^d and y_t^n denote (log) aggregate demand and the (log) natural output level, respectively. Let

$$y_{t+1}^n = \gamma_n y_t^n + \xi_{t+1},$$

where $0 \leq \gamma_n \leq 1$. Consider the model

$$\begin{aligned} \pi_t &= (1 - \alpha_\pi)\pi_{t+1|t} + \alpha_\pi \pi_{t-1} + \alpha_y \left(y_t^d - y_t^n \right) + \varepsilon_t \\ y_t^d &= (1 - \beta_y) y_{t+1|t}^d + \beta_y y_{t-1}^d - \beta_r \left(i_t - \pi_{t+1|t} \right) + \tilde{\eta}_t. \end{aligned}$$

Take expectations one and two periods earlier, respectively, lead the equations one period, and add a disturbance term,

$$\begin{aligned}\pi_{t+1} &= (1-\alpha_\pi)\pi_{t+2|t-1} + \alpha_\pi \pi_{t|t-1} + \alpha_y \left(y^d_{t+1|t-1} - y^n_{t+1|t-1} \right) + \varepsilon_{t+1} \\ y^d_{t+1} &= (1-\beta_y) y^d_{t+2|t} + \beta_y y^d_t - \beta_r \left(i_{t+1|t} - \pi_{t+2|t} \right) + \tilde{\eta}_{t+1}.\end{aligned}$$

Express this in terms of the output gap

$$y_t \equiv y^d_t - y^n_t.$$

We get

$$\begin{aligned}\pi_{t+1} &= (1-\alpha_\pi)\pi_{t+2|t-1} + \alpha_\pi \pi_{t|t-1} + \alpha_y y_{t+1|t-1} + \varepsilon_{t+1} \\ y_{t+1} &= (1-\beta_y) y_{t+2|t} + \beta_y y_t - \beta_r \left(i_{t+1|t} - \pi_{t+2|t} \right) + \left[-y^n_{t+1} + (1-\beta_y) y^n_{t+2|t} + \beta_y y^n_t \right] + \tilde{\eta}_{t+1} \\ &= (1-\beta_y) y_{t+2|t} + \beta_y y_t - \beta_r \left(i_{t+1|t} - \pi_{t+2|t} \right) + \beta_n y^n_t + \tilde{\eta}_{t+1} - \xi_{t+1},\end{aligned}$$

where

$$-y^n_{t+1} + (1-\beta_y) y^n_{t+2|t} + \beta_y y^n_t = \beta_n y^n_t - \xi_{t+1}.$$

Hence

$$\begin{aligned}\beta_n &= -\gamma_n + (1-\beta_y)\gamma_n^2 + \beta_y \\ &= (1-\gamma_n)[\beta_y(1+\gamma_n) - \gamma_n].\end{aligned}$$

We have

$$\begin{aligned}\beta_n &= 0 \quad \text{for } \gamma_n = 1 \\ \beta_n &> 0 \quad \text{for } \gamma_n < 1 \text{ and } \beta_y > \frac{\gamma_n}{1+\gamma_n} \\ \beta_n &< 0 \quad \text{for } 0 < \gamma_n < 1 \text{ and } \beta_y < \frac{\gamma_n}{1+\gamma_n}.\end{aligned}$$

Let us assume $\gamma_n = 1$, so $\beta_n = 0$.

Approximate $\alpha_\pi \pi_{t|t-1} + \alpha_y y_{t+1|t-1}$ by $\alpha_\pi \pi_t + \alpha_y y_t$, in order to get

$$\begin{aligned}\pi_{t+1} &= (1-\alpha_\pi)\pi_{t+2|t-1} + \alpha_\pi \pi_t + \alpha_y y_t + \varepsilon_{t+1} \\ y_{t+1} &= (1-\beta_y) y_{t+2|t} + \beta_y y_t - \beta_r \left(i_{t+1|t} - \pi_{t+2|t} \right) + \eta_{t+1},\end{aligned}$$

where $\eta_{t+1} = \tilde{\eta}_{t+1} - \xi_{t+1}$. What error is introduced by this approximation? The error in the inflation term is trivial,

$$\alpha_\pi \left(\pi_t - \pi_{t|t-1} \right) = \alpha_\pi \varepsilon_t.$$

With regard to the output gap term, we have

$$\begin{aligned}\alpha_y\left(y_t - y_{t+1|t-1}\right) &= \alpha_y y_t - \frac{\alpha_y}{1-\beta_y}\left[y_{t|t-1} - \beta_y y_{t-1} + \beta_r\left(i_{t|t-1} - \pi_{t+1|t-1}\right)\right] \\ &= -\frac{\alpha_y\beta_y}{1-\beta_y}\left(y_{t|t-1} - y_{t-1}\right) - \frac{\alpha_y\beta_r}{1-\beta_y}\left(i_{t|t-1} - \pi_{t+1|t-1}\right) + \alpha_y\eta_t.\end{aligned}$$

The benefit of the approximation is to have only π_t and y_t as state variables, rather than the other more complicated terms in the expression above.

E.2 State-space form

In order to write the model in state-space form, take expectations of (7.5) and (7.6) at t, and move $\pi_{t+3|t}$ and $y_{t+2|t}$ to the left-hand side:

$$(1-\alpha_\pi)\pi_{t+3|t} = \pi_{t+2|t} - \alpha_\pi\pi_{t+1|t} - \alpha_y y_{t+1|t} \tag{E.1}$$

$$\beta_r\pi_{t+2|t} + (1-\beta_y)y_{t+2|t} = y_{t+1|t} - \beta_y y_t + \beta_r i_{t+1|t} \tag{E.2}$$

$$\beta_r(1-\alpha_\pi)\pi_{t+3|t} + (1-\beta_y)y_{t+2|t} = -\beta_r\alpha_\pi\pi_{t+1|t} + (1-\beta_r\alpha_y)y_{t+1|t} - \beta_y y_t + \beta_r i_{t+1|t}. \tag{E.3}$$

Here we must have

$$\beta_r\alpha_y \neq 1. \tag{E.4}$$

Introduce the predetermined variable

$$X_{3t} = \pi_{t+1|t}$$

and the forward-looking variables

$$\begin{aligned} x_{1t} &= \pi_{t+2|t} \\ x_{2t} &= y_{t+1|t},\end{aligned}$$

and write the model as

$$\begin{aligned}\pi_{t+1} &= X_{3t} + \varepsilon_{t+1} \\ y_{t+1} &= x_{2t} + \eta_{t+1} \\ X_{3,t+1} &= x_{1t} + \alpha_\pi\varepsilon_{t+1} + \alpha_y\eta_{t+1} \\ (1-\alpha_\pi)x_{1,t+1|t} &= -\alpha_\pi X_{3t} + x_{1t} - \alpha_y x_{2t} \\ \beta_r(1-\alpha_\pi)x_{1,t+1|t} + (1-\beta_y)x_{2,t+1|t} &= -\beta_y y_t - \beta_r\alpha_\pi X_{3t} + (1-\beta_r\alpha_y)\,x_{2t} + \beta_r i_{t+1|t}.\end{aligned}$$

This can be written as

$$\begin{bmatrix} X_{t+1} \\ Cx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bu_t + \begin{bmatrix} \varepsilon_{X,t+1} \\ 0 \end{bmatrix},$$

where

$$\begin{bmatrix} X_t \\ x_t \end{bmatrix} = \begin{bmatrix} \pi_t \\ y_t \\ X_{3t} \\ x_{1t} \\ x_{2t} \end{bmatrix}, \; C = \begin{bmatrix} 1-\alpha_\pi & 0 \\ \beta_r(1-\alpha_\pi) & 1-\beta_y \end{bmatrix}, \; A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\alpha_\pi & 1 & -\alpha_y \\ 0 & -\beta_y & -\beta_r\alpha_\pi & 0 & 1-\beta_r\alpha_y \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \beta_r \end{bmatrix}, \; u_t = i_{t+1|t}, \; \varepsilon_{Xt} = \begin{bmatrix} \varepsilon_t \\ \eta_t \\ \alpha_\pi\varepsilon_t + \alpha_y\eta_t \end{bmatrix}.$$

We must have A_{22} non-singular; this requires (E.4).

The loss function (2.7) can be written as

$$L_t = \begin{bmatrix} X_t' & x_t' \end{bmatrix} Q \begin{bmatrix} X_t \\ x_t \end{bmatrix}$$

where

$$Q = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The model is now formulated as a standard linear stochastic regulator problem with rational expectations and forward-looking variables (see, for instance, Backus and Driffill (1986), Currie and Levine (1994) and Svensson (1994)). (The problem is slightly generalized, since C need not be an identity matrix.)

With forward-looking variables, there is a difference between the case of discretion and the case of commitment to an optimal rule, as discussed in the above references. In the discretion case, the forward-looking variables will be linear functions of the predetermined variables,

$$x_t = GX_t,$$

where the 2×3 matrix G is endogenously determined. The optimal policy rule will be of the form

$$u_t = fX_t.$$

In the commitment case, the optimal policy also depends on the shadow prices of the forward-looking variables.

E.3 The solution for a given instrument rule

The relevant state variables are $\pi_{t+1|t}$ and y_t. Restrict the instrument to be linear in these state variables,

$$i_{t+1|t} = f_\pi \pi_{t+1|t} + f_y y_t.$$

We realize from (E.1) and (E.3) that both $\pi_{t+2|t}$ and $y_{t+1|t}$ will be linear functions of the state variables $\pi_{t+1|t}$ and y_t,

$$\begin{aligned} \pi_{t+2|t} &= a\pi_{t+1|t} + by_t \\ y_{t+1|t} &= c\pi_{t+1|t} + dy_t, \end{aligned}$$

where a, b, c and d remain to be determined. Then we have

$$\begin{aligned} \pi_{t+3|t} &= a\pi_{t+2|t} + by_{t+1|t} \\ &= (a^2 + bc)\pi_{t+1|t} + (a + d)by_t \\ y_{t+2|t} &= c\pi_{t+2|t} + dy_{t+1|t} \\ &= (a + d)c\pi_{t+1} + (bc + d^2)y_t. \end{aligned}$$

Substitution of this into (E.1) and (E.2) leads to

$$(1 - \alpha_\pi) \left[(a^2 + bc)\pi_{t+1|t} + (a + d)by_t\right] = a\pi_{t+1|t} + by_t - \alpha_\pi \pi_{t+1|t} - \alpha_y \left(c\pi_{t+1|t} + dy_t\right)$$

$$\beta_r \left(a\pi_{t+1|t} + by_t\right) + (1 - \beta_y) \left[(a + d)c\pi_{t+1|t} + \left(bc + d^2\right) y_t\right] =$$

$$c\pi_{t+1|t} + dy_t - \beta_y y_t + \beta_r \left(f_\pi \pi_{t+1|t} + f_y y_t\right).$$

Identification of the coefficients for $\pi_{t+1|t}$ and y_t gives four equations,

$$\begin{aligned}(1-\alpha_\pi)\left(a^2+bc\right) &= a-\alpha_\pi-\alpha_y c\\ (1-\alpha_\pi)(a+d)b &= b-\alpha_y d\\ \beta_r a+(1-\beta_y)(a+d)c &= c+\beta_r f_\pi\\ \beta_r b+(1-\beta_y)\left(bc+d^2\right) &= d-\beta_y+\beta_r f_y,\end{aligned}$$

which can in principle be solved (numerically) for a, b, c and d.

Note that the case $a=b=0$ implies the equations

$$\begin{aligned}0 &= -\alpha_\pi-\alpha_y c\\ 0 &= -\alpha_y d\\ (1-\beta_y)dc &= c+\beta_r f_\pi\\ (1-\beta_y)d^2 &= d-\beta_y+\beta_r f_y,\end{aligned}$$

which implies

$$\begin{aligned}c &= -\frac{\alpha_\pi}{\alpha_y}\\ d &= 0\\ f_\pi &= \frac{\alpha_\pi}{\beta_r\alpha_y}\\ f_y &= \frac{\beta_y}{\beta_r}.\end{aligned}$$

This is the solution above for strict inflation targeting ($\lambda=0$).

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