Local Robustness Analysis: Theory and Application

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Discussion by
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Summary

- Model
$$x_{t+1} = A(L)x_t + B(L)u_t + \xi_{t+1},$$
$$\xi_t = w(L)\nu_t,$$
- Policy rule
$$u_t = -F(L)x_t.$$
- Loss
$$J = \mathrm{E}[x_t^2].$$

- Frequency domain: Spectrum of x_t,
$$f_x(\omega) \equiv p(\omega)\frac{f_\xi(\omega)}{\left|e^{i\omega} - A(e^{-i\omega})\right|^2},$$
where $p(\omega)$ is the squared "gain" of the "sensitivity function,"
$$p(\omega) \equiv \left|\frac{1}{1 + [e^{i\omega} - A(e^{-i\omega})]^{-1}B(e^{-i\omega})F(e^{-i\omega})}\right|^2. \tag{1}$$
- Loss in frequency domain,
$$\mathrm{E}[x_t^2] = \int_{-\pi}^{\pi} f_x(\omega)d\omega \equiv \int_{-\pi}^{\pi} p(\omega)\frac{f_\xi(\omega)}{\left|e^{i\omega} - A(e^{-i\omega})\right|^2}d\omega \equiv J(p, f_\xi),$$
- Local model uncertainty in frequency domain,
$$\int_{-\pi}^{\pi}[f_\xi(\omega) - \bar{f}_\xi(\omega)]^2 \leq \varepsilon^2. \tag{2}$$
- Robust control (Nash),
$$F^* = \arg\min_F \max_{f_\xi} J[p, f_\xi] \text{ subject to (1) and (2).}$$

- Bode's integral constraint,
$$\int_{-\pi}^{\pi} \ln|p(\omega)|\, d\omega = K, \tag{3}$$
where
$$K \equiv 4\pi \sum_j \ln\left|p_{u_j}\right| \geq 0,$$
where p_{u_j} are the unstable poles of $[e^{i\omega} - A(e^{-i\omega})]^{-1}B(e^{-i\omega})F(e^{-i\omega})$
  - Fundamental tradeoff across frequencies
- Robust control (Nash),
$$p^* = \arg\min_p \max_{f_\xi} J(p, f_\xi) \text{ subject to (2) and (3).}$$
- First-order condition for p,
$$f_x(\omega) \equiv p(\omega)\frac{f_\xi(\omega)}{\left|e^{i\omega} - A(e^{-i\omega})\right|^2} = \lambda.$$
Hence, optimal x_t is white noise.

Comments

- Elegant and concise description of local robust control
  - But is it practical?

- Limitations of Brock-Durlauf approach
  - Only scalar system—not multivariate states, targets, and controls
  - Only simple loss function—not multivariate, conditional
  - Only backward-looking system—not forward looking
  - Only local model uncertainty—not global (backward/forward looking, lag differences, ...)
  - Model uncertainty about shocks only, not $A(L)$, $B(L)$
    - ∗ Restrictive and problematic: Small perturbations in $A(L)$ and $B(L)$ may have dramatic effects (Onatski-Williams)
    - ∗ *Structural* modeling of model uncertainty essential (Onatski-Williams)
    - ∗ Friedman's "long and variable lags:" $A(L)$ and $B(L)$, not ξ_t
  - When $K > 0$, it depends on $F(e^{-i\omega})$, not "constant"?
    - ∗ Use F to eliminate unstable poles p_{u_j}?

- Robust control often on boundary of feasible set of models (Svensson 00)
  - ∗ Assumptions about feasible set of models crucial (ε)
  - ∗ Very unlikely models may determine robust control
- Robust policy rules need not be restricted to instrument rules, allow targeting rules (may be more robust)
(Walsh, Svensson-Woodford, Giannoni-Woodford)

- Advantages of Brock-Durlauf approach?
  - Elegance and conciseness (but revise notation somewhat)
  - Frequency domain more relevant/practical/intuitive than time domain?
    - ∗ Some frequencies more relevant (Onatski-Williams)
  - Design limits? Bode's constraint?
    - ∗ Tradeoff across frequencies (but when $K > 0$ not constant, ...)
    - ∗ Design limits (imperfect control) also in time domain
      - · Optimal x_t obvious in time domain

- Better for practical (monetary) policy: Robust decision process
  - "Forecast targeting" rather than commitment to particular instrument rule (Svensson JEL 03, 04)
    - ∗ No central bank will seriously consider commitment to particular instrument rule
    - ∗ Bayesian (adaptive) optimal control: Use all relevant information, optimize in a timeless perspective, Bayesian updating/learning of model