

Local Robustness Analysis: Theory and Application

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Discussion by
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Summary

- Model
- Policy rule
- Loss

$$x_{t+1} = A(L)x_t + B(L)u_t + \xi_{t+1},$$

$$\xi_t = w(L)v_t,$$

$$u_t = -F(L)x_t.$$

$$J = E[x_t^2].$$

- Frequency domain: Spectrum of x_t ,

$$f_x(\omega) \equiv p(\omega) \frac{f_\xi(\omega)}{|e^{i\omega} - A(e^{-i\omega})|^2},$$

$$p(\omega) \equiv \frac{1}{1 + [e^{i\omega} - A(e^{-i\omega})]^{-1} B(e^{-i\omega}) F(e^{-i\omega})}. \quad (1)$$

where $p(\omega)$ is the squared “gain” of the “sensitivity function,”

- Loss in frequency domain,

$$E[x_t^2] = \int_{-\pi}^{\pi} f_x(\omega) d\omega \equiv \int_{-\pi}^{\pi} p(\omega) \frac{f_\xi(\omega)}{|e^{i\omega} - A(e^{-i\omega})|^2} d\omega \equiv J(p, f_\xi),$$

- Local model uncertainty in frequency domain,

$$\int_{-\pi}^{\pi} [f_\xi(\omega) - \bar{f}_\xi(\omega)]^2 \leq \varepsilon^2. \quad (2)$$

- Robust control (Nash),

$$F^* = \arg \min_p \max_{J[p, f_\xi]} J$$

subject to (1) and (2).

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Comments

- Elegant and concise description of local robust control
 - But is it practical?

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- Limitations of Brock-Durlauf approach

- Only scalar system—not multivariate states, targets, and controls
- Only simple loss function—not multivariate, conditional
- Only backward-looking system—not forward looking
- Only local model uncertainty—not global (backward/forward looking, lag differences, ...)
- Model uncertainty about shocks only, not $A(L)$, $B(L)$
- * Restrictive and problematic: Small perturbations in $A(L)$ and $B(L)$ may have dramatic effects (Onatski-Williams)
- * *Structural* modeling of model uncertainty essential (Onatski-Williams)
- * Friedman’s “long and variable lags:” $A(L)$ and $B(L)$, not ξ_t
- When $K > 0$, it depends on $F(e^{-i\omega})$, not “constant”?
- * Use F to eliminate unstable poles p_{uj} ?

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- Robust control often on boundary of feasible set of models (Svensson 00)
 - * Assumptions about feasible set of models crucial (ε)
 - * Very unlikely models may determine robust control
- Robust policy rules need not be restricted to instrument rules, allow targeting rules (may be more robust) (Walsh, Svensson-Woodford, Giannouli-Woodford)

- Advantages of Brock-Durlauf approach?

- Elegance and conciseness (but revise notation somewhat)
- Frequency domain more relevant/practical/intuitive than time domain?
 - * Some frequencies more relevant (Onatski-Williams)
- Design limits? Bode’s constraint?
 - * Tradeoff across frequencies (but when $K > 0$ not constant, ...)
 - * Design limits (imperfect control) also in time domain
 - Optimal x_t obvious in time domain

- Better for practical (monetary) policy: Robust decision process
 - “Forecast targeting” rather than commitment to particular instrument rule (Svensson JEL 03, 04)
 - * No central bank will seriously consider commitment to particular instrument rule
 - * Bayesian (adaptive) optimal control: Use all relevant information, optimize in a timeless perspective, Bayesian updating/learning of model

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- Bode’s integral constraint,

$$\int_{-\pi}^{\pi} \ln |p(\omega)| d\omega = K, \quad (3)$$

where

$$K \equiv 4\pi \sum_j \ln |p_{uj}| \geq 0,$$

where p_{uj} are the unstable poles of $[e^{i\omega} - A(e^{-i\omega})]^{-1} B(e^{-i\omega}) F(e^{-i\omega})$

- Fundamental tradeoff across frequencies

- Robust control (Nash),

$$p^* = \arg \min_p \max_{J[p, f_\xi]} J$$

subject to (2) and (3).

- First-order condition for p ,

$$f_x(\omega) \equiv p(\omega) \frac{f_\xi(\omega)}{|e^{i\omega} - A(e^{-i\omega})|^2} = \lambda.$$

Hence, optimal x_t is white noise.

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