

# Monetary Policy Trade-Offs in an Estimated Open-Economy DSGE Model\*

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## Abstract

This paper studies the trade-offs between stabilizing CPI inflation and alternative measures of the output gap in Ramses, the Riksbank's estimated dynamic stochastic general equilibrium (DSGE) model of a small open economy. Our main finding is that the trade-off between stabilizing CPI inflation and the output gap strongly depends on which concept of potential output in the output gap between output and potential output is used in the loss function. If potential output is defined as a smooth trend this trade-off is much more pronounced compared to the case when potential output is defined as the output level that would prevail if prices and wages were flexible.

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## Contents

1	Introduction . . . . .	1
2	Model and parameters . . . . .	5
2.1	Model overview . . . . .	5
2.1.1	Domestic goods firms . . . . .	5
2.1.2	Importing and exporting firms . . . . .	6
2.1.3	Households . . . . .	7
2.1.4	Structural shocks, government, foreign economy . . . . .	10
2.1.5	Monetary policy . . . . .	10
2.2	Parameterization . . . . .	12
3	Variance trade-offs for the central bank . . . . .	13
3.1	Benchmark results . . . . .	13
3.2	Discerning the benchmark results: monetary policy and the transmission of shocks . . . . .	17
3.3	Limiting the volatility of short-term nominal interest rates . . . . .	22
4	Conclusions . . . . .	23
	Appendix . . . . .	25
A	Model solution . . . . .	25
B	Parameters . . . . .	26
C	Unconditional variances . . . . .	28

## 1. Introduction

In this paper, we use an estimated open economy model to study the trade-off between stabilizing CPI inflation and the output gap, and how this trade-off depends on alternative definitions of the output gap. Specifically, we compare variance trade-offs under optimal monetary policy and under an estimated instrument rule. We do this analysis in Ramses, the main model used at Sveriges Riksbank for forecasting and policy analysis. Ramses is a small open-economy dynamic stochastic general equilibrium (DSGE) model estimated with Bayesian techniques and is described in Adolfson, Laséen, Lindé, and Villani (ALLV) [4] and [5].

The notion that alternative definitions of the output gap can have important implications for the conduct of monetary policy is visualized in figure 1.1, which depicts one statistical and three model-based output gaps in Sweden 1997-2007.<sup>1</sup> As expected, the correlation is highest between the statistical HP-filtered output gap and the model trend output gap (where the trend is the model's unit-root technology shock). Even so, the upper panel of the figure demonstrates that the correlation between the routinely-used statistical HP-filtered output gap and all three model based gaps is well below unity, and that their variances are also clearly different.<sup>2</sup> By implication, adhering to one of these measures should have non-trivial implications for monetary policy.

We define optimal monetary policy as a central bank that minimizes an intertemporal loss function under commitment. We assume the central bank adopts a quadratic loss function that corresponds to flexible inflation targeting and is the weighted sum of three terms: the squared inflation gap between 4-quarter CPI inflation and the inflation target, the squared output gap (measured as the deviation between output and potential output), and the squared quarterly change in the central bank's policy rate. To get an idea about how inefficient the empirically estimated rule is compared with optimal policy and about the policy preferences implied by the estimated rule, we compare the optimal policy with policy following the estimated instrument rule.

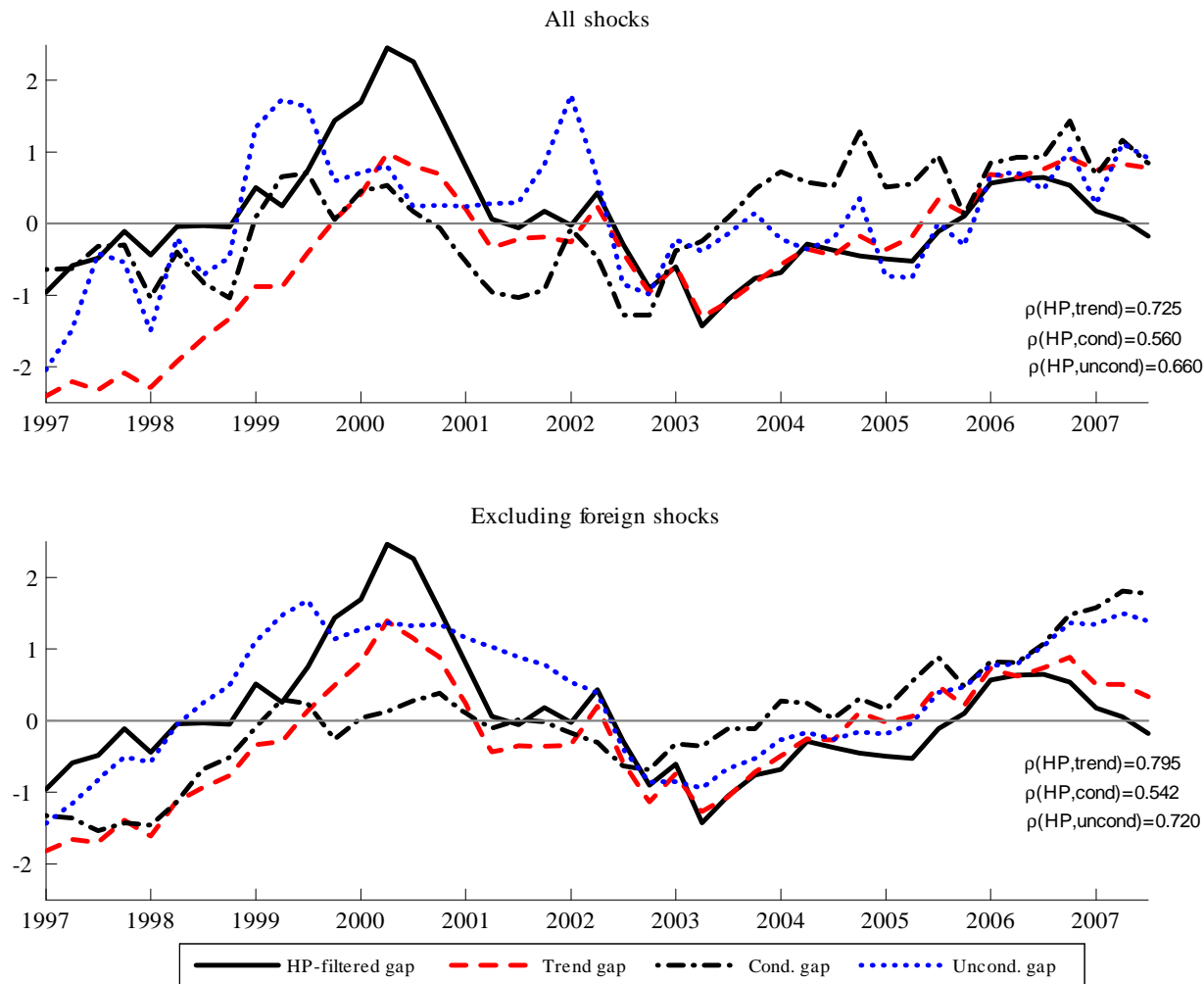
The definition of potential output is important since this latent variable is used to compute the output gap (the difference between output and potential output) in the loss function. A conventional measure of potential output is a smooth trend, such as the result of a Hodrick-Prescott (HP) filter.

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<sup>1</sup> We use Swedish data on seasonally adjusted GDP per capita 1980Q2 -2007Q3 as our measure of output. Potential output computed with the HP-filter uses a smoothing coefficient of  $\lambda = 1600$  on actual data, whereas the trend, flexible price conditional and unconditional potential output is computed via Kalman filtering techniques using the estimated model in section 2. Exact definitions of the various concepts of potential output in the model are provided in section 2.1.5.

<sup>2</sup> The correlation coefficients between the HP-filtered output gap and the estimated DSGE model's output gaps are not computed on data after 2005Q4 to avoid the well-known endpoint problems of the HP-filter (which causes the HP-filtered gap to drop notably towards the end of the sample in Figure 1.1).

Figure 1.1: Output gaps for Sweden 1997Q1 - 2007Q3 using different measures of potential output.



A second definition of potential output, promoted in the recent academic literature, is defined as the level of output that would prevail if prices and wages were flexible, see for instance Woodford [25] and Galí [14]. This latter measure of potential output is in line with the work of Kydland and Prescott [19], since it incorporates efficient fluctuations of output due to technology shocks.

Using an approach similar to ours, subsequent work by Justiniano, Primiceri and Tambalotti [17], and Edge, Kiley and Laforte [13] present measures of potential output for the US economy within closed-economy frameworks. Justiniano, Primiceri and Tambalotti [17] study the inflation and output stabilization trade-off in the US using an estimated DSGE model. They find that the gap between optimal output (maximizing the household's utility function) and potential output (the fully competitive equilibrium) is virtually zero once they treat the observed high-frequency

movement in wages as measurement errors rather than variations in workers' market power. Therefore, they conclude that inefficient movements in US output could have been eliminated without increasing price and wage inflation. To the extent that the welfare function is a good representation of the actual monetary policy objectives, they find that the historical conduct of monetary policy - as described by an estimated interest rate rule - has not performed well. We extend their analysis to an open-economy setting by using an estimated DSGE model with trade channels. By comparing the upper and lower panels in figure 1.1 above, we see that open economy aspects matter importantly for the computed output gaps.<sup>3</sup> Another important difference is that we build on the recent empirical results in Gali, Smets and Wouters [15], and assume that observed movements in real wage represent variations in workers' market power. Finally, and as mentioned above, we do not use the model's welfare function, but model the monetary policy objectives directly.

Our analysis focuses on the variance trade-off the central bank is facing under various specifications of the loss function, comparing the different output-gap definitions. Results for the estimated instrument rule are also reported. The efficient variance frontiers are computed with a given weight on interest-rate smoothing. As a benchmark, we use a weight of 0.37 on the squared changes in the nominal interest rate in the loss function.<sup>4</sup> However, it turns out that the volatility of the nominal interest rate in this case heavily violates the zero lower bound (ZLB) of the interest rate. Therefore, we also follow the suggestion by Woodford [25] and Levine, Pearlman, and Yang [20] and investigate to what extent the efficient variance frontier is affected by increasing the weight on the squared interest rate in the loss function, in order to limit the volatility of policy rates and thus ensure a low probability of the nominal interest rate falling below zero. In addition, we quantify to what extent the estimated instrument rule can be improved by optimizing the response coefficients of the simple instrument rule to minimize the loss function. Finally, we examine how different sets of shocks (technology, markup, preference, and foreign shocks) affect the variance trade-offs faced by the central bank for different definitions of the output gap in the loss function.

Our main findings are as follows. First, the stationary productivity shocks create a sharp trade-off between stabilizing CPI inflation and stabilizing the output gap when trend output is computed with a smooth trend. The estimated model assigns a dominant role to shocks to total factor

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<sup>3</sup> The foreign shocks are those defined in Section 3, with the exception that the unit root technology shock (which is common to both the domestic and foreign economies to ensure balanced growth in the model) is here treated as a domestic shock for comparison with the closed economy literature. If we include the permanent technology shock among the set of foreign shocks, we would see more noticeable differences in the low frequency components of the output gaps in Figure 1.1. Finally, notice that the statistical HP-filtered gap is kept unchanged in both panels to provide a basis of reference.

<sup>4</sup> This number stems from estimating the model on Swedish data under the assumption that the Riksbank conducted monetary policy according to the loss function with the trend output gap, see ALLS [2].

productivity as a driver of business cycles in Sweden in order to explain the fact that the correlation between GDP growth and CPI inflation is about  $-0.5$  for the years 1950–2007. Productivity shocks have also been shown by ALLV [6] to play a key role for understanding the episode with low inflation and high output growth in Sweden 2003–2006 (at least for policy under a simple instrument rule). Second, using an output gap in the loss function where potential output is defined as the level of output under flexible prices and wages improves the policy trade-off, but the trade-off still remains significant for some shocks, notably labor supply shocks (which are isomorphic to wage markup shocks) and price markup shocks. The specification of potential output in the output-gap definition is of key importance for the transmission of stationary technology shocks: if potential output is defined as trend output, the output response after a technology shock will be substantially smaller than if potential output is specified as the level of output under flexible prices and wages.<sup>5</sup> A labor shock, on the other hand, creates a trade-off between inflation and output-gap stabilization regardless of which output-gap definition is used.

Third, we find that the estimated instrument rule is clearly inefficient relative to optimal policy. Our analysis documents that most of this ineffectiveness is driven by the fact that the estimated policy rule responds very inefficiently to fluctuations induced by foreign shocks. Fourth, optimizing the coefficients in the simple instrument rule closes about half the distance relative to optimal policy. Finally, limiting the volatility of the short-term nominal interest rate shifts out the variance frontiers somewhat, but the conclusions regarding the trend output gap and the flexible price-wage output gap are – at least in our approximative approach – robust to introducing this constraint.<sup>6</sup>

The outline of the paper is as follows: Section 2 presents the model and very briefly discusses the data and the estimation of the model. Section 3 illustrates the variance trade-offs the central bank is facing under different output-gap definitions and attempt to examine their origins. Finally, section 4 presents a summary and some conclusions. An appendix contains some technical details. More technical details are reported in Adolfson, Laséen, Lindé and Svensson (ALLS) [3].

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<sup>5</sup> As in ALLS [2], we consider both a conditional and an unconditional measure of potential output under flexible prices and wages. Conditional potential output is contingent upon the existing current predetermined variables, whereas unconditional potential output is computed assuming the flexible price equilibrium has lasted forever, see Section 2.1.5 for further details.

<sup>6</sup> This conclusion is supported by the findings in Hebden, Lindé and Svensson [16] which show, by means of stochastic simulations in the standard hybrid New Keynesian model, that the difference between unconstrained (no zero bound constraint) and constrained (respecting the non-linear zero lower bound constraint) optimal monetary policy under commitment differs very little for empirically plausible probabilities of hitting the zero lower bound.

## 2. Model and parameters

### 2.1. Model overview

Ramses is a small open-economy DSGE model developed in a series of papers by ALLV [4] and [5], and shares its basic closed economy features with many new Keynesian models, including the benchmark models of Christiano, Eichenbaum and Evans [9], Altig, Christiano, Eichenbaum and Lindé [7], and Smets and Wouters [23]. The model economy consists of households, domestic goods firms, importing consumption and importing investment firms, exporting firms, a government, a central bank, and an exogenous foreign economy. Within each manufacturing sector there is a continuum of firms that each produces a differentiated good and sets prices according to an indexation variant of the Calvo model. Domestic as well as global production grows with technology that contains a stochastic unit-root, see Altig et al. [7]. In what follows we provide the optimization problems of the different firms and the households, and describe the behavior of the central bank.<sup>7</sup>

#### 2.1.1. Domestic goods firms

The domestic goods firms produce their goods using capital and labor inputs, and sell them to a retailer which transforms the intermediate products into a homogenous final good that in turn is sold to the households.

The final domestic good is a composite of a continuum of differentiated intermediate goods, each supplied by a different firm. Output,  $Y_t$ , of the final domestic good is produced with the constant elasticity of substitution (CES) function

$$Y_t = \left[ \int_0^1 (Y_{it})^{\frac{1}{\lambda_t^d}} di \right]^{\lambda_t^d}, \quad 1 \leq \lambda_t^d < \infty, \quad (2.1)$$

where  $Y_{it}$ ,  $0 \leq i \leq 1$ , is the input of intermediate good  $i$  and  $\lambda_t^d$  is a stochastic process that determines the time-varying flexible-price markup in the domestic goods market. The production of the intermediate good  $i$  by intermediate-good firm  $i$  is given by

$$Y_{it} = z_t^{1-\alpha} \epsilon_t K_{it}^\alpha H_{it}^{1-\alpha} - z_t \phi, \quad (2.2)$$

where  $z_t$  is a unit-root technology shock common to the domestic and foreign economies,  $\epsilon_t$  is a domestic covariance stationary technology shock,  $K_{it}$  the capital stock and  $H_{it}$  denotes homogeneous

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<sup>7</sup> For a complete list of the log-linearized equations in the model we refer to ALLS [2].

labor hired by the  $i^{th}$  firm. A fixed cost  $z_t\phi$  is included in the production function. We set this parameter so that profits are zero in steady state, following Christiano et al. [9].

We allow for working capital by assuming that a fraction  $\nu$  of the intermediate firms' wage bill has to be financed in advance through loans from a financial intermediary. Cost minimization yields the following nominal marginal cost for intermediate firm  $i$ :

$$\text{MC}_{it}^d = \frac{1}{(1-\alpha)^{1-\alpha}} \frac{1}{\alpha^\alpha} (R_t^k)^\alpha [W_t(1 + \nu(R_{t-1} - 1))]^{1-\alpha} \frac{1}{z_t^{1-\alpha}} \frac{1}{\epsilon_t}, \quad (2.3)$$

where  $R_t^k$  is the gross nominal rental rate per unit of capital,  $R_{t-1}$  the gross nominal (economy wide) interest rate, and  $W_t$  the nominal wage rate per unit of aggregate, homogeneous, labor  $H_{it}$ .

Each of the domestic goods firms is subject to price stickiness through an indexation variant of the Calvo [8] model. Each intermediate firm faces in any period a probability  $1 - \xi_d$  that it can reoptimize its price. The reoptimized price is denoted  $P_t^{d,new}$ . For the firms that are not allowed to reoptimize their price, we adopt an indexation scheme with partial indexation to the current inflation target,  $\bar{\pi}_{t+1}^c$ , since there is a perceived (time-varying) CPI inflation target in the model, and partial indexation to last period's inflation rate in order to allow for a lagged pricing term in the Phillips curve

$$P_{t+1}^d = \left(\pi_t^d\right)^{\kappa_d} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_d} P_t^d, \quad (2.4)$$

where  $P_t^d$  is the price level,  $\pi_t^d = P_{t+1}^d/P_t^d$  is gross inflation in the domestic sector, and  $\kappa_d$  is an indexation parameter. The different firms maximize profits taking into account that there might not be a chance to optimally change the price in the future. Firm  $i$  therefore faces the following optimization problem when setting its price

$$\max_{P_t^{d,new}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\xi_d)^s v_{t+s} [((\pi_t^d \pi_{t+1}^d \dots \pi_{t+s-1}^d)^{\kappa_d} (\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c \dots \bar{\pi}_{t+s}^c)^{1-\kappa_d} P_t^{d,new}) Y_{i,t+s} - \text{MC}_{i,t+s}^d (Y_{i,t+s} + z_{t+s}\phi^j)], \quad (2.5)$$

where the firm is using the stochastic household discount factor  $(\beta\xi_d)^s v_{t+s}$  to make profits conditional upon utility.  $\beta$  is the discount factor, and  $v_{t+s}$  the marginal utility of the households' nominal income in period  $t + s$ , which is exogenous to the intermediate firms.

### 2.1.2. Importing and exporting firms

The importing consumption and importing investment firms buy a homogenous good at price  $P_t^*$  in the world market, and convert it into a differentiated good through a brand naming technology. The exporting firms buy the (homogenous) domestic final good at price  $P_t^d$  and turn this into



a differentiated export good through the same type of brand naming. The nominal marginal cost of the importing and exporting firms are thus  $S_t P_t^*$  and  $P_t^d/S_t$ , respectively, where  $S_t$  is the nominal exchange rate (domestic currency per unit of foreign currency). The differentiated import and export goods are subsequently aggregated by an import consumption, import investment and export packer, respectively, so that the final import consumption, import investment, and export good is each a CES composite according to the following:

$$C_t^m = \left[ \int_0^1 (C_{it}^m)^{\frac{1}{\lambda_t^{mc}}} di \right]^{\lambda_t^{mc}}, \quad I_t^m = \left[ \int_0^1 (I_{it}^m)^{\frac{1}{\lambda_t^{mi}}} di \right]^{\lambda_t^{mi}}, \quad X_t = \left[ \int_0^1 (X_{it})^{\frac{1}{\lambda_t^x}} di \right]^{\lambda_t^x}, \quad (2.6)$$

where  $1 \leq \lambda_t^j < \infty$  for  $j = \{mc, mi, x\}$  is the time-varying flexible-price markup in the import consumption ( $mc$ ), import investment ( $mi$ ) and export ( $x$ ) sector. By assumption the continuum of consumption and investment importers invoice in the domestic currency and exporters in the foreign currency. To allow for short-run incomplete exchange rate pass-through to import as well as export prices we introduce nominal rigidities in the local currency price. This is modeled through the same type of Calvo setup as above. The price setting problems of the importing and exporting firms are completely analogous to that of the domestic firms in equation (2.5).<sup>8</sup> In total there are thus four specific Phillips curve relations determining inflation in the domestic, import consumption, import investment and export sectors.

### 2.1.3. Households

There is a continuum of households which attain utility from consumption, leisure and real cash balances. The preferences of household  $j$  are given by

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t^c \ln(C_{jt} - bC_{j,t-1}) - \zeta_t^h A_L \frac{(h_{jt})^{1+\sigma_L}}{1+\sigma_L} + A_q \frac{\left(\frac{Q_{jt}}{z_t P_t^d}\right)^{1-\sigma_q}}{1-\sigma_q} \right], \quad (2.7)$$

where  $C_{jt}$ ,  $h_{jt}$  and  $Q_{jt}/P_t^d$  denote the  $j^{th}$  household's levels of aggregate consumption, labor supply and real cash holdings, respectively. Consumption is subject to habit formation through  $bC_{j,t-1}$ , such that the household's marginal utility of consumption is increasing in the quantity of goods consumed last period.  $\zeta_t^c$  and  $\zeta_t^h$  are persistent preference shocks to consumption and labor supply,

<sup>8</sup>Total export demand satisfies  $C_t^x + I_t^x = \left[\frac{P_t^x}{P_t^*}\right]^{-\eta_f} Y_t^*$ , where  $C_t^x$  and  $I_t^x$  is demand for consumption and investment goods, respectively;  $P_t^x$  the export price;  $P_t^*$  the foreign price level;  $Y_t^*$  foreign output and  $\eta_f$  the elasticity of substitution across foreign goods.

respectively. Households consume a basket of domestically produced goods ( $C_t^d$ ) and imported products ( $C_t^m$ ) which are supplied by the domestic and importing consumption firms, respectively. Aggregate consumption is assumed to be given by the following CES function:

$$C_t = \left[ (1 - \omega_c)^{1/\eta_c} (C_t^d)^{(\eta_c-1)/\eta_c} + \omega_c^{1/\eta_c} (C_t^m)^{(\eta_c-1)/\eta_c} \right]^{\eta_c/(\eta_c-1)},$$

where  $\omega_c$  is the share of imports in consumption, and  $\eta_c$  is the elasticity of substitution across consumption goods.

The households can invest in their stock of capital, save in domestic bonds and/or foreign bonds and hold cash. The households invest in a basket of domestic and imported investment goods to form the capital stock, and decide how much capital to rent to the domestic firms given costs of adjusting the investment rate. The households can increase their capital stock by investing in additional physical capital ( $I_t$ ), taking one period to come in action. The capital accumulation equation is given by

$$K_{t+1} = (1 - \delta)K_t + \Upsilon_t [1 - \tilde{S}(I_t/I_{t-1})] I_t, \quad (2.8)$$

where  $\tilde{S}(I_t/I_{t-1})$  determines the investment adjustment costs through the estimated parameter  $\tilde{S}''$ , and  $\Upsilon_t$  is a stationary investment-specific technology shock. Total investment is assumed to be given by a CES aggregate of domestic and imported investment goods ( $I_t^d$  and  $I_t^m$ , respectively) according to

$$I_t = \left[ (1 - \omega_i)^{1/\eta_i} (I_t^d)^{(\eta_i-1)/\eta_i} + \omega_i^{1/\eta_i} (I_t^m)^{(\eta_i-1)/\eta_i} \right]^{\eta_i/(\eta_i-1)}, \quad (2.9)$$

where  $\omega_i$  is the share of imports in investment, and  $\eta_i$  is the elasticity of substitution across investment goods.

Each household is a monopoly supplier of a differentiated labor service which implies that they can set their own wage, see Erceg, Henderson and Levin [12]. After having set their wage, households supply the firms' demand for labor,

$$h_{jt} = \left[ \frac{W_{jt}}{W_t} \right]^{\frac{\lambda_w}{1-\lambda_w}} H_t,$$

at the going wage rate. Each household sells its labor to a firm which transforms household labor into a homogenous good that is demanded by each of the domestic goods producing firms. Wage stickiness is introduced through the Calvo [8] setup, where household  $j$  reoptimizes its nominal

wage rate  $W_{jt}^{new}$  according to the following<sup>9</sup>

$$\max_{W_{jt}^{new}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \xi_w)^s [-\zeta_{t+s}^h A_L \frac{(h_{j,t+s})^{1+\sigma_L}}{1+\sigma_L} + v_{t+s} \frac{(1-\tau_{t+s}^y)}{(1+\tau_{t+s}^w)} \left( (\pi_t^c \dots \pi_{t+s-1}^c)^{\kappa_w} (\bar{\pi}_{t+1}^c \dots \bar{\pi}_{t+s}^c)^{(1-\kappa_w)} (\mu_{z,t+1} \dots \mu_{z,t+s}) W_{jt}^{new} \right) h_{j,t+s}], \quad (2.10)$$

where  $\xi_w$  is the probability that a household is not allowed to reoptimize its wage,  $\tau_t^y$  a labor income tax,  $\tau_t^w$  a pay-roll tax (paid for simplicity by the households), and  $\mu_{zt} = z_t/z_{t-1}$  is the growth rate of the unit-root technology shock.

The choice between domestic and foreign bond holdings balances into an arbitrage condition pinning down expected exchange rate changes (that is, an uncovered interest rate parity condition). To ensure a well-defined steady-state in the model, we assume that there is premium on the foreign bond holdings which depends on the aggregate net foreign asset position of the domestic households, see, for instance, Schmitt-Grohé and Uribe [22]. Compared to a standard setting the risk premium is allowed to be negatively correlated with the expected change in the exchange rate (that is, the expected depreciation), following the evidence discussed in for example Duarte and Stockman [11]. For a detailed discussion and evaluation of this modification see ALLV [5]. The risk premium is given by:

$$\Phi(a_t, S_t, \tilde{\phi}_t) = \exp \left( -\tilde{\phi}_a(a_t - \bar{a}) - \tilde{\phi}_s \left( \frac{E_t S_{t+1}}{S_t} \frac{S_t}{S_{t-1}} - 1 \right) + \tilde{\phi}_t \right), \quad (2.11)$$

where  $a_t \equiv (S_t B_t^*) / (P_t z_t)$  is the net foreign asset position,  $S_t$  the nominal exchange rate, and  $\tilde{\phi}_t$  is a shock to the risk premium.

To clear the final goods market, the foreign bond market, and the loan market for working capital, the following three constraints must hold in equilibrium:

$$C_t^d + I_t^d + G_t + C_t^x + I_t^x \leq z_t^{1-\alpha} \epsilon_t K_t^\alpha H_t^{1-\alpha} - z_t \phi, \quad (2.12)$$

$$S_t B_{t+1}^* = S_t P_t^x (C_t^x + I_t^x) - S_t P_t^* (C_t^m + I_t^m) + R_{t-1}^* \Phi(a_{t-1}, \tilde{\phi}_{t-1}) S_t B_t^*, \quad (2.13)$$

$$\nu W_t H_t = \mu_t M_t - Q_t, \quad (2.14)$$

where  $G_t$  is government expenditures,  $C_t^x$  and  $I_t^x$  are the foreign demand for export goods which follow CES aggregates with elasticity  $\eta_f$ , and  $\mu_t = M_{t+1}/M_t$  is the monetary injection by the central bank. When defining the demand for export goods, we introduce a stationary asymmetric (or foreign) technology shock  $\tilde{z}_t^* = z_t^*/z_t$ , where  $z_t^*$  is the permanent technology level abroad, to allow for temporary differences in permanent technological progress domestically and abroad.

<sup>9</sup>For the households that are not allowed to reoptimize, the indexation scheme is  $W_{j,t+1} = (\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w)} \mu_{z,t+1} W_{jt}$ , where  $\kappa_w$  is an indexation parameter.

### 2.1.4. Structural shocks, government, foreign economy

The structural shock processes in the model are given by the univariate representation

$$\hat{\varsigma}_t = \rho_\varsigma \hat{\varsigma}_{t-1} + \varepsilon_{\varsigma t}, \quad \varepsilon_{\varsigma t} \stackrel{iid}{\sim} N(0, \sigma_\varsigma^2) \quad (2.15)$$

where  $\varsigma_t = \{ \mu_{zt}, \epsilon_t, \lambda_t^j, \zeta_t^c, \zeta_t^h, \Upsilon_t, \tilde{\phi}_t, \varepsilon_{Rt}, \bar{\pi}_t^c, \tilde{z}_t^* \}$ ,  $j = \{d, mc, mi, x\}$ , and a hat denotes the deviation of a log-linearized variable from a steady-state level ( $\hat{v}_t \equiv dv_t/v$  for any variable  $v_t$ , where  $v$  is the steady-state level).  $\lambda_t^j$  and  $\varepsilon_{Rt}$  are assumed to be white noise (that is,  $\rho_{\lambda^j} = 0$ ,  $\rho_{\varepsilon_R} = 0$ ).

The government spends resources on consuming part of the domestic good, and collects taxes from the households. The resulting fiscal surplus/deficit plus the seigniorage are assumed to be transferred back to the households in a lump sum fashion. Consequently, there is no government debt. The fiscal policy variables – taxes on labor income ( $\hat{\tau}_t^y$ ), consumption ( $\hat{\tau}_t^c$ ), and the payroll ( $\hat{\tau}_t^w$ ), together with (HP-detrended) government expenditures ( $\hat{g}_t$ ) – are assumed to follow an identified VAR model with two lags,

$$\Theta_0 \tau_t = \Theta_1 \tau_{t-1} + \Theta_2 \tau_{t-2} + S_\tau \varepsilon_{\tau t}, \quad (2.16)$$

where  $\tau_t \equiv (\hat{\tau}_t^y, \hat{\tau}_t^c, \hat{\tau}_t^w, \hat{g}_t)'$ ,  $\varepsilon_{\tau t} \sim N(0, I_\tau)$ ,  $S_\tau$  is a diagonal matrix with standard deviations and  $\Theta_0^{-1} S_\tau \varepsilon_{\tau t} \sim N(0, \Sigma_\tau)$ .

Since Sweden is a small open economy we assume that the foreign economy is exogenous. Foreign inflation,  $\pi_t^*$ , output (HP-detrended),  $\hat{y}_t^*$ , and interest rate,  $R_t^*$ , are exogenously given by an identified VAR model with four lags,

$$\Phi_0 X_t^* = \Phi_1 X_{t-1}^* + \Phi_2 X_{t-2}^* + \Phi_3 X_{t-3}^* + \Phi_4 X_{t-4}^* + S_{x^*} \varepsilon_{x^* t}, \quad (2.17)$$

where  $X_t^* \equiv (\pi_t^*, \hat{y}_t^*, R_t^*)'$ ,  $\varepsilon_{x^* t} \sim N(0, I_{x^*})$ ,  $S_{x^*}$  is a diagonal matrix with standard deviations and  $\Phi_0^{-1} S_{x^*} \varepsilon_{x^* t} \sim N(0, \Sigma_{x^*})$ . Given our assumption of equal substitution elasticities in foreign consumption and investment, these three variables suffice to describe the foreign economy in our model setup.

### 2.1.5. Monetary policy

Monetary policy is modeled in two different ways. First, we assume that the central bank minimizes an intertemporal loss function under commitment. Let the intertemporal loss function in period  $t$  be

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}, \quad (2.18)$$

where  $0 < \delta < 1$  is a discount factor, and  $L_t$  is the period loss given by

$$L_t = (p_t^c - p_{t-4}^c - \bar{\pi}^c)^2 + \lambda_y (y_t - \bar{y}_t)^2 + \lambda_{\Delta i} (i_t - i_{t-1})^2, \quad (2.19)$$

where the central bank's target variables are; the model-consistent year-over-year CPI inflation,  $p_t^c - p_{t-4}^c$ , where  $p_t^c$  denotes the log of the CPI and  $\bar{\pi}^c$  is the 2% inflation target; a measure of the output gap,  $y_t - \bar{y}_t$ , where  $y_t$  denotes output and  $\bar{y}_t$  denotes potential output; and the first difference of the instrument rate,  $i_t - i_{t-1}$ , where  $i_t$  denotes the Riksbank's instrument rate, the repo rate, and  $\lambda_y$  and  $\lambda_{\Delta i}$  are nonnegative weights on output-gap stabilization and instrument-rate smoothing, respectively.<sup>10,11</sup>

We compare results from two different measures of the output gap ( $y_t - \bar{y}_t$ ) in the loss function. The first measure, the *trend output gap* uses the trend production level as potential output ( $\bar{y}_t$ ), which is growing stochastically due to the unit-root stochastic technology shock in the model. This definition of potential output will resemble a potential output computed with an HP filter.<sup>12</sup> The second measure, the *unconditional output gap*, specifies potential output as the hypothetical output level that would arise if prices and wages were completely flexible and had been so for a very long time. Unconditional potential output therefore presumes different levels of the predetermined variables, including the capital stock, from those in the actual economy. In precise form the two different concepts of *potential* output are

$$\begin{aligned} \bar{y}_t^{\text{trend}} &= z_t, \\ \bar{y}_t^{\text{uncond}} &= F_y^f X_t^f, \end{aligned}$$

where  $z_t$  is the unit-root technology shock, the row vector  $F_y^f$  expresses output as a function of the predetermined state variables in the flex-price economy, and  $X_t^f$  is the state vector in the economy

<sup>10</sup> We use year-over-year inflation as a target variable rather than quarterly inflation, since the Riksbank and other inflation-targeting central banks normally specify their inflation target as a 12-month rate.

<sup>11</sup> The inflation target variable is assumed to be model-consistent CPI inflation since this measure more accurately captures the true import content in the consumption basket. In the model, where total consumption is a CES function of imported and domestic goods, model-consistent CPI inflation is

$$p_t^{c,\text{model}} - p_{t-4}^{c,\text{model}} = (1 - \omega_c) (p^c/p^d)^{-(1-\eta_c)} (p_t^d - p_{t-4}^d) + \omega_c (p^c/p^{m,c})^{-(1-\eta_c)} (p_t^{m,c} - p_{t-4}^{m,c}), \quad (2.20)$$

where  $\omega_c$  is the share of expenditures in the CPI spent on imported goods,  $p_t^d$  the (log) domestic price level and  $p_t^{m,c}$  the (log) price of imported goods that the consumer has to pay. The weights used to calculate the model-consistent inflation differ from those in the data by the steady-state relative prices ( $p^c/p^d$  and  $p^c/p^{m,c}$ ), which lower the import share in consumption. This definition of CPI inflation is consequently consistent with the notion that due to distribution costs etc., the import share of consumption is somewhat exaggerated in the official statistics.

<sup>12</sup> The correlation between the trend output gap and an output gap computed with the HP-filter is about 0.65 using 5000 observations of simulated data from the model.

with flexible prices and wages (see Appendix A for a description of the model solution and these matrices).<sup>13</sup>

We define the *flexprice equilibrium* under the assumption that prices and wages are completely flexible in the domestic economy (thus keeping the foreign economy distorted), and determine the nominal variables by assuming that CPI inflation is kept constant at its steady-state level ( $\hat{\pi}_t^c = 0$ ). When computing the two cases of flexprice potential output we also disregard markup shocks and fiscal shocks, and set these to zero in the flexprice economy.

Second, we assume monetary policy obeys an instrument rule, following Smets and Wouters [23], where the central bank adjusts the short term interest rate in response to deviations of CPI inflation from the perceived inflation target, the trend output gap (measured as actual minus trend output)<sup>14</sup>, the real exchange rate ( $\hat{x}_t \equiv \hat{S}_t + \hat{P}_t^* - \hat{P}_t^c$ ) and the interest rate set in the previous period. The instrument rule (expressed in log-linearized terms) follows:

$$\begin{aligned} i_t = & \rho_{Rt} i_{t-1} + (1 - \rho_{Rt}) \left[ \bar{\pi}_t^c + r_{\pi t} (p_t^c - p_{t-1}^c - \bar{\pi}_t^c) + r_{yt} (y_{t-1} - \bar{y}_{t-1}) + r_{xt} \hat{x}_{t-1} \right] \\ & + r_{\Delta\pi,t} \Delta(p_t^c - p_{t-1}^c) + r_{\Delta y,t} \Delta(y_t - \bar{y}_t) + \varepsilon_{Rt}, \end{aligned} \quad (2.21)$$

where  $\Delta$  denotes the first-difference operator,  $\bar{\pi}_t^c$  is a time-varying inflation target, a hat denotes log-deviations from steady-state, and  $\varepsilon_{Rt}$  is an uncorrelated monetary-policy shock.

## 2.2. Parameterization

The model's parameters are estimated using Bayesian techniques on 15 Swedish macroeconomic variables during the period 1980Q1–2007Q3. We refer the reader to ALLS [2] for a detailed description of the estimation. To make the paper self-contained we report in appendix B the values for the calibrated parameters (table B.1), the prior distributions we use in the estimation and the obtained posterior results (table B.2). In the subsequent analysis the estimated posterior mode values under the estimated instrument rule are used for all the non-policy parameters. The estimates of the model parameters suggest that they are invariant with respect to our alternative assumptions about monetary policy during the inflation targeting period (1993-), so we treat them as structural and

<sup>13</sup> Figure 1.1 also contains a third model-based measure, the conditional output gap, which makes potential output contingent upon the existing current predetermined variables. Conditional potential output is thus defined as the hypothetical output level that would arise if prices and wages suddenly become flexible in the current period and are expected to remain flexible in the future, i.e.  $\bar{y}_t^{\text{cond}} = F_y^f X_t$  where  $X_t$  is the vector of predetermined state variables. For a detailed description on how to calculate the unconditional and conditional potential output, see appendix C in ALLS [2].

<sup>14</sup> The trend output gap, rather than the unconditional output gap, seems to more closely correspond to the measure of resource utilization that the Riksbank has been responding to historically, see ALLV [5]. Del Negro, Schorfheide, Smets, and Wouters [10] report similar results for the US.

independent of the monetary policy (see table B.2). Clearly, this assumption is more of a stretch in the subsequent analysis when the deviations from past policy behavior is particularly large, i.e. for very small or large values of  $\lambda_y$  (see section 3).

### 3. Variance trade-offs for the central bank

After having described the model environment, we now turn to an examination of the trade-offs the central bank is facing under optimal policy and under a simple instrument rule. As shown in Rudebusch and Svensson [21], when the intertemporal loss function (2.18) is scaled by  $1 - \delta$ , the expected (conditional) intertemporal loss becomes equal to the unconditional mean of the period loss function when the discount factor approaches unity ( $\lim_{\delta \rightarrow 1} E_t \sum_{\tau=0}^{\infty} (1 - \delta)\delta^\tau L_{t+\tau} = E[L_t]$ ). The unconditional mean of the period loss function satisfies

$$E[L_t] = \text{Var} [p_t^c - p_{t-4}^c] + \lambda_y \text{Var}[y_t - \bar{y}_t] + \lambda_{\Delta i} \text{Var}[i_t - i_{t-1}] \quad (3.1)$$

under the assumption that the unconditional mean of 4-quarter CPI inflation equals the inflation target ( $E[p_t^c - p_{t-4}^c] = \bar{\pi}^c$ ) and the unconditional mean of the output gap equals zero ( $E[y_t - \bar{y}_t] = 0$ ). Under these assumptions, optimal policy for different loss-function weights  $\lambda_y$  and  $\lambda_{\Delta i}$  results in efficient combinations of (unconditional) variances of inflation, the output gap, and the first-difference of the nominal interest rate. These variances for different loss-function weights provide the efficient relevant policy trade-offs between stabilization of inflation and the output gap and interest-rate smoothing. Appendix C shows how the unconditional variances are computed. To investigate the role of alternative measures of the output gap in the loss function, we show the variance trade-offs for either the trend output gap or the unconditional output gap in the loss function. We first study the variance trade-offs when all shocks are active (figure 3.1) in Section 3.1, and then move on to an analysis of which type of shock influences the trade-offs most (figures 3.2 - 3.4) in Section 3.2.<sup>15</sup> The curves referring to ZLB concern the case when an upper bound on the volatility of short-term nominal interest rate is imposed. They are discussed in section 3.3.

#### 3.1. Benchmark results

In figure 3.1, the second row of the left column shows the variance of the trend output gap plotted against the variance of inflation, where inflation is 4-quarter CPI-inflation. The curve is obtained

<sup>15</sup> Since we want to explore what would happen if the central bank either follows an optimized simple instrument rule or commits to a loss function, we set the policy and inflation target shocks to zero in this section of the paper (i.e.,  $\varepsilon_t^R = 0$ , and  $\bar{\pi}_t^c = 0$ ).

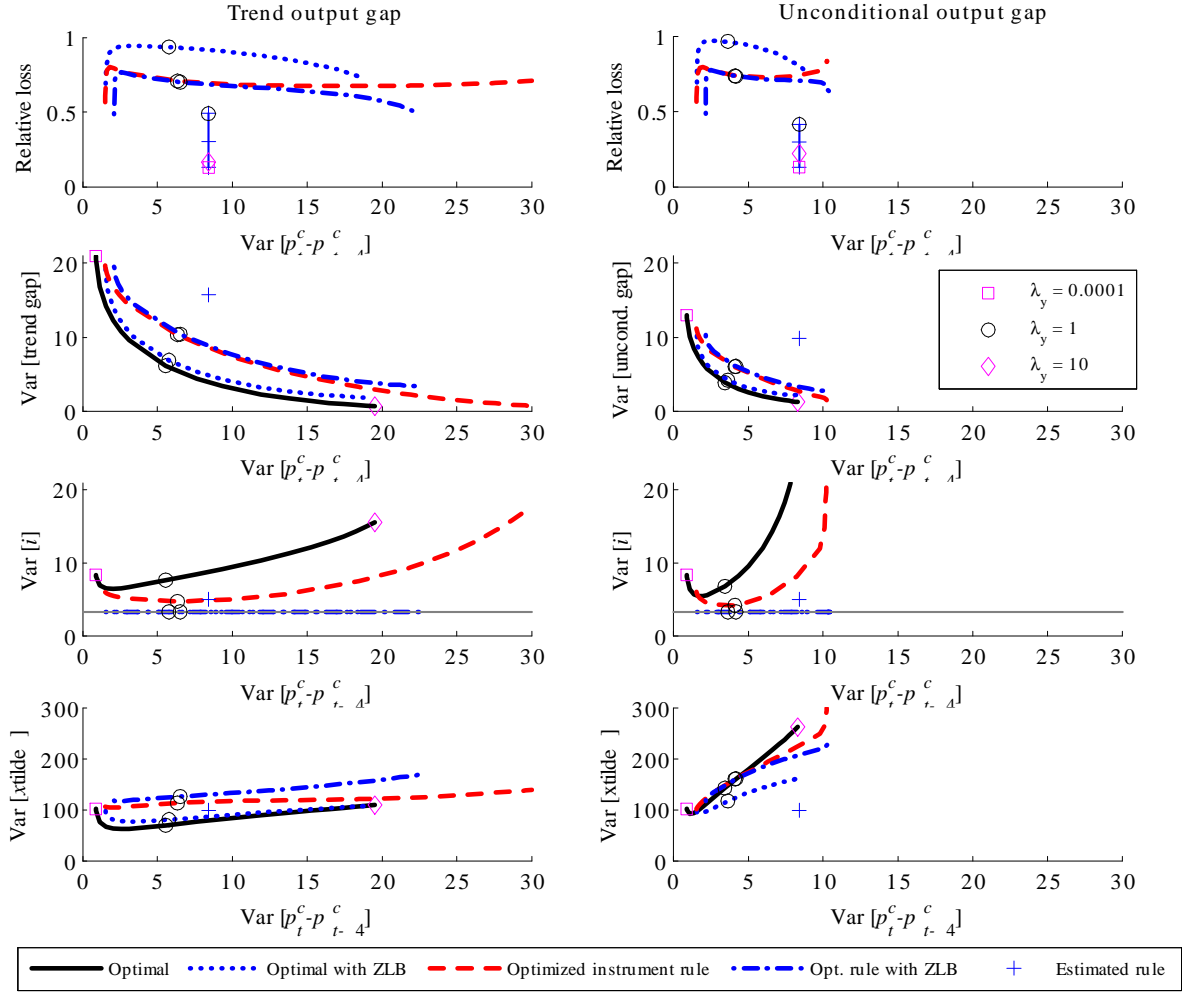
when varying the weight on output stabilization ( $\lambda_y$ ) in the loss function with the trend output gap, given a fixed weight on interest-rate smoothing ( $\lambda_{\Delta i} = 0.37$ ). The third row of the left column shows the corresponding variance of the nominal interest rate plotted against the variance of inflation, and the fourth row of the same column shows the variance of the real exchange rate plotted against the variance of inflation. Each  $\lambda_y$  results in a particular variance of inflation, and the figure should thus be read as if a vertical line through that level of inflation variance connects the three subplots. The curves are plotted for  $\lambda_y$  between 0.0001 and 10. A circle denotes the combination of variances resulting from  $\lambda_y = 1$ . On the solid curve only, the extreme low and high values for  $\lambda_y$  are marked by a square and diamond, respectively. The right column shows the variances when the unconditional output gap is used in the loss function instead of the trend output gap.

The top row of figure 3.1 shows the relative loss for the alternative policies we consider, expressed as the ratio between the unconditional mean loss under the optimal policy and the unconditional mean loss under the non-optimal (alternative) policies, plotted for each  $\lambda_y$  against the inflation variance of the non-optimal policy. Thus, the relative loss is bounded between zero and unity and shows what fraction of the loss for the non-optimal policy the loss for the optimal policy is. The vertical line marked with + shows the relative loss for the estimated instrument rule plotted against the (in this case constant) inflation variance for each  $\lambda_y$  of the loss function. Since the loss for the estimated rule is calculated according to equation (3.1) in this case the total loss will depend on the degree of output stabilization.

The figure shows that the gains from adhering to optimal policy instead of following the estimated instrument rule are substantial, especially for very small or large values of  $\lambda_y$ . For values of  $\lambda_y$  between 0.5 and 1.5, which seems most empirically relevant given the estimation results in Table B.2 in Appendix B, the estimated rule performs best relative to optimal policy for the trend output gap, but the loss is still about 50% higher relative to optimal policy. For the unconditional output gap, the estimated rule performs somewhat worse. Given that the rule is estimated on the trend output gap, this finding is not surprising. As noted previously, we assume that the model parameters are invariant to the way we model monetary policy. Hence, the results far away from the past policy behavior should be interpreted more cautiously. Nevertheless, according to our estimated model, there are thus considerable gains in conducting optimal policy instead of adhering to the estimated rule for both measures of the output gap. To examine to which extent the gap between the estimated simple rule in (2.21) and optimal policy can be closed upon by simply optimizing the



Figure 3.1: Variance trade-offs when using either the trend or unconditional output gap in loss function and optimized simple instrument rule.



response coefficients in the rule, we consider a slightly simplified version of the instrument rule,

$$i_t = \rho_R i_{t-1} + (1 - \rho_R) [\rho_\pi (p_t^c - p_{t-1}^c - \bar{\pi}^c) + \rho_y (y_t - \bar{y}_t)], \quad (3.2)$$

where the response coefficients  $\rho_R$ ,  $\rho_\pi$ , and  $\rho_y$  are chosen to minimize the unconditional mean of the central bank loss function (3.1) for each given  $\lambda_y$ .<sup>16</sup> We use the same output gap (trend or unconditional) in the simple instrument rule (3.2) and the unconditional mean of the loss function (3.1). The resulting optimized response coefficients in the simple instrument rule (3.2) are reported in table 3.1. We include forward-looking variables dated in period  $t$  in the simple instrument rule above, which means that the instrument rule not only depends on predetermined variables and is

<sup>16</sup> We use Matlab's optimizers 'fminunc' and 'fminsearch' repeatedly to find the global optimum for the different response coefficients.

hence an implicit rather than explicit instrument rule. Consequently, since the interest rate then depends on forward-looking variables which in turn depend on the interest rate, the instrument rule is an equilibrium relation rather than an operational realistic instrument rule. We include forward-looking variables in the simple instrument rule as a way to allow the interest rate also in this case to respond to some current shocks and hence to be less at a disadvantage compared with the optimal policy.

Table 3.1: Optimized simple instrument rule

$$i_t = \rho_R i_{t-1} + \rho_\pi (\pi_t - \bar{\pi}^c) + \rho_y (y_t - \bar{y}_t)$$

$\lambda_y$	Trend output gap					Unconditional output gap				
	$\rho_R$	$\rho_\pi$	$\rho_y$	Loss	Optimal loss	$\rho_R$	$\rho_\pi$	$\rho_y$	Loss	Optimal loss
0.0001	1.03	0.51	0.003	1.97	1.11	1.01	0.51	-0.004	1.98	1.11
0.01	1.03	0.51	0.004	2.16	1.32	1.01	0.50	-0.003	2.09	1.24
0.11	1.02	0.47	0.01	4.07	3.17	1.01	0.47	0.004	3.17	2.37
0.21	1.01	0.44	0.02	5.88	4.71	1.01	0.44	0.01	4.19	3.29
0.31	1.01	0.42	0.02	7.59	6.03	1.02	0.41	0.02	5.15	4.08
0.41	1.01	0.39	0.02	9.21	7.18	1.02	0.39	0.02	6.05	4.76
0.51	1.02	0.36	0.03	10.73	8.20	1.02	0.37	0.03	6.90	5.36
1	1.05	0.25	0.05	16.84	11.94	1.03	0.32	0.06	10.38	7.65
1.5	1.07	0.19	0.07	21.19	14.53	1.00	0.32	0.13	12.90	9.35
2	1.09	0.17	0.09	24.31	16.46	0.96	0.32	0.21	14.60	10.73
3	1.10	0.16	0.14	28.51	19.23	0.90	0.25	0.33	16.91	12.99
4	1.12	0.16	0.20	31.24	21.20	0.90	0.24	0.40	18.74	14.89
5	1.13	0.17	0.27	33.20	22.73	0.91	0.25	0.45	20.46	16.58
6	1.13	0.17	0.34	34.72	23.97	0.92	0.26	0.49	22.10	18.13
7	1.13	0.17	0.40	35.95	25.03	0.93	0.27	0.54	23.68	19.59
8	1.13	0.17	0.47	36.98	25.95	0.93	0.29	0.58	25.20	20.97
9	1.12	0.17	0.53	37.87	26.76	0.93	0.30	0.62	26.69	22.29
10	1.12	0.17	0.59	38.66	27.49	0.93	0.31	0.65	28.14	23.57

Note: “Loss” is the loss from equation (3.1) under the instrument rule with optimized coefficients, and “Optimal loss” is the loss under optimal policy.

Table 3.1 shows that the optimized coefficients of the instrument rule are such that the optimized simple instrument rules are generally “super-inertial,” that is,  $\rho_R$  is above unity. The exception is that  $\rho_R$  is lower than unity for the unconditional output gap when  $\lambda_y$  is sufficiently high (roughly above 1). Another property is that the response coefficients on the inflation (output gap) decreases (increases) as the weight on output-gap stabilization is increased. This property is not obvious and not general, since the mapping from loss function weights to optimal instrument-rule response coefficients is complicated and model-dependent.

From the top row in figure 3.1 we see that optimizing the response coefficients of the simple instrument rule closes a substantial part of the gap between the estimated instrument rule and the optimal policy.

For  $\lambda_y = 1$ , we find that the optimized instrument rule closes 20 percentage points of the gap to optimal policy relative to the estimated rule for the trend output gap, but the loss fraction between optimal policy and the optimized rule is despite this only about 0.7, implying that inflation and output-gap variances are still inefficient and can be further reduced by optimal policy. By and large, similar findings apply for the unconditional output gap. Interestingly, the nominal interest rate variance is larger under optimal policy than under the estimated instrument rule, which contributes to the more favorable inflation output trade-off relative to the estimated instrument rule. Figure 3.1 also shows that the central bank appears to face a relatively sharp trade-off between stabilizing inflation and the output gap. If the central bank wants to decrease the variance of inflation from 20 to 1, then it has to accept an increase in the variance of output of about 20, that is, the variance frontier has an “average” slope of about  $-1$ .<sup>17</sup> As can be seen from the right column of the second row in figure 3.1, the slope of the trade-off is about the same if the unconditional output gap is used in the loss function instead of the trend output gap. However, we see that the variance trade-off is more favorable and the variance curve is closer to origin for the unconditional output gap compared with that for the trend output gap. Thus, it is easier to stabilize the unconditional output gap than the trend output gap. Finally, the bottom row shows that lower output gap variance and higher inflation variance go with higher variance of the real exchange rate, so the real exchange rate is apparently implicitly adjusted to stabilize the output gap.

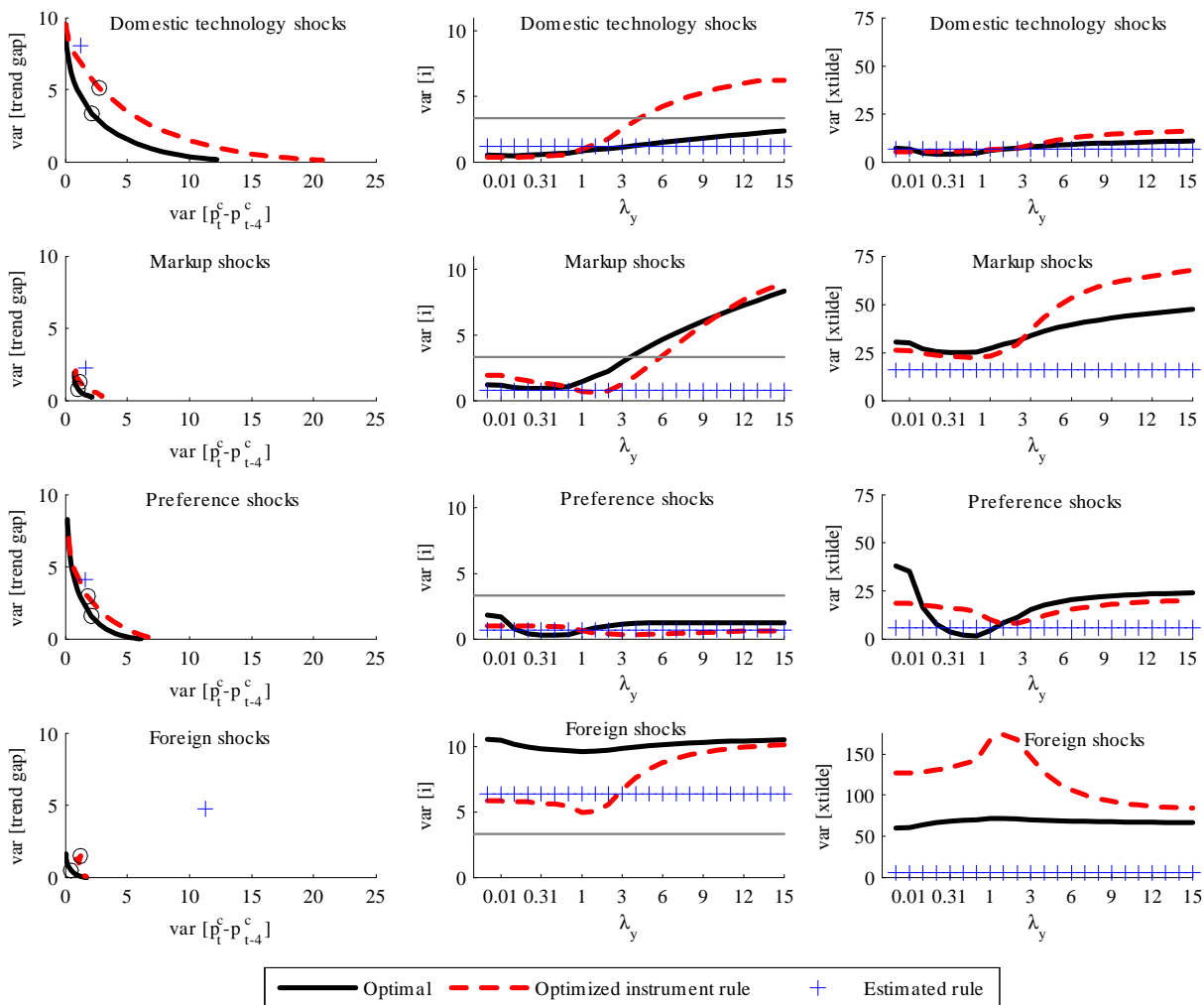
### 3.2. Discerning the benchmark results: monetary policy and the transmission of shocks

To examine which shocks create the trade-offs in figure 3.1, figures 3.2 and 3.4 plot the variance trade-offs between inflation and the output gap for different subsets of active shocks, as well as the corresponding variances of the nominal interest rate and the real exchange rate plotted against each  $\lambda_y$ . Following the analysis in Section 3.1, we condition on our estimated value  $\lambda_{\Delta i} = 0.37$ . We have divided the shocks into four different categories: domestic technology shocks (that is, stationary technology and investment specific technology shocks), markup shocks (that is, domestic, imported consumption, imported investment and export markup shocks), preference shocks (that is,

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<sup>17</sup> We measure both inflation and the output gap in per cent, which implies that the variance is defined in terms of squared %.

Figure 3.2: Variance trade-offs between when different shocks are active. Trend output gap in the loss function.



consumption preference and labor supply shocks), and foreign shocks (that is, unit-root technology, asymmetric technology, risk premium and foreign VAR shocks, which are foreign inflation, output and interest rate shocks). It is important to notice that the parameters in the optimized simple instrument rule are optimized on the full set of shocks, not on each subset separately. Therefore, it is possible that the variance trade-offs between inflation and the output gap are not always downward sloping for a particular subset of shocks for the optimized simple instrument rule (but they are always downward sloping for the optimal policy, which responds optimally to each shock separately (see (A.3) and (A.4)).

Figure 3.2 refers to the case with the trend output gap in the loss function and the variance of

the trend output gap. It shows that, in that case, the variance trade-offs between inflation and the trend output gap is predominantly driven by the domestic technology shocks. The reason is that the stationary technology shock affects actual output but not trend output as can be seen from the first row in Figure 3.3, which plots the impulse response function to the stationary technology shock under alternative assumptions about the conduct of monetary policy (optimal vs. estimated rule and alternative potential output definitions).<sup>18</sup> Focusing on the results for the trend output gap (black dotted lines and pink dash-dotted lines), we see that as trend productivity is not affected and the shock is efficient in the sense that it lowers inflation pressure and increases actual output, the trend output gap (i.e. actual minus potential output) increases and thus creates a trade-off between stabilizing inflation and the output gap. For this parameterization of the loss function and the rule, the central bank decides to tighten policy – i.e. raise real rates – both under optimal policy and the simple instrument rule in order to avoid an even larger surge in the output gap, and inflation therefore falls considerably.

Returning to Figure 3.2, we also see that the central bank needs to balance inflation stabilization against output-gap stabilization for most of the different sets of shocks as the variance frontiers are downward sloping for all subsets of shocks. Most notable of the other shocks are the two preference shocks, of which the labor supply shock is most important. The second row of Figure 3.3 shows the impulse responses to a positive (one standard deviation) labor supply shock.<sup>19</sup> As with the stationary technology shock, the figure documents that this shock induces a sizable trade-off between the trend output gap stabilization and inflation stabilization as the increase in labor supply puts downward pressure on real wages and inflation and thereby stimulate higher output.

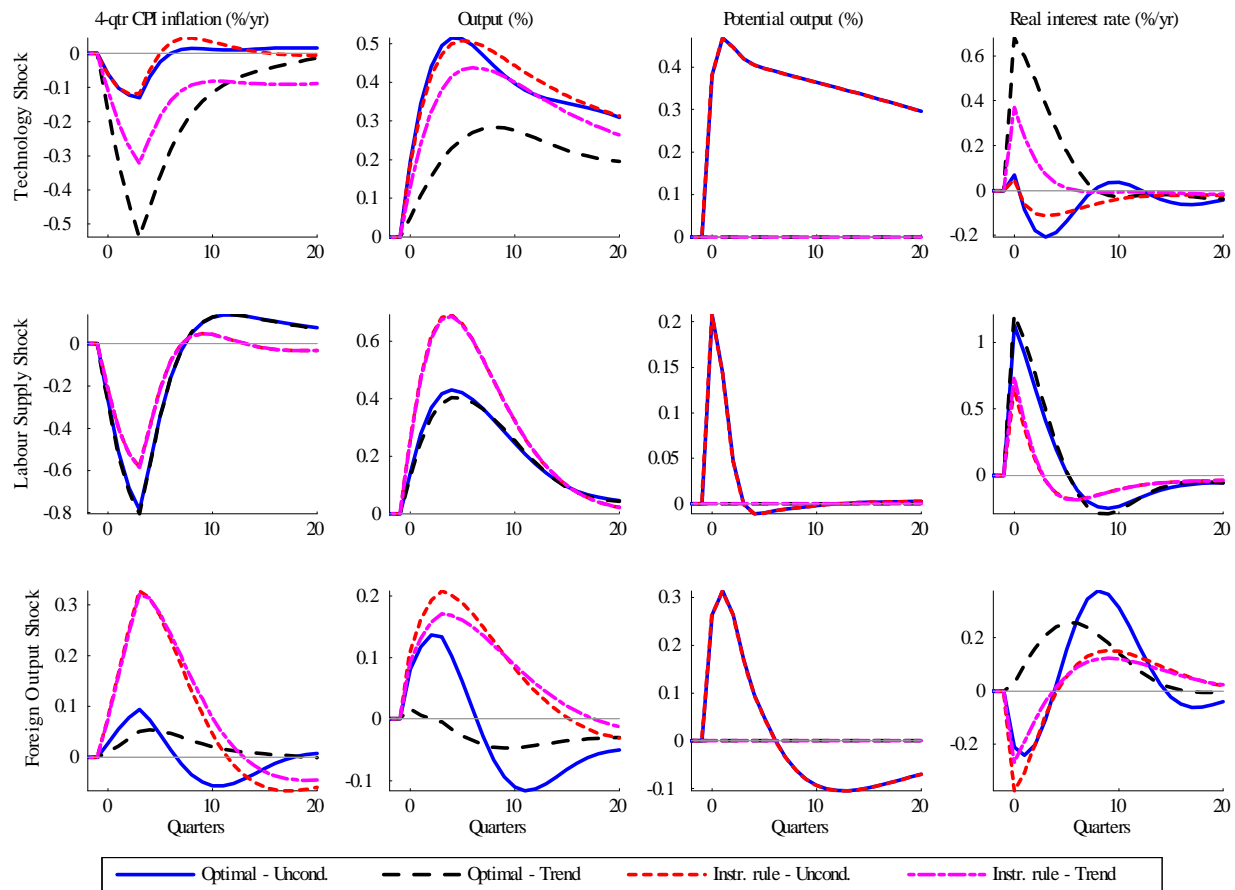
Another interesting finding in the left-hand column in Figure 3.2 is that the estimated instrument rule is rather close to the variance trade-off frontier for the optimal policy for all shock categories except the foreign shocks, for which the estimated instrument rule is found to be very inefficient. Among the foreign shocks, variations in foreign demand are of key importance and the impulse response functions to a foreign demand shock are plotted in the last row in Figure 3.3. From the figure, we see that optimal monetary policy targeting the trend output gap will respond

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<sup>18</sup> In Figure, the impulse occurs in quarter 0. Before quarter 0, the economy is in the steady state with  $X_t = 0$  and  $\Xi_{t-1} = 0$  for  $t \leq 0$  and  $x_t = 0$  and  $i_t = 0$  for  $t \leq -1$ . Under optimal policy, we use the estimated loss function ( $\lambda_y = 1.10$ ,  $\lambda_{\Delta i} = 0.37$ ) with the trend output gap (Optimal - Trend) and the unconditional output gap (Optimal - Uncond.) and for the instrument rule we use the estimated coefficients regardless if we use the trend (Instr. rule - Trend) or the unconditional (Instr. rule - Uncond.) gap.

<sup>19</sup> For the set of observables we use to estimate our model, this shock is up to a scaling factor observationally equivalent to a (negative) wage markup shock. But consistent with the specification of the utility function 2.7 and the results in Galí, Smets and Wouters [15], who distinguish between labor supply and wage markup shocks by using data on the real wage, employment and the unemployment rate and find that labor supply shocks dominate, we treat this shock as a genuine labor supply shock.

Figure 3.3: Impulse response functions to a (one-standard deviation) shock for stationary technology, foreign output, and labour supply under optimal policy ( $\lambda_y = 1.1$  and  $\lambda_{\Delta i} = 0.37$ ) and estimated instrument rule for different output gaps.



vigorously by raising real rates and inducing an appreciation of the real exchange rate in order to keep actual output close to its (unaffected) potential level and inflation in check. The estimated rule, however, features a high-degree of interest rate smoothing and relatively small short-run coefficients on inflation, output and the real exchange rate. Therefore, the instrument rule based policy only reacts to the extent that the boost in foreign output, inflation and interest rates push up domestic output and core CPI inflation. Consequently, the real exchange rate depreciates and contributes to further boost export demand. The boost to export demand and the weaker currency will trigger a large increase in domestic output and inflation, with domestic real interest rates only rising gradually as can be seen from the last row in Figure 3.3. This reasoning makes clear that open-economy aspects of the model matter importantly in the conduct of optimal monetary policy since all variations in the interest rate also lead to fluctuations in the exchange rate. The third col-

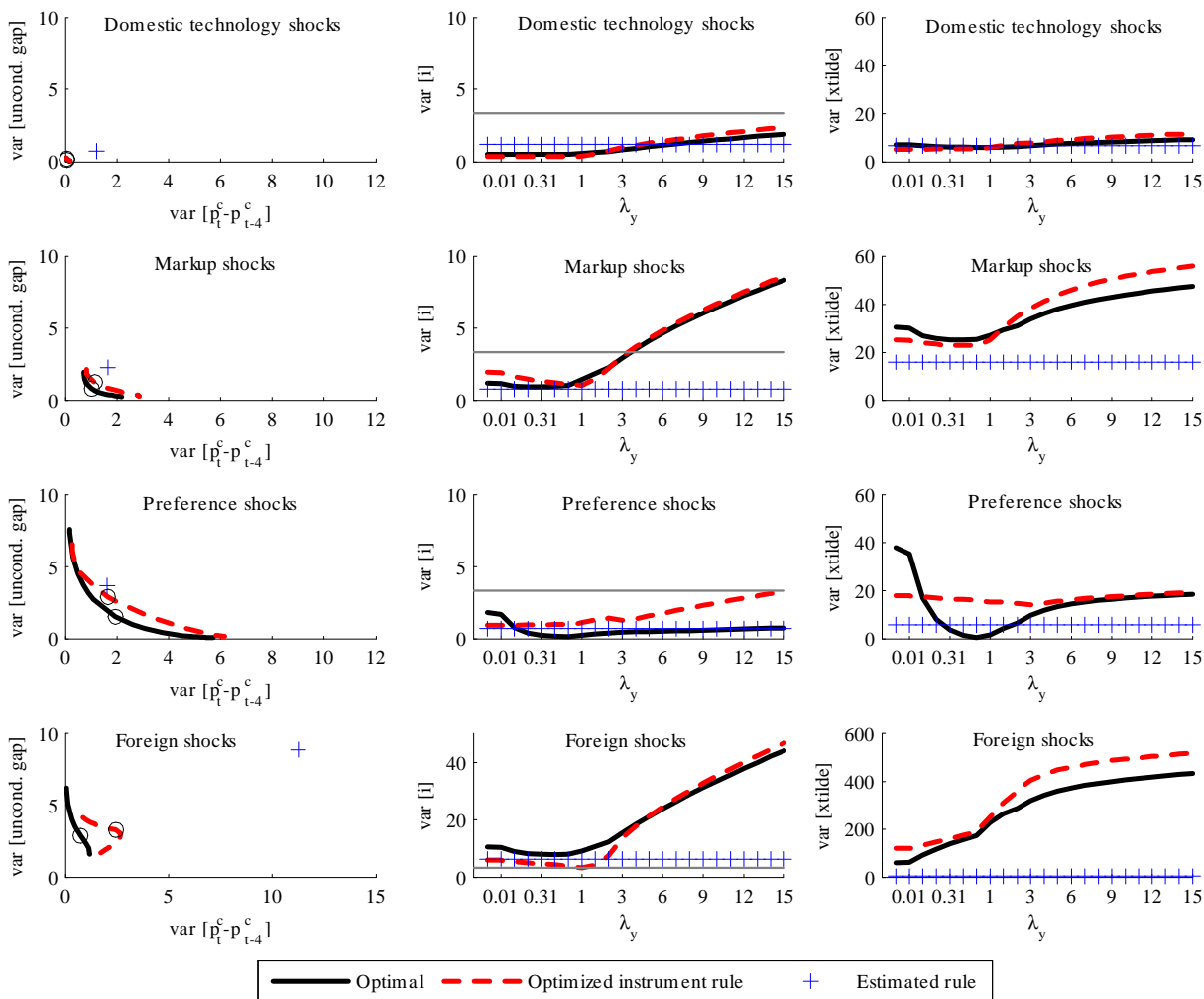
umn of Figure 3.2 shows that optimal monetary policy induces a large variance of the real exchange rate for all the different categories of shocks, especially for the cost-push shocks and foreign shocks. Hence, optimal monetary policy uses the real exchange channel to stabilize economic activity and inflation to a larger extent than the estimated rule.

Figure 3.4 refers to the case when the trend output gap is replaced by the unconditional output gap in the loss function and instrument rule, and the variance of the unconditional output gap is shown on the y-axis in the panels in the left column (instead of the trend output gap variance in Figure 3.2). In this case, the variance trade-off is mainly caused by preference shocks rather than domestic technology shocks. Since stationary technology shocks in this case influence potential output, the output gap will be less affected by this type of shocks compared with the trend output gap. This means that the central bank does not have to trade off inflation stability for output-gap stability to the same extent when using the unconditional output gap in the loss function. This mechanism is evident from the results in the first row in Figure 3.3 (blue solid lines and red dotted lines show results for the unconditional gap in the loss function and rule, respectively). The labor supply shock and the consumption preference shock, on the other hand, affect inflation and the output gap with opposite signs irrespective of which output gap definition is used in the loss function. This can be seen from the second row in Figure 3.3. Because both wages and prices are sticky, the downward adjustment of real wages that would occur if prices and wages were flexible will be muted, and actual output will therefore increase more than potential output. The positive output gap causes the central bank to tighten real interest rates, and inflation falls.<sup>20</sup> So this shock, along with the price markup shocks are the main sources of the trade-off between inflation and the unconditional output gap under the loss function and the optimized simple policy rule. For the foreign demand shock, optimal monetary policy targeting the unconditional output gap will engineer a fall in real rates initially in order to allow actual output to rise in tandem with potential output. But since the boost in potential output is temporary, real rates are subsequently tightened more relative to the case when optimal monetary policy targets the trend output gap. For the estimated instrument rule, the results are quite similar regardless of which output gap measure is used, since the estimated rule features a high degree of smoothing and small short-run response coefficients on the level and the change in the output gap.

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<sup>20</sup> Notice that the effects on flexprice potential output and actual output are more short-lived compared to the technology shock, since the persistence of the labor supply shock is much lower ( $\rho_{\varsigma_n} = 0.38$ ).

Figure 3.4: Variance trade-offs when different subsets of shocks are active. Unconditional output gap in loss function.



### 3.3. Limiting the volatility of short-term nominal interest rates

As is evident in figure 3.1, the interest-rate variance is relatively high under both the optimized instrument rule and the optimal policy. This means that the zero lower bound (ZLB) for the nominal interest rate may occasionally bind when shocks hit the economy. An approximation to the (non-linear) constraint that the nominal interest rate must be non-negative is to limit the variance of the nominal interest rate and thereby reduce the probability that the interest rate violates the ZLB. This approximation allows us to keep the linear-quadratic approach and focus on the second moments, but a potential drawback is of course that the approximation also limits upward movements in the nominal interest rate. We nevertheless adopt this approximation (see



Woodford [25] for a discussion).<sup>21</sup>

When optimizing the response coefficients of the simple instrument rule we therefore add the restriction there is only a 1% probability of hitting the ZLB. With an assumed steady state value for the nominal interest rate of 4.25% this implies that the variance of the nominal interest rate is not allowed to be larger than 3.34%. The dashed-dotted curves in figure 3.1 show the outcome of this procedure. Limiting the variance in the nominal interest rate implies the central bank can not stabilize output and inflation to the same extent, and the variance frontier moves slightly further out compared with when the ZLB is not imposed. For small  $\lambda_y$  the difference is not particularly pronounced, but for large  $\lambda_y$  the ZLB restriction results in a much larger output-gap variance that is not compensated by the decrease in inflation variance and hence the loss increases substantially. A larger output variability also feeds into a somewhat higher variance of the real exchange rate when the trend output gap is considered. It should, however, also be noted that the restriction on the variance of the nominal interest rate is strongly binding. For large weights on output-gap stabilization, the interest rate variance is almost six (nine) times as high when the ZLB is not imposed in the trend (unconditional) output-gap case. From this perspective, the impact on the inflation-output variance trade-off seems rather modest.

We also impose the zero lower bound on the nominal interest rate on the optimal policy, in this case by adding an extra interest-rate variance argument,  $\lambda_i i_t^2$ , to the loss function in (2.19) and gradually increasing  $\lambda_i$  until the variance of the nominal interest rate is not larger than 3.34%. The resulting variance trade-offs are depicted as dotted curves in figure 3.1. We see that the trade-off between inflation and output-gap variance shifts out a bit but not much. So even if the interest-rate variance in the unrestricted case is larger with optimal policy than with the optimized simple instrument rule, the zero lower bound appears to have about the same impact on the trade-off between inflation and output-gap variance.<sup>22</sup>

## 4. Conclusions

Within a small open economy framework, this paper has examined how the trade-offs between stabilizing CPI inflation and alternative measures of the output gap depend on the conduct of monetary policy. We have shown that it matters substantially which output-gap definition the

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<sup>21</sup> In a smaller model it would be possible to deal with the consequences of the zero lower bound in a more rigorous fashion, for example along the lines of Adam and Billi [1] and Hebden, Lindé and Svensson [16].

<sup>22</sup> The interest-rate variance can also be reduced by increasing the weight on interest-rate smoothing,  $\lambda_{\Delta i}$ . However, this deteriorates the trade-off between inflation and output-gap variance quite a bit (not shown) and is hence an inefficient way of reducing the interest-rate variance compared to increasing the weight on interest-rate variance,  $\lambda_i$ .

central bank uses in its loss function. Depending on whether it is the trend output gap (between output and trend output) or the unconditional output gap (between output and unconditional flexprice potential output, the hypothetical output level that would prevail if prices and wages were entirely flexible and had been so forever) that is included in the loss function, the central bank faces different trade-offs between stabilizing inflation and the output gap. According to our analysis, the trade-off between stabilizing inflation and the output gap is more favorable for the unconditional output gap than for the commonly used trend output gap. However, abandoning the trend output gap in favor of the unconditional output gap would also be associated with an increase in the variance of output since unconditional potential output fluctuates more than trend output due to the fact that stationary but persistent technology shocks are important to explain business cycle fluctuations in the Swedish economy. On the other hand, because the trade-off between output-gap stabilization and inflation stabilization is more favorable for unconditional output gaps than for trend output gaps, abandoning the trend output gap in favor of the unconditional output gap should be associated with lower inflation variability.<sup>23</sup>

The sensitivity of the results when limiting the volatility of short-term nominal interest rate was also examined. While we acknowledge that our approach to address the effects of imposing the zero lower bound is a crude approximation and should therefore be treated with grain of salt, our results do suggest that this assumption has similar implications for optimized simple instrument rules and optimal policy.

In future work, it would be of interest to extend our analysis to other small open economies (e.g. Canada). It would also be of interest to allow for financial frictions and a more developed banking sector in the model. Finally, it would be of interest to study the influence of monetary transmission lags (i.e. assume that firms and households make current period pricing and consumption decisions before the central bank adjust interest rates following Christiano, Eichenbaum and Evans [9]) and allow for imperfect information about the state of the economy.

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<sup>23</sup> Analysis not reported shows that the trade-off between inflation and the conditional output gap (defined in footnote 13) is similar to that between inflation and the unconditional output gap, and hence more favorable than between inflation and the trend output gap.

## Appendix

### A. Model solution

After log-linearization, Ramses is a log-linear model with forward-looking variables. It can be written in the following state-space form,

$$\begin{bmatrix} X_{t+1} \\ Hx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}. \quad (\text{A.1})$$

Here,  $X_t$  is an  $n_X$ -vector of *predetermined* variables in period  $t$  (where the period is a quarter);  $x_t$  is an  $n_x$ -vector of *forward-looking* variables;  $i_t$  is an  $n_i$ -vector of *instruments* (the forward-looking variables and the instruments are the *nonpredetermined* variables);<sup>24</sup>  $\varepsilon_t$  is an  $n_\varepsilon$ -vector of i.i.d. shocks with mean zero and covariance matrix  $I_{n_\varepsilon}$ ;  $A$ ,  $B$ , and  $C$ , and  $H$  are matrices of the appropriate dimension; and  $y_{t+\tau|t}$  denotes  $E_t y_{t+\tau}$  for any variable  $y_t$ , the rational expectation of  $y_{t+\tau}$  conditional on information available in period  $t$ . The variables are measured as differences from steady-state values, so their unconditional means are zero. The elements of the matrices  $A$ ,  $B$ ,  $C$ , and  $H$  are estimated with Bayesian methods and considered fixed and known for the policy simulations. Hence the conditions for certainty equivalence are satisfied. The appendix of ALLS [2] provides details on Ramses, including the elements of the vectors  $X_t$ ,  $x_t$ ,  $i_t$ , and  $\varepsilon_t$ .

First we assume monetary policy can be described as minimizing an intertemporal loss function under commitment. Let  $Y_t$  be an  $n_Y$ -vector of *target* variables, measured as the difference from an  $n_Y$ -vector  $Y^*$  of time invariant *target levels*. We assume that the target variables can be written as a linear function of the predetermined, forward-looking, and instrument variables,

$$Y_t = D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} \equiv [D_X \quad D_x \quad D_i] \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}, \quad (\text{A.2})$$

where  $D$  is an  $n_Y \times (n_X + n_x + n_i)$  matrix and partitioned conformably with  $X_t$ ,  $x_t$ , and  $i_t$ .

Under the assumption of optimization under commitment in a timeless perspective, the optimal policy and resulting equilibrium can be described by the following difference equations,

$$\begin{bmatrix} x_t \\ i_t \end{bmatrix} = F \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}, \quad (\text{A.3})$$

$$\begin{bmatrix} X_{t+1} \\ \Xi_t \end{bmatrix} = M \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix} + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}, \quad (\text{A.4})$$

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<sup>24</sup> A variable is predetermined if its one-period-ahead prediction error is an exogenous stochastic process (Klein [18]). For (A.1), the one-period-ahead prediction error of the predetermined variables is the stochastic vector  $C\varepsilon_{t+1}$ .

for  $t \geq 0$ , where  $X_0$  and  $\Xi_{-1}$  are given. The Klein algorithm returns the matrices  $F$  and  $M$ . These matrices depend on  $A$ ,  $B$ ,  $H$ ,  $D$ ,  $W$ , and  $\delta$ , but they are independent of  $C$ . The independence of  $C$  demonstrates the certainty equivalence of the optimal policy and equilibrium. The  $n_X$ -vector  $\Xi_t$  consists of the Lagrange multipliers of the lower block of (A.1), the block determining the forward-looking variables. The initial value  $\Xi_{-1}$  for  $t = 0$  is given by the optimization for  $t = -1$  (or equal to zero in the case of commitment from scratch in  $t = 0$ ). The choice and calculation of the initial  $\Xi_{-1}$  is further discussed in ALLS [2].

## B. Parameters

In table B.1 we report the parameters we have chosen to calibrate. These parameters are mostly related to the steady-state values of the observed variables (that is, the great ratios:  $C/Y$ ,  $I/Y$  and  $G/Y$ ). Table A.2 shows the prior and posterior distributions obtained in ALLS [2].

Table B.1: Calibrated parameters

Parameter	Description	Calibrated value
$\beta$	Households' discount factor	0.999999
$\alpha$	Capital share in production	0.25
$\eta_c$	Substitution elasticity between $C_t^d$ and $C_t^m$	5
$\sigma_a$	Capital utilization cost parameter	1,000,000
$\mu$	Money growth rate (quarterly rate)	1.010445
$\mu_z$	Technology growth rate (quarterly rate)	1.005455
$\sigma_L$	Labor supply elasticity	1
$\delta$	Depreciation rate	0.025
$\lambda_w$	Wage markup	1.30
$\omega_i$	Share of imported investment goods	0.50
$\omega_c$	Share of imported consumption goods	0.35
$\nu$	Share of wage bill financed by loans	1
$\tau^y$	Labor income tax rate	0.30
$\tau^c$	Consumption tax rate	0.24
$\tau^k$	Capital income tax rate	0.00
$\rho_{\bar{\pi}}$	Inflation target persistence	0.975
$g_r$	Government expenditures-output ratio	0.30

Table B.2: Prior and posterior distributions

Parameter		Prior distribution			Posterior distribution					
		type	mean	std.d. /df	Policy rule		Commitment		Loss params.	
					mode	std.d. Hess.	mode	std.d. Hess.	mode	std.d. Hess.
Calvo wages	$\xi_w$	beta	0.750	0.050	0.719	0.045	0.719	0.042		
Calvo domestic prices	$\xi_d$	beta	0.750	0.050	0.712	0.039	0.737	0.043		
Calvo import cons. prices	$\xi_{m,c}$	beta	0.750	0.050	0.868	0.018	0.859	0.016		
Calvo import inv. prices	$\xi_{m,i}$	beta	0.750	0.050	0.933	0.010	0.929	0.011		
Calvo export prices	$\xi_x$	beta	0.750	0.050	0.898	0.019	0.889	0.025		
Indexation wages	$\kappa_w$	beta	0.500	0.150	0.445	0.124	0.422	0.115		
Indexation prices	$\kappa_d$	beta	0.500	0.150	0.180	0.051	0.173	0.050		
Markup domestic	$\lambda_d$	truncnormal	1.200	0.050	1.192	0.049	1.176	0.050		
Markup imported cons.	$\lambda_{m,c}$	truncnormal	1.200	0.050	1.020	0.028	1.021	0.029		
Markup imported invest.	$\lambda_{m,i}$	truncnormal	1.200	0.050	1.137	0.051	1.154	0.049		
Investment adj. cost	$\tilde{S}''$	normal	7.694	1.500	7.951	1.295	7.684	1.261		
Habit formation	$b$	beta	0.650	0.100	0.626	0.044	0.728	0.035		
Subst. elasticity invest.	$\eta_i$	invgamma	1.500	4.0	1.239	0.031	1.238	0.030		
Subst. elasticity foreign	$\eta_f$	invgamma	1.500	4.0	1.577	0.204	1.794	0.318		
Risk premium	$\tilde{\phi}$	invgamma	0.010	2.0	0.038	0.026	0.144	0.068		
UIP modification	$\tilde{\phi}_s$	beta	0.500	0.15	0.493	0.067	0.488	0.029		
Unit root tech. shock	$\rho_{\mu_z}$	beta	0.750	0.100	0.790	0.065	0.765	0.072		
Stationary tech. shock	$\rho_\varepsilon$	beta	0.750	0.100	0.966	0.006	0.968	0.005		
Invest. spec. tech shock	$\rho_\Upsilon$	beta	0.750	0.100	0.750	0.077	0.719	0.067		
Asymmetric tech. shock	$\rho_{\tilde{z}^*}$	beta	0.750	0.100	0.722	0.052	0.736	0.058		
Consumption pref. shock	$\rho_{\zeta_c}$	beta	0.750	0.100	0.919	0.034	0.881	0.038		
Labour supply shock	$\rho_{\zeta_h}$	beta	0.750	0.100	0.382	0.082	0.282	0.064		
Risk premium shock	$\rho_{\tilde{\phi}}$	beta	0.750	0.100	0.852	0.059	0.885	0.041		
Unit root tech. shock	$\sigma_{\mu_z}$	invgamma	0.200	2.0	0.127	0.025	0.201	0.039		
Stationary tech. shock	$\sigma_\varepsilon$	invgamma	0.700	2.0	0.457	0.051	0.516	0.054		
Invest. spec. tech. shock	$\sigma_\Upsilon$	invgamma	0.200	2.0	0.441	0.069	0.470	0.065		
Asymmetric tech. shock	$\sigma_{\tilde{z}^*}$	invgamma	0.400	2.0	0.199	0.030	0.203	0.031		
Consumption pref. shock	$\sigma_{\zeta_c}$	invgamma	0.200	2.0	0.177	0.035	0.192	0.031		
Labour supply shock	$\sigma_{\zeta_h}$	invgamma	1.000	2.0	0.470	0.051	0.511	0.053		
Risk premium shock	$\sigma_{\tilde{\phi}}$	invgamma	0.050	2.0	0.454	0.157	0.519	0.067		
Domestic markup shock	$\sigma_{\lambda_d}$	invgamma	1.000	2.0	0.656	0.064	0.667	0.068		
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	invgamma	1.000	2.0	0.838	0.081	0.841	0.084		
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	invgamma	1.000	2.0	1.604	0.159	1.661	0.169		
Export markup shock	$\sigma_{\lambda_x}$	invgamma	1.000	2.0	0.753	0.115	0.695	0.122		
Interest rate smoothing	$\rho_{R,1}$	beta	0.800	0.050	0.912	0.019	0.900	0.023		
Inflation response	$r_{\pi,1}$	truncnormal	1.700	0.100	1.676	0.100	1.687	0.100		
Diff. infl response	$r_{\Delta\pi,1}$	normal	0.300	0.100	0.210	0.052	0.208	0.053		
Real exch. rate response	$r_{x,1}$	normal	0.000	0.050	-0.042	0.032	-0.053	0.036		
Output response	$r_{y,1}$	normal	0.125	0.050	0.100	0.042	0.082	0.043		
Diff. output response	$r_{\Delta y,1}$	normal	0.063	0.050	0.125	0.043	0.133	0.042		
Monetary policy shock	$\sigma_{R,1}$	invgamma	0.150	2.0	0.372	0.061	0.360	0.059		
Inflation target shock	$\sigma_{\pi^c,1}$	invgamma	0.050	2.0	0.465	0.108	0.647	0.198		
Interest rate smoothing 2	$\rho_{R,2}$	beta	0.800	0.050	0.882	0.019				
Inflation response 2	$r_{\pi,2}$	truncnormal	1.700	0.100	1.697	0.097				
Diff. infl response 2	$r_{\Delta\pi,2}$	normal	0.300	0.100	0.132	0.024				
Real exch. rate response 2	$r_{x,2}$	normal	0.000	0.050	-0.058	0.029				
Output response 2	$r_{y,2}$	normal	0.125	0.050	0.081	0.040				
Diff. output response 2	$r_{\Delta y,2}$	normal	0.063	0.050	0.135	0.029				
Monetary policy shock 2	$\sigma_{R,2}$	invgamma	0.150	2.0	0.100	0.012				
Inflation target shock 2	$\sigma_{\pi^c,2}$	invgamma	0.050	2.0	0.081	0.037				
Output stabilization	$\lambda_y$	truncnormal	0.5	100.0			1.091	0.526	1.102	0.224
Interest rate smoothing	$\lambda_{\Delta i}$	truncnormal	0.2	100.0			0.476	0.191	0.369	0.061
Log marg likelihood laplace						-2631.56		-2654.45		

### C. Unconditional variances

As shown in Svensson [24], the model solution satisfies

$$\tilde{X}_{t+1} = M\tilde{X}_t + \tilde{C}\varepsilon_{t+1}, \quad (\text{C.1})$$

$$\tilde{x}_t = F\tilde{X}_t, \quad (\text{C.2})$$

where

$$\tilde{X}_t \equiv \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}, \quad \tilde{x}_t \equiv \begin{bmatrix} x_t \\ i_t \end{bmatrix}, \quad \tilde{C} \equiv \begin{bmatrix} C \\ 0 \end{bmatrix}$$

(note that  $\tilde{x}_t$  here does not denote the real exchange rate). The variance-covariance matrices of the predetermined variables,  $\Sigma_{\tilde{X}\tilde{X}}$ , and the forward-looking variables,  $\Sigma_{\tilde{x}\tilde{x}}$ , therefore satisfy the equations

$$\Sigma_{\tilde{X}\tilde{X}} = M\Sigma_{\tilde{X}\tilde{X}}M' + \tilde{C}\Sigma_{\varepsilon\varepsilon}\tilde{C}', \quad (\text{C.3})$$

$$\Sigma_{\tilde{x}\tilde{x}} = F\Sigma_{\tilde{X}\tilde{X}}F', \quad (\text{C.4})$$

where  $\Sigma_{\varepsilon\varepsilon}$  is the variance-covariance matrix of the i.i.d. shocks  $\varepsilon_t$ .

The solution for the target variables and the observed variables are also functions of the predetermined variables,

$$Y_t = D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} = D \begin{bmatrix} I_{n_X} & 0 \\ & F \end{bmatrix} \tilde{X}_t \equiv \tilde{D}\tilde{X}_t,$$

$$Z_t = \bar{D} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} + \eta_t = \bar{D} \begin{bmatrix} I_{n_X} & 0 \\ & F \end{bmatrix} \tilde{X}_t + \eta_t \equiv \bar{\bar{D}}\tilde{X}_t + \eta_t.$$

Then their variance-covariance matrices,  $\Sigma_{YY}$  and  $\Sigma_{ZZ}$ , can be determined from the variance-covariance matrix of the predetermined variables,

$$\Sigma_{YY} = \tilde{D}\Sigma_{\tilde{X}\tilde{X}}\tilde{D}', \quad (\text{C.5})$$

$$\Sigma_{ZZ} = \bar{\bar{D}}\Sigma_{\tilde{X}\tilde{X}}\bar{\bar{D}}' + \Sigma_{\eta\eta}, \quad (\text{C.6})$$

where  $\Sigma_{\eta\eta}$  is the variance-covariance matrix of the measurement errors  $\eta_t$ .

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