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Transparency and Credibility: Monetary Policy with Unobservable Goals*

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Abstract

We define and study transparency, credibility, and reputation in a model where the central bank's characteristics are unobservable to the private sector and inferred from the policy outcome. A low-credibility bank optimally conducts a *more* expansionary policy than a high-credibility bank, in the sense that it induces higher inflation, but a *less* expansionary policy in the sense that it induces lower inflation and employment than expected. Increased transparency makes the bank's reputation and credibility more sensitive to its actions. This moderates the bank's policy, and induces the bank to follow a policy closer to the socially optimal one. Full transparency of the central bank's intentions is generally socially beneficial, but frequently not in the interest of the bank. Somewhat paradoxically, direct observability of idiosyncratic central bank goals removes the moderating influence on the bank and leads to the worst equilibrium.

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1 Introduction

In December 1989, as U.S. inflation was cresting 5 percent for the first time in 6 years, the Federal Open Market Committee (FOMC) held discussions regarding whether the Fed should more firmly pursue "price stability."[1] FOMC members generally agreed that price stability was their inflation goal, with FOMC Vice Chairman Corrigan referring to their [18, p. 45], "collective zeal" on this point. When Atlanta Fed president Forrestal questioned public support for increasing unemployment to reduce inflation from its then level of just under 5 percent, Dr. Prell, director of the Fed's Division of Research and Statistics, immediately responded that [p. 14–15] "...if the public thinks that the FOMC is thinking this way, then that means there is no credibility to the disinflationary commitment...So we're in that credibility bind..." Several members offered views similar to those of Corrigan [p. 30–31]: "...I don't think it's prudent for this institution...to bet the ranch on that [credibility] because if we're wrong we've got a heck of a problem on our hands..." Minneapolis Fed president Stern stated [p. 21]: "...I personally would start with the weak credibility case.... [I]f you start with something as pessimistic as that I think you have a difficult challenge in a rigorous way to justify [the pursuit of price stability]." The FOMC chose not to pursue its zealously-held goal at that time.

Credibility and transparency are now centerpieces of policy discussions by both academics and policymakers. This paradigm—first codified by Kydland and Prescott [26] and Barro and Gordon [6]—rose to favor because it offered an account of why industrialized countries chose such high inflation rates from the 1960s through the early 1980s and offered important predictions about the economics of reducing inflation in these economies and in economies facing hyperinflations. This literature has made great strides in these areas.

While a number of countries have now returned to extended periods of relatively low inflation—below, say, five percent—the issues of credibility and transparency that came to the fore during high inflation remain prominent. Credibility was clearly viewed to be of central importance by the FOMC in December 1989 after the 6 years of low U.S. inflation, and as documented by Blinder [4], such issues have not faded after a further decade of low inflation. Several authors have recently argued that there would be significant benefits to the U.S. and other countries from adopting a more transparent policy such as inflation targeting or some simple rule (see, for instance, Bernanke, Laubach, Mishkin and Posen [3] and Blinder [5]). The existing literature

[1] We put price stability in quotation marks; when central bankers refer to price stability they may mean low or zero inflation, which implicitly or explicitly allows base drift of the price level. In this case, the price level has a unit root and would probably not be considered as "stable", outside central banking circles.

offers only limited help in analyzing such claims, however, since its focus has primarily been on analyzing the level of the steady-state inflation bias or on the transition between high and low inflation.

This paper focuses on the role of credibility, transparency, and reputation in the context of a stationary, low-inflation equilibrium. The model of Cukierman and Meltzer [14] (CM) is an excellent starting point for this work. It is the simplest model we know of in which credibility and transparency can be clearly defined and in which the credibility and reputation have rich dynamics around a low-inflation steady-state.

While the CM model has great potential, and deserves far more attention than it has received, it has drawbacks that we attempt to address. First, the central-bank loss function is objectionable, since it can be interpreted as being linear in output: the central bank would accept arbitrary increases in employment variance for tiny reductions in inflation. This led CM to the strongly counterfactual prediction that central banks will ignore their own reputations in setting policy. Second, the effects of transparency in CM are inextricably linked with control-error variance—unavoidable error in implementing policy decisions—so that improving transparency also means improving monetary control. We seek to capture aspects of certain real world efforts to improve transparency, such as the issuance of inflation reports, which may increase transparency without directly altering the degree of monetary control. Thus, the notion of transparency we focus on is the ease with which the private sector can deduce the central bank's *intentions*, at unchanged degree of control.

Solving the CM model under a more standard loss function, we note that the central bank cares about its reputation and find a number of important results. For example, even in low-inflation steady-state equilibria, transparency and reputation have a modest but important role to play. Furthermore, we find that reputation dynamics can mitigate or eliminate the time-consistency problem for patient banks with very persistent goals. Increased transparency is generally good for society (though not for all parameter values), but we identify a potential conflict between society and the central bank regarding transparency. For a possibly relevant range of parameters the general public wants full transparency and the central bank wants minimal transparency.

Section 2 specifies the main building blocks and the basic features of our model. Section 3 presents solutions for several different policy regimes. Section 4 compares and contrasts the regimes; sections 5 and 6 focus in detail on the roles of credibility and transparency, respectively.

Section 7 summarizes and concludes. The appendices contain technical details.

2 Building blocks

2.1 The model

The model only differs formally from the CM model in the period loss function and in the specification of the central bank's inflation control error. The model has two agents, the private sector (also called the public) and the central bank. Private-sector behavior is summarized by a standard Phillips curve,

$$l_t = (\pi_t - \pi_{t|t-1}) + \varepsilon_t, \tag{2.1}$$

where l_t is (log) employment in period t, and π_t is the inflation rate in period t (the change in the log price level between period $t-1$ and t) and ε_t is an employment shock (a supply shock). The average rate of employment, $\mathrm{E}[l_t]$, is normalized to equal zero. Private-sector inflation expectations are rational: $\pi_{t|t-1} = \mathrm{E}^p_{t-1}\pi_t$, where E^p denotes the rational expectation with respect to private-sector information. Throughout, the rational expectation with respect to central-bank information is denoted by E. Subscripts like $_{t|t-1}$ always indicate the conditional expectation for period t based on the public's information at the end of period $t-1$.

The central bank has imperfect control over inflation,

$$\pi_t = i_t + \eta_t, \tag{2.2}$$

where i_t is the central bank's *intention* for inflation, and η_t is a mean-zero control error. Since we will generally assume that i_t is not observed by the public, we emphasize that i_t is the bank's intended policy outcome and not its instrument. In a simple way, this captures the fact that observable outcomes do not flawlessly reveal central-bank intentions. The control error satisfies

$$\eta_t = \xi_t + \nu_t, \tag{2.3}$$

where ξ_t and ν_t are independent mean-zero normal shocks. The private sector observes ξ_t at the end of period t, whereas the component ν_t remains unobservable.

The central bank's loss function at the end of period $t-1$ is

$$\mathrm{E}_{t-1} \sum_{j=t}^{\infty} \beta^{j-t} L_j, \tag{2.4}$$

where β $(0 < \beta < 1)$ is a discount factor and the period t loss function is

$$L_t \equiv \frac{1}{2}\left[\pi_t^2 + (l_t - l_t^*)^2\right]. \tag{2.5}$$

The central bank's total employment target, l_t^*, fulfills

$$\begin{aligned} l_t^* &= l^* + z_t, && (2.6) \\ z_t &= \rho z_{t-1} + \theta_t, && (2.7) \end{aligned}$$

where $l^* \geq 0$ is the long-run employment target, z_t is a time-varying preference parameter that we call the employment target, $0 \leq \rho < 1$, and θ_t is a shock to the target.

These preferences can be interpreted as representing a central bank with an explicit zero inflation target, and an implicit, unobservable, and time-varying employment target. We interpret the stochastic portion of the loss function as arising from shifts in the way the central-banking structure aggregates heterogeneous societal preferences over inflation and employment.[2] Thus, the central bank's taste for inflation surprises may fluctuate due to an altered composition of the policymaking board, shifting political fortunes, or other economic factors that might have uncertain effects on the central bank's taste for employment.[3]

Since it is commonly assumed that central banks have preferences that are in some way *unrepresentative* of the public,[4] we follow Lewis [29] in considering what we stipulate to be a more *representative* social loss function. This function is of the form (2.4) but with the period loss given by

$$L_t^p \equiv \frac{1}{2}\left[\pi_t^2 + (l_t - l^*)^2\right], \tag{2.8}$$

which simply removes z_t from the central-bank loss. This reflects the view that the private sector appoints a central banker that it agrees with on average, but the central banker's preferences have an idiosyncratic component not shared by the public.[5]

While we examine several regimes, the central bank has full information about its preferences in all regimes and, at the end of period t, it has full information about all period t shocks. The time line in each period is as follows. At the end of period $t-1$, the public forms its expectations of period t variables. The central bank observes those expectations.[6] At the beginning of period

[2] It might seem natural that l_t^* is fixed but that the relative weights on the inflation and employment terms vary stochastically. Under this formulation, however, the solution to the problem is not a linear decision rule.

[3] For example, in 1998 the public were clearly uncertain about how the Fed's relative taste for employment versus inflation had shifted due to the crises in Asia, Russia, and Brazil.

[4] See, for instance, Rogoff [35], Walsh [41] and [42], Persson and Tabellini [34] and Svensson [40].

[5] The interpretation of loss functions in models of monetary policy is always complicated. There are standard justifications of (2.5) as a true social loss function. More in line with our preferred interpretation, one can arrive at both (2.5) and (2.8) as different aggregations of heterogeneous individual losses with (2.8) involving more representative weights. We prefer to interpret the loss functions less literally as approximations intended to capture some broad features of the problem.

[6] Under our assumption that the private sector has no private information, the central bank can always construct those expectations. In the real world, central banks extract private-sector expectations from a number of different sources, such as surveys and prices of financial instruments (see the survey in Söderlind and Svensson [37]).

t, the central bank observes its employment target, z_t, and the supply shock, ε_t, and chooses its intention, i_t. Next, the control error, η_t, is realized, giving π_t, and the public observes ε_t, giving l_t. Then the cycle begins again. All shocks in the model are normal, mutually uncorrelated, and have zero mean and fixed variance. The variance of any particular shock v is denoted σ_v^2.

2.2 Reputation, credibility, and transparency

One of the benefits of this framework is that it allows fairly natural definitions of the key notions of credibility, reputation, and transparency. In equilibrium, we will show that $z_{t|t-1}$—the public's best guess as to the bank's employment target—summarizes everything the public has learned about central-bank preferences from economic outcomes. Thus, $z_{t|t-1}$ summarizes the bank's *reputation* in period t.

As Blinder [4] emphasizes, there is no unanimously agreed-upon definition of *credibility* in the literature. Blinder's favorite definition, with which we agree, is that deeds are expected to match words. In the present context, it is natural to assume that the central bank in each period $t-1$ announces a zero inflation target for period t. There are two reasons for this. First, as noted above, the central-bank loss function might be interpreted as being consistent with a zero inflation target. Second, as we show below, the socially optimal policy implies zero expected inflation. Thus, we measure credibility of the zero-inflation announcement in period $t-1$ by the negative of the absolute value of the deviation of inflation expectations from zero,

$$c_{t-1} \equiv - \left|\pi_{t|t-1}\right|. \tag{2.9}$$

This definition is called the "average credibility of announcements" by Cukierman and Meltzer [13] and Cukierman [11]; the further inflation expectations are from zero, the lower is credibility.[7]

Transparency is connected to how easily the public can deduce central-bank goals and intentions from observables. In this model, the central bank's goals and intentions are private information, and the unobservable portion of the inflation control error, ν_t, prevents the public from being able to perfectly infer this information. For a given level of control error variance, the higher is the variance of ν_t, the more difficult will it be for the public to discern central-bank intentions and, hence, the lower is transparency. Remembering that $\eta_t = \xi_t + \nu_t$, we set

$$\sigma_\xi^2 \;=\; \tau\sigma_\eta^2$$

[7] CM ([14], p. 1108) gives a second, different definition of credibility as minus the absolute difference between the banks intention and the public's perception of it: $-\left|i_t - i_{t|t-1}\right|$. We prefer the first definition, since standard usage of the term seems, in principle, to allow that a bank could credibly announce a policy it did not intend to follow. CM's second definition disallows this.

$$\sigma_\nu^2 = (1-\tau)\sigma_\eta^2. \tag{2.10}$$

and let τ $(0 \le \tau \le 1)$ denote (the degree of) transparency. Thus, transparency is identified with the share of the control-error variance arising from the observed component: $\tau = 1$ gives "full transparency of intention," under which the public perfectly infers the bank's intention each period; $\tau = 0$ gives minimum transparency.[8] [9]

One example of increased transparency is the immediate release of FOMC transcripts and the blue and green books. This would not *directly* alter monetary control, but would *ceteris paribus* make it easier for the public to deduce the Fed's intentions. Similarly, in inflation-targeting countries, the regular publication of informative Inflation Reports and Monetary Policy Statements, especially when supplemented with Monetary Policy Committee minutes, makes it easier for the public to deduce the central bank's intentions.

2.3 Three regimes

We study three monetary policy regimes, which differ in the degree of transparency, but have a lack of a commitment technology in common, so that the central bank minimizes its loss function (2.4) under discretion. These are:

U Unobservable goal and intention: In this regime, $0 \le \tau < 1$, and z_t and i_t are not observed by the private sector. In period t, the private sector observes only π_t, l_t, ξ_t and ε_t.

OI Observable intention: This is regime U but with $\tau = 1$, full transparency of *intention*. The private sector does not observe z_t directly, but it observes π_t, l_t, ε_t, and η_t, from which it can deduce i_t and, in equilibrium, z_t, without error.

OG Observable goal and intention: "Extreme" transparency. In period t, the private sector directly observes z_t, π_t, η_t, ε_t, and l_t.

Regime U is our baseline case. Regime OI is the limit of regime U when transparency of intention reaches its maximum. We show that the public can infer the bank's goal perfectly in regime OI, but the equilibrium is remarkably different from the equilibrium in regime OG, where the goal is directly observed rather than perfectly inferred.

[8] Stein [38] analyzes a different sense of transparency that may also be important. Stein derives equilibria where the central bank makes announcements about its private information that are a deterministic function of information, but the function is not invertible. Thus, the announcements do not reveal all information. This interesting work builds on Crawford and Sobel's [10] more general work on costless signalling. Unfortunately, those results are static and extending them to the context of dynamic, repeated games—which we think is the appropriate context for monetary policymaking—is well beyond the scope of this paper. We discuss this issue more extensively in [16]. Palmqvist [32] incorporates excplicit signalling in a simplified variant of our model.

[9] In [16] ,we show that this formulation is equivalent to one where the entire control error, η_t, is not observed but the central bank makes a noisy announcement about η_t at the end of period t in the form of a variable that has a squared correlation of τ with η_t.

As a basis of comparison, we consider regime S (the social optimum) where the central bank is forced to commit to a policy rule that minimizes the social loss function, (2.4) with (2.8). This results in the standard commitment solution,

$$i_t = -\frac{1}{2}\varepsilon_t. \tag{2.11}$$

The policy optimally smooths the effect of the supply shock between inflation and employment, and disregards z_t, which does not enter the social loss function.[10]

2.4 Generic economic dynamics for all regimes

The analysis of these regimes is greatly simplified by the fact that in our model, the dynamics of the economy, up to the parameters of the central-bank policy rule, are the same in each regime. In all regimes, we assume that the private sector believes that the central bank's policy follows

$$i_t = k_0 + k_1\varepsilon_t + k_2 z_t + k_3 z_{t|t-1}, \tag{2.12}$$

for some coefficients k_0, ..., k_3.[11]

We confirm in section 3 that, if the private sector believes the policy is given by (2.12), the central bank will optimally behave according to (2.12). This assumption has the effect of making a simple linear learning scheme optimal for the private sector and, in particular, rules out signalling equilibria where small changes in policy can signal sharp differences in central-bank preferences.

Given the private sector's belief in (2.12), expected inflation is given by

$$\pi_{t|t-1} = k_0 + (k_2 + k_3) z_{t|t-1}, \tag{2.13}$$

and employment evolves according to

$$l_t = i_t + \eta_t - k_0 - (k_2 + k_3) z_{t|t-1} + \varepsilon_t. \tag{2.14}$$

These expressions will hold notwithstanding if the private sector's beliefs about policy are rational. Thus, in a rational-expectations equilibrium, the central bank behaves according to (2.12),

[10] It is relevant to ask why the other regimes are of interest when the optimal rule (2.11) could be imposed. We believe that, in the real world, policy under discretion arises because the complexity of the economic and political environment make codification, adoption, and verification of a good policy rule difficult. In any formal model that can be solved, a forcing rule may seem the obvious answer. Nevertheless, we believe that studying discretion and transparency in a tractable model may yield important lessons.

[11] None of our results change if we extend (2.12) to include $k_4 z_{t-1}$ so that policy can depend separately on θ_t. For all the cases considered, optimality implies $k_4 = 0$.

and equilibrium dynamics are,

$$
\begin{aligned}
\pi_t &= k_0 + k_1\varepsilon_t + k_2 z_t + k_3 z_{t|t-1} + \eta_t && (2.15)\\
\pi_{t|t-1} &= k_0 + (k_2 + k_3)\, z_{t|t-1} && (2.16)\\
\pi_t - \pi_{t|t-1} &= k_1\varepsilon_t + k_2(z_t - z_{t|t-1}) + \eta_t && (2.17)\\
l_t &= (1 + k_1)\varepsilon_t + k_2(z_t - z_{t|t-1}) + \eta_t && (2.18)\\
l_t - l_t^* &= (1 + k_1)\varepsilon_t + k_2(z_t - z_{t|t-1}) + \eta_t - l^* - z_t && (2.19)
\end{aligned}
$$

The only endogenous variable not determined here is the key to the analysis: reputation, $z_{t|t-1}$. The next section completes the derivation of the rational-expectations equilibria for the various regimes.

3 Solving the model

3.1 Regime U: unobservable goals and intentions

We solve the model by noting that the Kalman filter provides the optimal solution to the public learning problem and by casting the central-bank optimization as a dynamic programming problem. For the CM model our approach naturally gives the same solution that CM find by more direct means. Their direct approach is intractable under our standard loss function.

We first derive the public's learning rule about z_t, and then the optimal ks in the policy function. Since the public does not directly observe z_t or i_t directly, it forms its expectation of inflation for period t at the end of period $t-1$ based only on the history of π_t, l_t and ξ_t. At the end of period t, the public can construct the variable

$$x_t \equiv \pi_t - k_0 - k_1\varepsilon_t - k_3 z_{t|t-1} - \xi_t = i_t + \nu_t - k_0 - k_1\varepsilon_t - k_3 z_{t|t-1}, \tag{3.1}$$

where we have used (2.3). Under the public's assumption that policy is made according to (2.12), we have

$$x_t = k_2 z_t + \nu_t. \tag{3.2}$$

Furthermore, under (2.12), x_t contains all the new private-sector information about z_t that arrives in period t: $\mathrm{E}^p[z_t|x_t, z_{t|t-1}] = \mathrm{E}^p[z_t|\text{all private-sector information in period } t]$. Believing that it observes $k_2 z_t$ plus a normal error, the private sector's learning problem is optimally solved using the Kalman filter, treating (2.7) as the transition equation and (3.2) as the measurement equation. The steady-state solution to this problem gives the dynamics of reputation:[12]

$$
\begin{aligned}
z_{t+1|t} &= (\rho - gk_2)z_{t|t-1} + gx_t && (3.3)\\
&= \rho z_{t|t-1} + g\left[k_2(z_t - z_{t|t-1}) + \nu_t\right], && (3.4)
\end{aligned}
$$

where g is the Kalman gain and can be expressed in terms of k_2 and the exogenous parameters only.[13] Equation (3.4) makes it clear that reputation is a first order autoregressive process with the same persistence, ρ, as z_t.[14]

Under the private sector's belief (2.12), π_t, l_t, z_t and $z_{t|t-1}$ evolve as in (2.2), (2.14), (2.7), and (3.3), respectively. There are two state variables in this economy, and for our purposes, it is natural to take the employment target, z_t, and reputation, $z_{t+1|t}$ as state variables. We recursively define the central bank's (steady-state) value function as

$$V(z_{t|t-1}, z_{t-1}) \equiv \mathrm{E}_{t-1} \min_{i_t} \mathrm{E}_{t-} \left[L_t + \beta V(z_{t+1|t}, z_t)\right], \tag{3.5}$$

where E_{t-} denotes the expectation of the central bank given its information at the *beginning* of period t, after it has observed ε_t and θ_t, but before η_t, π_t, and l_t have been realized. Because the loss function is quadratic and the two state variables evolve linearly (according to (2.7) and (3.3)), the value function is quadratic,

$$V(z_{t|t-1}, z_{t-1}) = \delta_0 + \delta_1 z_{t|t-1} + \frac{1}{2}\delta_2 z_{t|t-1}^2 + \delta_3 z_{t-1} + \frac{1}{2}\delta_4 z_{t-1}^2 + \delta_5 z_{t|t-1} z_{t-1}, \tag{3.6}$$

where the coefficients δ_0, ..., δ_5 remain to be determined.

In period t, the central bank solves

$$\min_{i_t} \mathrm{E}_{t-} \left[L_t + \beta V(z_{t+1|t}, z_t)\right]. \tag{3.7}$$

The first-order condition with respect to i_t is

$$i_t + \mathrm{E}_{t-} l_t - l^* - z_t + \beta \mathrm{E}_{t-} \left[(\delta_1 + \delta_2 z_{t+1|t} + \delta_5 z_t)\frac{\partial z_{t+1|t}}{\partial i_t}\right] = 0, \tag{3.8}$$

where the derivative $\frac{\partial z_{t+1|t}}{\partial i_t}$ enters because current policy affects future reputation through (3.1) and (3.3). The expectations and the partial derivative in this expression can be evaluated using

[12] That is, when the forecast error variance has converged. See appendix A.

[13] Two credibility definitions by CM were discussed above. CM [14] also take $(\rho - gk_2)$ as a measure of credibility. While this term is important in the dynamics of reputation, and, hence, credibility, it does not seem to be a natural *definition* of credibility.

[14] Note that the second term in (3.4) is not serially correlated and is uncorrelated with $z_{t|t-1}$ and its history.

expressions already shown, and the resulting expression can be solved for i_t, obtaining a policy rule of the form (2.12) with coefficients that fulfill (see appendix B)

$$\begin{aligned} k_0 &= l^* - \beta g \delta_1 && (3.9) \\ k_1 &= -\frac{1}{2} && (3.10) \\ k_2 &= \frac{1 - \beta g \delta_5}{2 + \beta g^2 \delta_2} && (3.11) \\ k_3 &= k_2 - \beta g(\rho - g k_2)\delta_2. && (3.12) \end{aligned}$$

In appendix B, we prove that there is a solution to these equations and provide numerical evidence in favor of the uniqueness of the solution.[15]

We solve the model for all regimes before discussing the economic interpretation, but it is useful to give some intuition for one central property driving results in each of the regimes. When $l^* > 0$, the bank, on average, has some incentive to use positive inflation surprises to increase employment. In equilibrium, one factor that prevents this is what we will call the *reputation cost*. Take a bank with $l_t^* > 0$ and suppose, for simplicity, that its reputation at t truly reflects its preferences: $z_t = z_{t|t-1}$. If the bank considers a marginal increase in i_t above the equilibrium value, it will find benefits in terms of higher employment at t. Increasing i_t increases employment through the Phillips curve by pushing inflation higher than expected (as in (2.14)). The reputation cost is due to the fact that this inflation surprise at t will increase $z_{t+1|t}$ (through (3.3)). The marginal effect is given by $\partial E_{t-} z_{t+1|t} / \partial i_t = g$. The larger is the Kalman gain in the learning problem, the greater is the sensitivity of the bank's reputation to its action.

We can see how the reputation affect figures in the bank's decision by re-writing the first-order condition, (3.8), as

$$-\ \mathrm{E}_{t-} \frac{\partial L_t}{\partial i_t} = \beta \mathrm{E}_{t-} \frac{\partial V(z_{t+1|t}, z_t)}{\partial z_{t+1|t}} \frac{\partial z_{t+1|t}}{\partial i_t}, \tag{3.13}$$

This equation reveals that the bank trades off the future reputation cost of an inflation surprise on the righthand side of the equation against the current net benefits on the lefthand side.

Key results below for each regime are driven by how various factors affect the magnitude of the reputation cost of inflation. We now summarize the solution for regimes OI (observable instrument) and OG (observable goal).

[15] The derivation naturally rests on the assumption that the private sector assumes that the central bank acts according to (2.12). There are almost certainly some other equilibria of the model without this assumption. CM implicitly make the same assumption as we do, and Rogoff [36] pointed out the likely existence of other equilibria without the assumption.

3.2 Regimes OI and OG

The solution in regime OI, observable instrument, is obtained from that of regime U by assuming full transparency of intentions ($\tau = 1$). Thus, the ks follow by letting σ_ν^2 go to zero in the expressions for the baseline regime, (3.9)–(3.12) and (B.9)–(B.13). Taking the relevant limits gives the policy rule coefficients for regime OI,

$$\begin{aligned} k_0 &= \frac{1-\beta\rho}{1+\beta\rho} l^* < l^* && (3.14) \\ k_1 &= -\frac{1}{2} && (3.15) \\ k_2 &= k_3 = \frac{1-\beta\rho^2}{2(1+\beta\rho^2)} < \frac{1}{2}, && (3.16) \end{aligned}$$

where we have used that $gk_2 = \rho$ when $\sigma_\nu^2 = 0$ (see appendix C).

In regime OG, we allow the private sector to observe z_t directly in period t. Thus, $z_{t|t-1} \equiv \rho z_{t-1}$ independent of the policy rule. The value function (3.5) from the baseline regime is still appropriate. Inflation expectations, $\pi_{t|t-1}$, are given by (2.16) after substituting for $z_{t|t-1}$,

$$\pi_{t|t-1} = k_0 + (k_2 + k_3)\rho z_{t-1}. \qquad (3.17)$$

Since $\frac{\partial z_{t+1|t}}{\partial i_t} \equiv 0$, the first-order condition with respect to i_t, (3.8), is now

$$\mathrm{E}_{t-}\left[(i_t + \eta_t) + (i_t + \eta_t - \pi_{t|t-1} + \varepsilon_t - l^* - z_t)\right] = 2i_t - \pi_{t|t-1} + \varepsilon_t - l^* - z_t = 0,$$

which, with (3.17), implies

$$i_t = \frac{1}{2}(k_0 + l^*) + \frac{1}{2}\varepsilon_t + \frac{1}{2}z_t + \frac{1}{2}(k_2 + k_3)\rho z_{t-1} + \frac{1}{2}\theta_t.$$

Thus, the rule for regime OG is,

$$\begin{aligned} k_0 &= l^* && (3.18) \\ k_1 &= -\frac{1}{2} && (3.19) \\ k_2 &= k_3 = \frac{1}{2}. && (3.20) \end{aligned}$$

3.3 Numerical analysis of the model

Because we do not have a closed form solution for regime U, we follow CM by studying a number of properties of the model numerically. We summarize the numerical approach here; for details, see appendices B and D. Judd [24] further explains and justifies this type of numerical analysis of models.

We study the properties of the model on a large parameter space comprised by $(\beta, \rho, \tau) \in [0,1]^3$, $(\sigma_\eta^2, \sigma_\theta^2, \sigma_\varepsilon^2, l^*) \in [0,10]^4$. Specifically, we solve the model for 100,000 points drawn uniformly from this parameter space.[16] Once the model is solved for a particular draw, we tally which among a large number of claims hold true for that parameter value, e.g.: Is central-bank loss in regime U lower than in OI? Is the derivative of the central-bank loss with respect to transparency positive?

Once all 100,000 draws are tallied, we can state the share of draws for which each claim holds true. If a property holds for some parameter values and not for others, the solutions for the particular parameter values constitute a constructive proof that the result is indeterminate.

If a property holds for all 100,000 points, we do not have proof that the property holds for all values, but we can make a very strong statement. If a claim holds for each of N draws, the probability that the claim is false on a fraction of the parameter space of at least size ω is less than or equal to $(1-\omega)^N$. Thus, with 100,000 draws, the probability that the claim is false for at least 0.01 percent of the parameter space is less than 0.005 percent.[17]

In what follows, we first compare and contrast the different regimes and then turn to the details of credibility and transparency in case U.

4 Comparing the regimes

Several important properties of equilibria under the various regimes are summarized in table 4.1. As in all Barro-Gordon-type models, the central bank has an incentive to use inflation surprises to stimulate employment. In equilibrium, average inflation expectations must be high enough so that the marginal employment benefit of a surprise is offset by the marginal inflation cost. In all regimes, the average inflation bias (k_0) is bounded by l^*, the average wedge between the central bank's long-run employment target and equilibrium employment.

In all regimes, the response to a supply shock is the same, $k_1 = -1/2$. In all regimes but U, the public learns z_t at the end of t so that the variance of the private-sector forecast of z_{t+1} is at the minimum possible value of σ_θ^2. Despite the fact that the public knows all there is to know about z_t at the end of period t in regimes OI, OG, and S, the outcomes for the average inflation bias span the range from zero in S to the upper bound of l^* in OG. This section explores

[16] Rather than using a uniform draw from the parameter space, with a meaningful *prior* density measuring the empirical relevance of various regions, more meaningful *posterior* measures of the empirical relevance of the computed properties can be produced.

[17] $0.9999^{100,000} \approx 0.000045$.

Table 4.1. Summary of regimes

Regime	k_0	k_1 ε_t	k_2 z_t	k_3 $z_{t\|t-1}$	$\rho - gk_2$	P
U	$< l^*$	$-\frac{1}{2}$	$< \frac{1}{2}$	$< k_2$	$< \rho$	$> \sigma_\theta^2$
OI	$< l^*$	$-\frac{1}{2}$	$< \frac{1}{2}$	$= k_2$	0	σ_θ^2
OG	l^*	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-	σ_θ^2
S	0	$-\frac{1}{2}$	0	0	-	σ_θ^2

Note: P denotes the variance of the forecast error, $\mathrm{E}[(z_t - z_{t|t-1})^2]$.

Table 4.2. A numerical example

						$\mathrm{E}[\cdot_t^2]$			Loss	
Regime	k_0	k_2	k_3	g	$\mathrm{Var}[z_{t\|t-1}]$	π_t	$l_t - l_t^*$	l_t	$\mathrm{E}[L_t]$	$\mathrm{E}[L_t^p]$
U	0.56	0.41	0.34	0.37	0.34	1.88	3.15	1.52	2.52	1.70
OI	0.20	0.18	0.18	3.84	0.96	1.45	3.88	1.28	2.66	1.36
OG	1.00	0.50	0.50	-	0.96	3.46	3.46	1.50	3.46	2.48
S	0.00	0.00	0.00	-	0.96	1.25	4.21	1.25	2.73	1.25

Note: $l^* = 1, \beta = 0.95, \rho = 0.7$ and $\sigma_\varepsilon^2 = \sigma_\theta^2 = \sigma_\eta^2 = 1$. In regime U, $\tau = 0$.

why these and some other important features come from the model. It begins with a numerical example, allowing us to get some notion of the economic magnitudes involved.

4.1 A numerical example

One of the central questions raised in the introduction is whether issues like credibility and transparency should continue to receive any significant attention in an economy that has solved the average inflation bias problem and attained a low-inflation steady-state. While this model is not sufficiently rich to calibrate to some real economy and make definite quantitative statements, some broad conclusions can be drawn.

Table 4.2 presents a numerical example for what we take to be conservative parameter values—values not intended to maximize the importance of transparency and credibility. Thus, the average employment target of the central bank (which bounds the average inflation bias) is $l^* = 1$ and the employment target is quite persistent ($\rho = 0.7$) but with a modest variance of 1.4 ($= \sigma_\varepsilon^2/(1 - \rho^2)$). In regime U, we set transparency to a minimum ($\tau = 0$) in order to demonstrate the maximum contrast from regime OI ($\tau = 1$).

From both the central bank and social perspectives, regime OG is the worst. The central bank ranks the other regimes from the best to the worst as U, OI, S; the societal ranking is exactly the opposite. We explore these relative rankings below; here, we emphasize that the differences among regimes are potentially of economic importance. For example, with these parameter values the variance of l_t at the social optimum is 1.25 and this optimum is nearly attained in regime OI. This employment gap variance is about 20 percent higher in regimes U and OG. Furthermore, the results imply that inflation will be 3 percentage points above the optimum inflation more than 10 percent of the time in regime OG, but less than 1 percent of the time in regime OI.

Thus, even though each of these regimes constitutes a low-inflation steady-state—inflation is almost never above 5 percent in any of the regimes—reputation, credibility, and transparency remain potentially important determinants of economic outcomes. We now turn to the reason for this.

4.2 A patient bank with very persistent goals is socially optimal

As the bank becomes more patient and the goal becomes more persistent, the central bank moves toward the social optimum in the limit:

Proposition 4.1. *In regimes U and OI, in the limit as $\beta\rho \to 1$, the coefficients of the policy rule converge to those of regime S, the social optimum: $k_0 = k_2 = k_3 = 0, k_1 = -1/2$.*

For regime OI, this limit is easy to see in (3.14)–(3.16); for regime U, see appendix B. This result is driven by the reputation cost of surprise inflation to the central bank. As noted above, a marginal unexpected increase in intended inflation at t leads to a positive inflation surprise worsening the banks reputation, which raises expected loss from $t+1$ onward. The effect of raising ρ and β is to make this reputation cost prohibitive. As noted in (3.4), $z_{t|t-1}$ is as persistent as z_t. As ρ approaches one, any change in reputation due to an inflation surprise approaches permanence. With β close to one, the future costs of this nearly permanent loss of reputation weigh heavily on current decisions by the bank. By setting $k_2 = 0$, the bank guarantees that the private sector will not attribute any inflation surprise to an increase in the employment target and insulates the bank from any reputation cost. In short, the potential reputation costs are so large that the bank completely ignores its idiosyncratic goals. If one believes, as we do, that central bank goals evolve slowly, this result provides an alternative to Rogoff's [35] "weight-conservative" central banker as a solution to time-consistency problems (see also Svensson [40]).

4.3 Regime OG: Should the public observe the goal directly or infer it from actions?

The reputation costs that drive the limiting result are absent in the observed goal regime, which makes OG the worst of all regimes. As noted above, the average inflation bias is the largest of all regimes (table 4.1); more generally, using the numerical method described above, we find

Proposition 4.2. *When the central bank's idiosyncratic goals are directly observed by the public (regime OG), average inflation, social loss, and central bank loss are each higher than under any level of transparency of intention with unobserved goal (regimes U and OI).*

The intuition for this is clear. In the OG regime, the public *directly* observes the bank's employment target and thus $z_{t|t-1} \equiv \rho z_{t-1}$ *independent of the bank's behavior*. Thus, $\partial z_{t+1|t}/\partial i_t \equiv 0$; there is no longer a reputation cost of inflation. No matter what the level of transparency in regime U, the constraining effect of reputation leads to a better outcome than in regime OG.

At first, it may seem puzzling that OG is also worse than OI, since z_t is perfectly known at the end of t in both these regimes. The important difference is that z_t is inferred from i_t in OI, whereas it is directly observed in OG. Since the perfect inference regarding z_t only occurs in equilibrium in OI, if the central bank were to implement higher-than-equilibrium inflation, its reputation would suffer. This cost of off-equilibrium-path behavior constrains the bank.

Thus, it matters how transparency is implemented: "extreme" transparency, in the sense that the public is no longer learning about the central bank's future intentions from current actions, is worse than no transparency at all.[18]

4.4 Response to the supply shock

As noted above, the central bank in all regimes responds to the supply shock by optimally spreading the effect between output and inflation: $k_1 = -1/2$ (see table 4.1). Thus, the marginal effect on intended inflation of an increase in the supply shock is $-1/2$ independent of regime and independent of the level of the goal, z_t, and of the level of credibility, c_t. This is contrary to the often stated intuition (Bernanke and Mishkin [7], Federal Reserve Bank of Kansas City [17]) that greater transparency and/or credibility increase the bank's flexibility in responding to supply shocks. Similarly, banks do not build up credibility to spend it (disproportionately) when the employment target is highest. These results will generally follow in any linear-quadratic model where the supply shock affects inflation and output linearly. These results should form an important baseline: conflicting results must rest on important nonlinearities.

[18] A similar result emerges in CM and recently Canavan [8] has generated the same result: uncertainty about the actions of the central bank can reduce the inflation bias.

If the supply shock were unobservable, the response coefficient k_1 would depend on the regime and the transparency. Still, with a linear reaction function, the response to supply shocks would remain independent of reputation and credibility.

5 Optimal acquisition of credibility in regime U

In all our regimes except U, the public knows the central bank's goals precisely in equilibrium and the bank does not face the commonly discussed problem of wanting low inflation, but having the public skeptical about that desire. The question of what a bank should do in this situation has, of course, been widely discussed. For example, it is sometimes claimed that a central bank with low credibility should follow a more restrictive policy than a fully credible bank in order to regain credibility.[19]

To explore this question, we compare two realizations of the economy, starting at the beginning of period t with the same value of the state variable z_{t-1}, but in one case credibility is low (ℓ) and in the other it is high (h), $c^{\ell}_{t-1} < c^{h}_{t-1}$. Credibility alone does not tell the sign of inflation expectations; we restrict the discussion of a low and a high credibility bank to situations of positive (that is, too high) inflation expectations: $\pi^{\ell}_{t|t-1} > \pi^{h}_{t|t-1} \geq 0$. By (2.13), the two banks' reputations will then fulfill $z^{\ell}_{t|t-1} > z^{h}_{t|t-1}$. For any variable v_t, define $\Delta v_t \equiv v^{\ell}_t - v^{h}_t$, that is, the low-credibility value minus the high-credibility value. We thus have

$$\Delta\pi_{t|t-1} > 0, \ \Delta z_{t|t-1} > 0. \tag{5.1}$$

Proposition 5.1. *In regime U,* ceteris paribus*:*
(i) *The low-credibility bank optimally implements* higher *inflation than the high-credibility bank, $\Delta\pi_t > 0$.*
(ii) *The low-credibility bank optimally implements* lower *inflation relative to private-sector expectations $\Delta(\pi_t - \pi_{t|t-1}) < 0$. This larger negative inflation surprise leads to* lower *employment in the low-credibility economy, $\Delta l_t < 0$.*

Part (i) and (ii) follow directly from (2.15), (2.13), (2.18) and (5.1), since

$$\Delta\pi_t = k_3\Delta z_{t|t-1} < (k_2 + k_3)\Delta z_{t|t-1} = \Delta\pi_{t|t-1}$$

and $k_3 < k_2 + k_3$. The low-credibility bank accommodates part of, but only part of, its higher inflation expectations, resulting in higher inflation. The negative inflation surprise is larger in absolute terms under low credibility, leading to lower employment,

$$\Delta l_t = \Delta(\pi_t - \pi_{t|t-1}) = -k_2\Delta z_{t|t-1} < 0.$$

[19] See, for instance, Federal Reserve Bank of Kansas City [17] for similar statements.

The low-credibility bank would, of course, gain reputation and credibility faster if it accommodated less of the inflation expectation, but this is not optimal due to the current employment cost. The cost of the negative inflation surprise is the opposite of the benefits of a positive surprise discussed above: lowering i_t leads to benefits in terms of lower inflation and better future reputation, but lowers employment through the Phillips curve. The optimal policy is a compromise between these concerns. This result is one formalization of results from the "gradualist versus cold turkey" debate regarding lowering inflation in the early 1980s.[20]

While the optimal speed of adjustment will vary depending on the model, the result that will generalize quite broadly is that with low credibility, the bank should allow higher inflation, but will generate greater negative inflation surprises, than with high credibility.[21]

Given this result, evidence of inflation and inflation expectations above some long-run target does not necessarily suggest that a bank is insufficiently attentive to the target; rather, the bank may be optimally responding to low credibility. Only the fact that the low-credibility bank is implementing smaller absolute inflation surprises would be evidence that it is behaving suboptimally.

6 The effects of transparency in regime U

In this section, we study the role of transparency in regime U. We report results on "welfare" under various transparency levels as measured by the unconditional expectation of the relevant loss function. Thus, we learn which regime is best on average, or which would be preferred without knowledge of the state variables. The social unconditional loss is proportional to $\mathrm{E}[L_t^p]$ with L_t^p given by (2.8), which can be written as

$$\mathrm{E}\left[L_t^p\right] = \frac{1}{2}\left(k_0^2 + \mathrm{Var}\left[\pi_t\right] + \mathrm{Var}[l_t] + l^{*2}\right). \tag{6.1}$$

The central bank's unconditional loss is proportional to $\mathrm{E}[L_t]$, with L_t as in (2.5), which can be written as a sum of six terms:

$$\mathrm{E}\left[L_t\right] = \frac{1}{2}\left(k_0^2 + \mathrm{Var}\left[\pi_t\right] + \mathrm{Var}[l_t] + l^{*2} + \mathrm{Var}[z_t] - 2\mathrm{Cov}[l_t, z_t]\right). \tag{6.2}$$

The central-bank loss differs from the social loss by the term $\frac{1}{2}(\mathrm{Var}[z_t] - 2\mathrm{Cov}[l_t, z_t])$, where only the covariance term is endogenous. The intuition for this is that the central-bank optimum

[20] See, e.g., Fuhrer [20] and Ball [2].

[21] In most standard models, the bank will be trading off the speed of learning against the cost of surprises. With (approximately) linear learning, the tradeoff will generally work as in this paper.

differs from the private-sector optimum only to the extent that the central bank can generate movements in employment that follow the target z_t. The central bank can only achieve this by using inflation surprises.

6.1 Increasing transparency in case U

The results are summarized in,

Proposition 6.1. *Consider increasing transparency, τ, in regime U.*

(i) Reputation. *Raising τ raises the variance of reputation, but decreases the variance of reputation errors ($z_t - z_{t|t-1}$). Raising τ raises the expected reputation cost to the bank of raising intended inflation (increases $\partial E_{t-} z_{t+1|t}/\partial i_t$).*

(ii) Inflation. *Raising τ lowers average inflation (strictly whenever $l^* > 0$), but may raise or reduce the variance of inflation and the inflation term in the unconditional loss,* $\mathrm{E}[\pi_t^2]$.

(iii) Employment. *Raising τ reduces the variance of employment and reduces the employment term in the social unconditional loss,* $\mathrm{E}[(l_t - l^*)^2]$*, but raises the employment term of the central-bank unconditional loss,* $\mathrm{E}[(l_t - l_t^*)^2]$.

(iv) Unconditional loss. *Raising τ may raise or reduce social and central-bank unconditional loss. For plausible discount factors ($\beta > 0.5$), social loss always falls.*

Each part of the claim concerns the derivative of some equilibrium value with respect to τ; the results were demonstrated numerically, as discussed in section 3.3. Despite the complexity of the model, some intuitively appealing properties seem to drive the results.

If raising τ deserves the interpretation as increasing transparency, the variance of reputational errors should fall. This is indeed the case; reducing the variance of the unobservable portion of the control error makes the central bank's reputation track its actual preferences more closely (thus, reduces $\mathrm{Var}[z_t - z_{t|t-1}] \equiv P$).[22] Given the smaller unobservable control-error variance, the public assumes that a greater share of any inflation surprise is due to intentional action by the bank, and the sensitivity of the bank's reputation to its intention rises: $\partial z_{t+1|t}/\partial i_t = g$ increases. This raises the reputation cost of inflation to the bank, which drives the remaining results.

In particular, the greater reputation cost leads to a decrease in k_0, k_2, and k_3.[23] Consider why k_0, average inflation, falls. In any Barro-Gordon-type model, inflation must be high enough, on average, to keep the central bank from engineering positive inflation surprises on average. The rise in transparency increases the marginal reputation cost of inflation and thereby reduces the average level of inflation required to keep the bank from using inflation surprises.

[22] If we held k_2 constant, this result could be demonstrated analytically. In principle, the central bank could reduce k_2 in response to increased transparency to the extent that its private goal would be harder to detect. This does not happen in equilibrium.

[23] Of course, $k_1 = -1/2$ remains optimal in all regimes.

Table 6.1. An increase in transparency, τ

Parm.				Var[$\cdot_t$]		E[$\cdot_t^2$]			Loss	
space	s	P	g	$z_{t\|t-1}$	π_t	π_t	$l_t - l_t^*$	l_t	E[L_t]	E[L_t^p]
Full	+	−	+	+	43.8	93.1	+	−	79.6	96.3
Small	+	−	+	+	87.5	87.5	+	−	+	−

Note: Plus and minus indicate unambiguous signs of the derivate with respect to τ. Numbers indicate the proportion of the parameter space for which the sign is negative. The "full" parameter space refers to the parameter space $(\beta, \rho, \tau) \in [0,1]^3$, $(\sigma_\eta^2, \sigma_\varepsilon^2, \sigma_\theta^2, l^*) \in [0,10]^4$. The "small" parameter space is the same except that $\beta = 0.99999$ and $l^* = 0$.

We call the fall in k_2 and k_3 a reduction in "activism" by the bank. The argument why $k_2 + k_3$ falls when transparency rises is very similar to the argument for k_0. While k_0 gives the unconditional inflation bias, $k_0 + (k_2 + k_3) z_{t|t-1}$ gives the conditional inflation bias for t seen from $t-1$. The conditional inflation bias falls for each level of $z_{t|t-1}$, for the same reason as the unconditional one falls: the marginal reputation cost to the bank of inflation has risen at each level of $z_{t|t-1}$.[24]

Most of the remaining results in the proposition follow from the how the reduction in k_0, k_2, and k_3 affect key components in the model listed in table 6.1 (we emphasize the row for the "full" parameter space at this point). As for part (i), we have explained the reduction in the variance of reputation errors. This also accounts for the increased variance of reputation as the predictor better tracks the predicted.

Part (ii), inflation. The fall in average inflation, k_0, was discussed above. One paradoxical result is that the unconditional variance of inflation may rise. From (2.15), this variance is[25]

$$\mathrm{Var}[\pi_t] = k_2^2 \mathrm{Var}[z_t] + [k_3^2 + 2(k_2 + k_3)] \mathrm{Var}[z_{t|t-1}] + \frac{1}{4}\sigma_\varepsilon^2 + \sigma_\eta^2. \tag{6.3}$$

The contribution to inflation variance of the control error and supply shock (the final two terms) are unchanged. The first term—the contribution of the employment target variance—falls with k_2. The change in the second term is ambiguous. The variance of the bank's reputation rises as noted above, while $(k_2 + k_3)$ falls. For $56.2 (= 100 - 43.8)$ percent of the parameter space, the rise in the variance of reputation dominates all other changes, and the overall variance of inflation rises. The inflation term in the loss function ($\mathrm{E}[\pi_t^2]$) rises for only $6.9 (= 100 - 93.1)$ percent of the parameter space, as the fall in k_0^2 offsets the rise in inflation variance.[26]

[24] The argument why k_2 and k_3 fall separately is more complicated, but seems to involve the same elements.
[25] Noting that $\mathrm{Cov}[z_t, z_{t|t-1}] = \mathrm{Var}[z_{t|t-1}]$.
[26] $\mathrm{E}[\pi_t^2] = \mathrm{Var}[\pi_t] - k_0^2$.

Part (iii), employment. The variance of employment clearly falls as the variance of inflation surprises falls. This unambiguously lowers the employment term in the social loss function.

In contrast, the rise in transparency unambiguously raises the employment term in the central-bank loss function, $\mathrm{E}[(l_t - l_t^*)^2]$. The fall in employment variance is due to the fact that, with greater transparency, the bank chooses to generate smaller inflation surprises. These surprises were, of course, used to make employment move with the target l_t^*, and the bank is made worse off without this co-movement.[27]

Part (iv), loss. Given the results for the components of the loss functions, it is natural that both the central bank and social loss can either rise or fall with transparency. Social loss generally falls (96.3 percent of the parameter space) with increased transparency, however. Further, for plausible discount rates ($\beta > 1/2$), greater transparency is uniformly socially good.

Rises in transparency are also good for the central bank on 79.5 percent of the parameter space. It is important to note, however, that the loss rises for plausible parameter values, e.g., $\beta = 0.97, \rho = 0.30, \sigma_\eta = 1.89, \sigma_\theta = 1.0, \sigma_\varepsilon = 1.15, l^* = 0.11, \tau = 0.36$. In this case, the target is moderately persistent and the control error has a standard deviation about twice that of the target shock and the real shock.

The general lessons from this section are that increases in transparency in a natural way cause the bank to be less activist. The average and conditional inflation biases in the model unambiguously fall. The variance of inflation may rise or fall, however, since the effect of the fall in activism may be dominated by the effect of reputation more clearly tracking actual central-bank preferences. The improved tracking reduces the variance of inflation surprises, which reduces the variance of employment. While this is good for the public, the reduction in employment variance is due to the component that was correlated with the bank's target, making the bank worse off.

[27] More formally, we have

$$\mathrm{E}[(l_t - l_t^*)^2] = \mathrm{Var}[l_t - z_t] + l^{*2} = \mathrm{Var}[l_t] + l^{*2} + \mathrm{Var}[z_t] - 2\mathrm{Cov}[l_t, z_t].$$

The second and third terms on the right side do not change. The first term falls. It is, however, the component of l_t that covaries with z_t that is diminished in variance; thus, the fall in the covariance between l_t and z_t more than offsets the fall in variance of l_t:

$$\mathrm{Var}[l_t] - 2\mathrm{Cov}[l_t, z_t] = (k_2^2 - 2k_2)P + \frac{1}{4}\sigma_\varepsilon^2 + \sigma_\eta^2,$$

where we have used $\mathrm{Cov}[l_t, z_t] = k_2 P$. The left side rises, since P falls and since $k_2^2 - 2k_2 < 0$ falls in magnitude.

6.2 Optimal transparency

Since loss does not change monotonically for all levels of τ, proposition 6.1 leaves open the question of which degree of transparency minimizes loss. This question is resolved in,

Proposition 6.2. *For the "full" parameter space (the top row in table 6.1):*

(i) *Full transparency of intention minimizes the social loss for* 97.3 *percent of the parameter space. The social loss is always minimized at either* $\tau = 1$ *or* $\tau = 0$.

(ii) *Full transparency of intention minimizes the central-bank loss for* 79.5 *percent of the parameter space, whereas minimum transparency minimizes it for* 18.6 *percent. An intermediate degree of transparency is best for central-bank loss for* 1.9 *percent.*

(iii) *The optimal transparency is always at least as high for society as for the central bank. For* 15.9 *percent of the parameter space,* $\tau = 0$ *minimizes central-bank loss while* $\tau = 1$ *minimizes social loss.*

This proposition is shown numerically. Full transparency of intention is generally best for society. For most of the full parameter space, it is best for the central bank as well. Throughout the full parameter space, the optimal degree of transparency is at least as high for society as for the central bank.

However, social and central-bank preferences sharply conflict on a strikingly large portion of the parameter space (15.8 percent): the bank wants minimum transparency and the public wants full transparency. We can shed further light on this phenomenon by considering the "small" parameter space which differs from the full space only by imposing that the bank is patient ($\beta = 0.99999$) and has no average bias ($l^* = 0$). This corresponds to the second row of table 6.1. For this case we have

Proposition 6.3. *With a patient central bank with no average inflation bias, central-bank loss is monotonically increasing with transparency, while social loss is monotonically decreasing with transparency. Thus,* $\tau = 0$ *minimizes central-bank loss and maximizes social loss, and* $\tau = 1$ *maximizes central-bank loss and minimizes social loss.*

This result is clear from table 6.1.[28] The public likes transparency for the same reason as in the full parameter space. The central bank's increased preference for minimum transparency in the small parameter space is related to the roles of both l^* and β. With $l^* > 0$ there is an average inflation bias, and one benefit to the bank of lower transparency is a reduced average bias. With $l^* = 0$ this benefit is gone. The primary cost to the bank of increasing transparency is a more limited ability to generate a correlation between employment and z_t. When a shock drives z_t up,

[28] We suspect, but have not confirmed, that for $\beta < 1$, there is some tiny portion of the parameter space for which the bank prefers nonzero transparency. Thus, this region is smaller than the tolerance for our absolute statements given in section 3.3.

z_t remains high and since the public learns slowly, the bank can increase l_t for several periods. Greater patience (β near one), raises the benefit from persistent increases employment in such periods. Higher transparency would increase the speed of learning and reduce the persistence of the increase in employment. Thus, a patient bank with no average bias prefers the minimum possible transparency.[29]

These results suggest a possible conflict between society and the central bank over transparency. Perhaps paradoxically, this conflict may be smaller in cases of high average inflation bias (high l^*): societies with large average bias problems may find it easier to adopt transparency than those with small average bias problems.

We emphasize that these results and those of CM preceding them, raise a host of complicated issues. Since we consider a central bank choosing the optimal τ once-and-for-all, we implicitly assume that the central bank has a commitment technology regarding transparency, but not with regard to the policy rule itself. While CM make a similar assumption, we believe the discretion case might also be of interest. Commitment might arise if we view the public as choosing τ and imposing it on the central bank, but then we must consider Lewis's [29] argument that the central bank could offset increased transparency by increasing the variance of the control error. This suggests a game between the central bank and those regulating it. Our related paper, [16], deals with these issues.

7 Conclusions

In this paper, we follow Cukierman and Meltzer in examining the importance of transparency and credibility in monetary policy by using a model where the central bank's employment target is stochastic and time-varying. We believe that our work improves on their seminal work by acknowledging an explicit stabilization objective for employment or output and by distinguishing transparency from control-error variance. The former makes central-bank policy depend on its reputation; the latter we believe to be necessary for avoiding confusion between transparency and control in monetary policy. Thus, increased transparency means that the private-sector can more easily infer the central bank's intentions, at given degrees of monetary control.

The model has implications for some frequent claims in the literature. One of these claims is that a low-credibility bank, everything else equal, should conduct a less expansionary policy

[29] As shown in proposition 4.1, in the limiting case as $\beta\rho \to 1$, all regimes converge to the social optimum under commitment. Thus, whereas a patient bank with no average bias always disagrees with the public over the optimum transparency, in the limit as the persistence of the goal goes to one, this disagreement becomes moot.

than a high-credibility bank. We find that a low-credibility bank—one facing higher inflation expectations—will generate a larger (negative) inflation surprise from the public's perspective, leading to lower employment and, in this sense, conducts a *less* expansionary policy. However, the low-credibility bank at the same time generates higher inflation than a high-credibility bank and, in this sense, conducts a *more* expansionary policy. Thus, it cannot be inferred from higher inflation alone that a bank is not optimally pursuing an inflation target.

A second claim is that a low-credibility bank has less flexibility to respond to shocks in order to avoid further erosion of credibility. In contrast, we find that low and high-credibility banks react in the same way to supply shocks and shocks to the employment target; it is not the case that the low-credibility bank has less scope to stabilize supply shocks, nor does it more urgently build up credibility than a high-credibility bank.

A third claim is that increased transparency increases credibility and improves policy outcomes. In our model, increased transparency makes the central bank's reputation and private-sector inflation expectations more sensitive to the central bank's actions. This increases the costs for the bank of deviating from the announced zero-inflation policy, and hence deters the bank from using inflation surprises to achieve its employment target. As a result, variability of both inflation and employment falls, and any average inflation bias is reduced. These changes generally (but not always) increase social welfare. In many cases, however, increased transparency leads to a worse outcome for the bank. Since the central bank's preferences and social preferences for transparency diverge, society might prefer to decide on the level of transparency in monetary policy, rather than delegate this decision to the central bank.[30]

The fact that increased transparency makes the bank's optimal policy closer to the social optimum may throw some light on McCallum's [31] criticism of discretion equilibria in monetary policy. McCallum argues that the problems arising in discretion equilibria may not be decisive in practice because central bankers see the value of the policy consistent with commitment and can *just do it*. If we are to maintain the equilibrium framework, this can only be interpreted as the belief that there is some heretofore unmodelled aspect of preferences or commitment mechanisms that alters the equilibrium outcomes.[31] We sympathize with this view: some implicit commitment mechanism may exist. We would like to see this mechanism specified and discussed,

[30] We recall Milton Friedman's response to Fischer [19], footnote 52, on central bankers' loss functions: "From revealed preference, I suspect that by far and away the two most important variables in their loss functions are avoiding accountability on the one hand and achieving public prestige on the other."

[31] For instance, the bank would have an incentive to deviate from the commitment policy in the current period and promise to follow the commitment policy from the next period onwards.

however, because we also sympathize with Canzoneri's [9] view that, in the presence of private information, commitment would be hard to sustain. In our model, private information exists, but the central bank's concern about its reputation creates an incentive to behave more in accordance with the socially optimal policy. Increased transparency makes this incentive stronger. Indeed, a very patient central bank with very persistent idiosyncratic deviations from the social employment goal would, in the limit, follow the socially optimal policy. Thus, credibility and transparency may push the discretion equilibrium toward the socially preferred equilibrium, in spite of the absence of an explicit commitment mechanism.[32]

With regard to transparency, we find an especially intriguing result. When the central bank's idiosyncratic goals can be directly observed, the central bank's preferences need not be inferred from the its actions. Since the central bank's reputation is then independent of its actions, the central bank loses an important constraint on its behavior. The resulting equilibrium has higher average inflation and higher variability of both inflation and employment than in any other case studied. Thus, this type of "extreme" transparency is counterproductive. In a richer model, such extreme transparency might be beneficial if, for example, directly observing the central bank's idiosyncratic goals allowed society to force its own goals on the central bank more effectively.

There are some obvious qualifications to our results, some of which may be suitable for future work. In a separate paper, [16], we treat the optimal transparency more thoroughly, considering the issues of commitment and the incentives of the central bank to renege. Further complicating the private sector's learning problem with confusions between supply shocks and shocks to goals as well as between temporary and persistent shocks would complicate the private sector's signal extraction problem and possibly modify some of our results. Finally, we have assumed that the private sector believes the central bank's policy rule to be linear; as a consequence, it is optimal for the central bank to choose a linear rule. While this is a natural starting point, and the existence of strong nonlinearities in learning seems implausible to us, work relaxing this assumption might create further insights.

[32] Others interpret McCallum as implicitly relying on trigger-strategy equilibria, an interpretation that is rejected by McCallum.

A The Kalman filter

Following Harvey [23], but using our notation, we have the transition equation (2.7) and measurement equation (3.2), which implies the updating equation (3.3). In steady state, the Kalman gain, g, is given by

$$g = g(k_2) \equiv \rho \frac{k_2 P(k_2)}{k_2^2 P(k_2) + \sigma_\nu^2}. \tag{A.1}$$

where P is the conditional variance of the optimal predictor:

$$P = P(k_2) \equiv \sqrt{\left(\frac{(1-\rho^2)\frac{\sigma_\nu^2}{k_2^2} - \sigma_\theta^2}{2}\right)^2 + \sigma_\theta^2 \frac{\sigma_\nu^2}{k_2^2}} - \frac{(1-\rho^2)\frac{\sigma_\nu^2}{k_2^2} - \sigma_\theta^2}{2} > 0. \tag{A.2}$$

If ρ and k_2 are both positive, from (A.1) it is clear that,

$$0 \le g(k_2)k_2 \le \rho. \tag{A.3}$$

B Unobservable goal

Using $\mathrm{E}_{t-} l_t = i_t - \pi_{t|t-1} + \varepsilon_t$, we can write the first-order condition (3.8) as

$$i_t = \frac{1}{2}\left\{ l^* + z_t + \pi_{t|t-1} - \beta \mathrm{E}_{t-}\left[(\delta_1 + \delta_2 z_{t+1|t} + \delta_5 z_t \ ) \frac{\partial z_{t+1|t}}{\partial i_t} \right] \right\} - \frac{1}{2}\varepsilon_t. \tag{B.1}$$

From (2.12), (3.2) and (3.3) and we have,

$$\begin{aligned} \pi_{t|t-1} &= k_0 + (k_2 + k_3) z_{t|t-1} && \text{(B.2)} \\ \frac{\partial z_{t+1|t}}{\partial i_t} &= g && \text{(B.3)} \\ \mathrm{E}_{t-} z_{t+1|t} &= (\rho - g k_2) z_{t|t-1} + g k_2 z_t. && \text{(B.4)} \end{aligned}$$

Substituting and collecting terms gives a function of the form (2.12) with (3.9)–(3.12):

$$\begin{aligned} i_t &= \frac{1}{2}\left(l^* + k_0 - \beta g \delta_1 \right) - \frac{1}{2}\epsilon_t + \frac{1}{2}\left[1 - \beta g \left(g k_2 \delta_2 + \delta_5 \right) \right] z_t \\ &\quad + \frac{1}{2}\left[k_2 + k_3 - \beta g \left(\rho - g k_2 \right) \delta_2 \right] z_{t|t-1}. \end{aligned}$$

Now, return to the value function using (2.12) and (2.18),

$$\begin{aligned} V(z_{t|t-1}, z_{t-1}) &= \mathrm{E}_{t-1}\left\{ \frac{1}{2}\left[\left(k_0 + k_1 \epsilon_t + k_2 z_t + k_3 z_{t|t-1} + \eta_t \right)^2 \right.\right. \\ &\quad \left.\left. + \left((k_1 + 1)\epsilon_t + k_2 (z_t - z_{t|t-1}) + \eta_t - l^* - z_t \right)^2 \right] + \beta V(z_{t+1|t}, z_t) \right\}. \end{aligned}$$

Expansion of the value function gives:

$$\begin{aligned} V(z_{t|t-1}, z_{t-1}) &= \mathrm{E}_{t-1}\left\{ \frac{1}{2}\left[k_0^2 + k_1^2 \epsilon_t^2 + k_2^2 z_t^2 + k_3^2 z_{t|t-1}^2 + \eta_t^2 \right] \right. \\ &\quad + k_0 k_2 z_t + k_0 k_3 z_{t|t-1} + k_2 k_3 z_{t|t-1} z_t \\ &\quad + \frac{1}{2}\left[(k_1+1)^2 \epsilon_t^2 + (k_2 - 1)^2 z_t^2 + k_2^2 z_{t|t-1}^2 + \eta_t^2 + l^{*2} \right] \\ &\quad - (k_2 - 1) k_2 z_{t|t-1} z_t - (k_2 - 1) l^* z_t + k_2 l^* z_{t|t-1} \\ &\quad \left. + \beta\delta_0 + \beta\delta_1 z_{t+1|t} + \frac{1}{2}\beta\delta_2 z_{t+1|t}^2 + \beta\delta_3 z_t + \frac{1}{2}\beta\delta_4 z_t^2 + \beta\delta_5 z_{t+1|t} z_t \right\}. \end{aligned}$$

The following expressions are useful in evaluating this expectation:

$$\mathrm{E}_{t-1} z_{t+1|t} = (\rho - gk_2) z_{t|t-1} + \rho g k_2 z_{t-1} \tag{B.5}$$

$$\mathrm{E}_{t-1} z_{t+1|t}^2 = (\rho - gk_2)^2 z_{t|t-1}^2 + (gk_2)^2 \left(\rho^2 z_{t-1}^2 + \sigma_\theta^2\right) + 2\rho(\rho - gk_2) g k_2 z_{t|t-1} z_{t-1} + g^2 \sigma_\nu^2 \tag{B.6}$$

$$\begin{aligned} \mathrm{E}_{t-1} z_{t+1|t} z_t &= \mathrm{E}_{t-1} \left[(\rho - gk_2) z_{t|t-1} + g (k_2 z_t + \eta_t) \right] z_t \\ &= \rho (\rho - gk_2) z_{t|t-1} z_{t-1} + g k_2 \left(\rho^2 z_{t-1}^2 + \sigma_\theta^2 \right). \end{aligned} \tag{B.7}$$

Thus, expanding expectations in the value function yields

$$\begin{aligned} V(z_{t|t-1}, z_{t-1}) &= \frac{1}{2} \left[k_0^2 + \frac{1}{4} \sigma_\epsilon^2 + k_2^2 \left(\rho^2 z_{t-1}^2 + \sigma_\theta^2 \right) + k_3^2 z_{t|t-1}^2 + \sigma_\eta^2 \right] \\ &\quad + k_0 k_2 \rho z_{t-1} + k_0 k_3 z_{t|t-1} + k_2 k_3 \rho z_{t|t-1} z_{t-1} + \\ &\quad \frac{1}{2} \left[\frac{1}{4} \sigma_\epsilon^2 + (k_2^2 - 2k_2 + 1)(\rho^2 z_{t-1}^2 + \sigma_\theta^2) + k_2^2 z_{t|t-1}^2) + \sigma_\eta^2 + l^{*2} \right] \\ &\quad - (k_2 - 1) k_2 \rho z_{t|t-1} z_{t-1} - (k_2 - 1) l^* \rho z_{t-1} + k_2 l^* z_{t|t-1} \\ &\quad + \beta\delta_0 + \beta\delta_1 (\rho - gk_2) z_{t|t-1} + \beta\delta_1 \rho g k_2 z_{t-1} \\ &\quad + \frac{1}{2} \beta\delta_2 \left[(\rho - gk_2)^2 z_{t|t-1}^2 + \rho^2 (gk_2)^2 z_{t-1}^2 + (gk_2)^2 \sigma_\theta^2 + g^2 \sigma_\nu^2 \right] \\ &\quad + \beta\delta_2 \rho (\rho - gk_2) g k_2 z_{t|t-1} z_{t-1} + \beta\delta_3 \rho z_{t-1} + \frac{1}{2} \beta\delta_4 (\rho^2 z_{t-1}^2 + \sigma_\theta^2) \\ &\quad + \beta\delta_5 \left[\left((\rho - gk_2) z_{t|t-1} + \rho g k_2 z_{t-1} \right) \rho z_{t-1} + g k_2 \sigma_\theta^2 \right]. \end{aligned}$$

Collecting the constants and rearranging using (2.10) gives,

$$\begin{aligned} \delta_0 &= \frac{1}{2} \frac{l^{*2} + k_0^2 + \frac{1}{2}\sigma_\epsilon^2 + \left(2k_2^2 - 2k_2 + 1 + \beta(gk_2)^2 \delta_2 + \beta\delta_4\right) \sigma_\theta^2}{1 - \beta} \\ &\quad + \frac{1}{2} \frac{[2 + \beta(1 - \tau) g^2 \delta_2 + 2\beta g k_2 \delta_5] \sigma_\eta^2}{1 - \beta}, \end{aligned} \tag{B.8}$$

Collect terms in $z_{t|t-1}$ and rearranging using (3.9) gives:

$$\delta_1 = \frac{l^* (k_2 + k_3)}{1 - \beta(\rho - gk_2) + \beta g k_3}. \tag{B.9}$$

Collect terms in $z_{t|t-1}^2$:

$$\delta_2 = \frac{k_2^2 + k_3^2}{1 - \beta(\rho - gk_2)^2}. \tag{B.10}$$

Collect terms in z_{t-1} and rearrange using (3.9):

$$\delta_3 = \frac{\rho l^*}{1 - \beta\rho}, \tag{B.11}$$

Similarly the terms in z_{t-1}^2 give:

$$\delta_4 = \rho^2 \frac{1 - 2k_2 + 2k_2^2 + \beta(gk_2)^2 \delta_2 + 2\beta g k_2 \delta_5}{1 - \beta\rho^2}. \tag{B.12}$$

Finally the terms in $z_{t|t-1} z_{t-1}$ with (3.12) give:

$$\delta_5 = \frac{\rho k_2}{1 - \beta\rho(\rho - gk_2)}, \tag{B.13}$$

B.1 Existence

We now have a simultaneous set of 9 equations, 3 for the ks and 6 for the δs ($k_1 = -\frac{1}{2}$ is known). We first show that we can rewrite that system as a single equation for k_2 in terms of itself,

$$k_2 = f(k_2),$$

and equations giving the eight remaining ks and δs in terms of k_2.

First, we get an expression for k_3 in terms of k_2 only. Since g by (A.1) depends only on k_2, and δ_2 by (B.10) depends only on k_2 and k_3, (3.12) can be written as an expression in k_2 and k_3 only. Taking k_2 as fixed, the equation is a quadratic in k_3:

$$k_3 = k_2 - \frac{\beta g(k_2)[\rho - g(k_2)k_2]}{1 - \beta(\rho - g(k_2)k_2)^2} k_2^2 - \frac{\beta g(k_2)[\rho - g(k_2)k_2]}{1 - \beta(\rho - g(k_2)k_2)^2} k_3^2$$

or

$$0 = \frac{A(k_2)}{k_2} k_3^2 + k_3 - k_2[1 - A(k_2)]$$

where

$$A(k_2) = \frac{\beta g(k_2)k_2[\rho - g(k_2)k_2]}{1 - \beta[\rho - g(k_2)k_2]^2}.$$

This has solutions of the form,

$$k_3 = \frac{-1 \pm \sqrt{1 + 4A(k_2)[1 - A(k_2)]}}{2A(k_2)} k_2 \tag{B.14}$$

We note that we can write $A(k_2)$ as

$$A(k_2) = \frac{\beta\left[\rho - (\rho - g\left(k_2\right)k_2)\right]\left[\rho - g(k_2)k_2\right]}{1 - \beta\left[\rho - g(k_2)k_2\right]^2} = \frac{\beta\rho\left[\rho - g(k_2)k_2\right] - \beta\left[\rho - g(k_2)k_2\right]^2}{1 - \beta\left[\rho - g(k_2)k_2\right]^2};$$

hence

$$0 \le A(k_2) < 1, \tag{B.15}$$

since $0 \le \rho - g(k_2)k_2 < 1$. Note that if we take the positive root in (B.14), this inequality implies that $0 < k_3 < k_2$.

We have two arguments for taking the positive root. First, for particular parameter values, one can rule out the negative root by showing that a one-period deviation from the implied policy rule decreases the central bank's loss. Using the approach described in Appendix D, we verified numerically that the negative root is not an equilibrium. Second, McCallum [30] argues that we should consider solutions for which the coefficients of the policy rule are continuous in the parameters of the problem. It is straightforward to see that this argument rules out the negative root.

Thus, we have k_3 in terms of k_2 alone, and substituting this expression for k_3 into the formulae for δ_2 gives an expression for δ_2 in terms of k_2 alone. By (B.13), δ_5 depends only on k_2. Recursive substitution using these results gives expressions for the other δs and k_0. Finally, substituting the expressions for δ_2 and δ_5 into (3.11) gives the desired equation for k_2:

$$k_2 = f(k_2) \equiv \frac{\frac{1-\beta\rho^2}{1-\beta\rho^2+\beta\rho g(k_2)k_2}}{2 + \beta g(k_2)^2\delta_2(k_2)}. \tag{B.16}$$

Now existence and uniqueness are only a question of whether there are zero, one, or more solutions to (B.16). Since $0 < \frac{1-\beta\rho^2}{1-\beta\rho^2+\beta\rho g(k_2)k_2} < 1$ and $\beta g(k_2)^2\delta_2(k_2) > 0$, we have

$$0 < f(k_2) < \frac{1}{2}$$

for all k_2. Thus, any solution to (B.16) must be in $[0, \frac{1}{2}]$. Further, since f is continuous for $k_2 \in [0, \frac{1}{2}]$, (B.16) must have at least one solution. Arguments about uniqueness are discussed in the text and appendix D.

B.2 Proof that k_2 and k_3 go to zero when $\beta\rho$ goes to one

First, we show that $\lim_{\beta\rho\to 1} k_2 = 0$. Denote the numerator in (B.16) by N, so that

$$\frac{1}{N} = \frac{1-\beta\rho^2+\beta\rho g(k_2)k_2}{1-\beta\rho^2} = 1 + \frac{\beta\rho g(k_2)k_2}{1-\beta\rho^2},$$

or, using $g(k_2)k_2 = \frac{\rho k_2^2 P(k_2)}{k_2^2 P(k_2)+\sigma_\nu^2}$,

$$\frac{1}{N} = 1 + \frac{\beta\rho^2 k_2^2 P(k_2)}{(1-\beta\rho^2)(k_2^2 P(k_2)+\sigma_\nu^2)}.$$

Now let $\beta\rho \to 1$, in which case $\beta \to 1$ and $\rho \to 1$ (since they are both bounded above by one), and $\beta\rho^2 \to 1$. Assume, contrary to the desired result, that k_2 is bounded away from zero as $\beta\rho^2 \to 1$. Since $P(k_2)$ is bounded below by σ_ν^2, we have $\frac{1}{N} \to \infty$ implying $N \to 0$. Since the denominator of (B.16) is bounded below by 2, $N \to 0$ implies that $k_2 \to 0$, a contradiction. Since (as noted above) $0 \leq k_3 < k_2$, it follows directly that $\lim_{\beta\rho\to 1} k_3 \to 0$.

C Regime OI

From (A.1) and (A.2) it is clear that $gk_2 = \rho$ when $\sigma_\nu^2 = 0$. Using this fact and (3.12), we have $k_2 = k_3$. Substituting in (B.9), (B.10), and (B.13), gives $\delta_1 = \frac{2l^* k_2}{1+\beta\rho}, \delta_2 = 2k_2^2, \delta_5 = \rho k_2$
Using these results, (3.9), and $gk_2 = \rho$,

$$k_0 = l^* - \frac{2\beta\rho l^*}{1+\beta\rho} = l^* \frac{1-\beta\rho}{1+\beta\rho}.$$

From (3.10), $k_1 = -\frac{1}{2}$, and from (3.11) using $gk_2 = \rho$ and the expressions for δ_2 and δ_3,

$$k_2 = \frac{1-\beta g\rho k_2}{2+\beta g^2(2k_2^2)} = \frac{1-\beta\rho^2}{2(1+\beta\rho^2)}.$$

These together imply the results in the text.

D The numerical analysis

For a given value of the parameters $(\beta, \rho, \tau, \sigma_\eta^2, \sigma_\theta^2, l^*)$, we solve regime U by searching over $k_2 \in [0, \frac{1}{2}]$ for a k_2 satisfying $k_2 = f(k_2)$. Regimes OI and OG can be solved by direct computation. After solving the models, any aspect of the models for which we have formulae can be computed directly. This includes all the results about the values of the ks, the loss functions, g, and P. In particular, the derivatives are all evaluated with analytic formulae.

Three items for which we state numerical results cannot be directly computed: (1) Uniqueness of the solution to $k_2 = f(k_2)$, and (2) The incentive to deviate from the possible equilibrium with the negative k_3 root, 3) Verifying what τ is optimal for any value for the other parameters.

Numerical uniqueness is checked by computing $f(k_2)$ for 100 evenly spaced points in the interval $[0.0001, 0.5]$ and checking whether f is monotonically declining over the range for those points.

To test the incentive to deviate under the negative k_3 root, we first solve the model for the ks, δs, and g taking the negative root for k_3. We then repeat the following steps for a wide range of the state variables z_t and $z_{t|t-1}$ (these are the state variables as of time $t-$, which is relevant in what follows): (i) Evaluate the central-bank loss under the implied policy rule seen from time $t-$, after θ and $\varepsilon = 0$ are drawn at t, but before η is drawn. (ii) Evaluate the central-bank loss seen from time $t-$ (with $\varepsilon = 0$) from setting i_t equal to various arbitrarily chosen values, but returning to the policy rule from $t + 1$ onward. If for any $(z_t, z_{t|t-1})$ pair, there is an i_t that dominates the policy rule, we have proved that the negative k_3 root is not an equilibrium for this parameter value. In 100,000 draws, about 3 percent of the draws would not solve at all with the negative k_3 root; for all the remaining draws, the negative root does not constitute an equilibrium.

For the optimal τ, we draw a value for the other parameters and check the value of the two loss functions at 100 evenly spaced points between zero and one. The smallest loss is taken as the optimum.

Four separate numerical experiments were performed: There was one run for the full parameter space and one for the small parameter space where all aspects except the optimal τ and the validity of the negative k_3 root were checked. There was one run checking the optimal τ for the full parameter space (since the derivatives of loss with respect to τ for the small parameter space were of one sign, the optimal τ results follow without further computation). Finally, there was one run checking the validity of the negative k_3 root. In each case, for a small number of draws, numerical instability for certain extreme parameter values kept us from solving the model at all. For the four experiments, this problem arose for 63, 66, 152, and 3,319 draws out of 100,000, respectively.

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