Time Consistency of Fiscal and Monetary Policy: A Solution*

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Abstract

This paper demonstrates how time consistency of the Ramsey policy—the optimal fiscal and monetary policy under commitment—can be achieved. Each government should leave its successor with a unique maturity structure for the nominal and indexed debt, such that the marginal benefit of a surprise inflation exactly balances the marginal cost. Unlike in earlier papers on the topic, the result holds for quite general Ramsey policies, including timevarying polices with positive inflation and positive nominal interest rates. We compare our results with those in Persson, Persson, and Svensson (1987), Calvo and Obstfeld (1990), and Alvarez, Kehoe, and Neumeyer (2004).

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1 Introduction

Time consistency of optimal monetary and fiscal policy has been extensively discussed in the literature on the macroeconomics of public finance. Calvo's [2] seminal paper pointed to the ex post incentives of a government to use a surprise inflation to reduce the real value of any outstanding fiat money, when other sources of finance distort economic activity. Lucas and Stokey [6] (henceforth LS) extended Calvo's analysis by showing how similar time-consistency problems arise in a real economy due to the government's ability to manipulate the market value of indexed debt. In addition, they showed that these problems can be avoided if every government undertakes a unique restructuring scheme of the maturity (and contingency) of the indexed debt left to its successor, under the assumption that outstanding government debt can not be subject to outright default. LS also argued, however, that the time-consistency problem is unavoidable in a monetary economy, where governments always have an ex post incentive to reduce (increase) the real value of net nominal government liabilities (assets) by a surprise inflation, so as to lower distortionary taxes.

Counter to this, Persson, Persson, and Svensson [9] (henceforth PPS) suggested that a unique restructuring of both nominal and indexed debt could resolve both types of time-consistency problems. More precisely, PPS suggested that the first-order conditions for optimal fiscal and monetary policy in a sequence of discretionary equilibria could be made identical to the corresponding first-order conditions for the Ramsey policy—the optimal policy under commitment. One of their conditions for the nominal debt structure is that each government leaves its successor with a total value of nominal claims on the private sector equal to the money stock, such that net nominal liabilities are zero, which appeared to remove the incentive for a surprise inflation. By applying an informal but innovative variation argument, however, Calvo and Obstfeld [3] (henceforth CO) could show that the solution proposed by PPS is in fact not an optimum.

A recent paper by Alvarez, Kehoe, and Neumeyer [1] (henceforth AKN) reexamined the time consistency of the optimal fiscal and monetary policy in a setting very similar to that of LS, PPS, and CO, except that they assumed that private-sector preferences satisfy conditions that imply that the Friedman rule, a zero nominal interest rate, is optimal (see section 5 for these conditions). Under the Friedman rule, AKN then demonstrated that the Ramsey policy can be made time consistent: this is achieved by the LS conditions on the indexed debt structure plus the PPS condition of zero government net nominal liabilities. As AKN noted, however, under the Friedman rule their monetary economy becomes isomorphic to a non-monetary economy, indeed the non-monetary economy examined by LS. The AKN result is thus to a large extent a restatement of the LS result.

Given the results in the literature, it would thus appear that the time-consistency problem of optimal policy is unavoidable in genuinely monetary economies where monetary instruments and nominal assets and liabilities play an essential role in shaping equilibrium allocations and raising some revenue for the government, in particular, in economies where the Friedman rule is not optimal.

In this paper, we show that such a conclusion is premature.[1] We show that time consistency can be restored under the assumption that surprise inflation entails some direct costs for the private sector, in addition to the indirect costs via lower wealth. We introduce such direct costs by tying the provision of liquidity services to beginning-of-period real balances rather than end-of period ones. There then exists a unique restructuring scheme for the nominal and indexed government debt, which makes the Ramsey policy time consistent. As in LS original paper and the following literature, however, the argument relies on the absence of outright government default.

Beyond demonstrating that time consistency of the Ramsey policy is possible in genuinely monetary economies, we think this result is valuable for at least two reasons. First, and most importantly, it is plainly unrealistic that surprise inflations entail no direct costs whatsoever. A neutral, unanticipated increase in the price level could never be done instantaneously because of various nominal rigidities and contract lags. Economic agents will thus have the opportunity to take costly action to reduce their losses or increase their gains. A surprise inflation will also normally have undesirable wealth redistribution effects, cause some bankruptcies, etc.[2] Second, the result enlarges the set of economic environments where time consistency can be achieved. One of AKN's necessary conditions for time-consistent policy implies a unitary income elasticity of real balances, which is far from universally observed in the data. Moreover, their assumption of no initial outstanding nominal liabilities is very strong. Perhaps it is not a coincidence that we rarely observe policies leading to zero nominal interest rates, as implied by these conditions.

The analysis is related to a study by Nicolini [7], who considers a monetary where money does not enter in the utility function, but through a cash-in-advance constraint as in Svensson [13]. In this environment, surprise inflation does indeed impose direct costs on the public. Nicolini shows how this alters the nature of the time inconsistency problem facing the policy maker – who may

[1] The basis of our analysis was first laid out in an unpublished reply to CO (Persson, Persson, and Svensson [10]).

[2] See Persson, Persson, and Svensson [11] for a case study of the possibilities for and consequences of an attempt to dramatically increase inflation in Sweden in order to reduce the real value of the nominal public debt.

actually have incentives to impose a surprise deflation – but does not study how time consistency may be restored by debt management.

Section 2 of our paper lays out a model of a monetary economy, where the Friedman rule need not be optimal, and where the government may thus optimally raise some revenue from anticipated inflation. The economy's Ramsey policy and equilibrium is characterized in section 3. We then demonstrate, in section 4, how a careful restructuring of the nominal and indexed debt makes the Ramsey policy time consistent under discretion. In section 5, we compare our analysis and results to those in the original PPS setup and suggestion, the CO comment, and the recent AKN paper. Section 6 concludes.

2 The model

Our model follows quite closely those in LS and PPS, although the notation is somewhat modified.[3] Thus, we consider an economy with a representative consumer and a government. Time is discrete and separated into periods, $t = 0, 1, 2, \ldots$. For simplicity, all uncertainty is assumed away and the consumer and the government have perfect foresight; our results can be easily generalized to an economy with uncertainty and state-contingent debt. A single good is produced with a simple linear technology, according to the resource constraint,

$$c_t + x_t + g_t \leq 1. \tag{1}$$

Given a unitary endowment of time in each period, c_t is consumption of the representative consumer in period t, x_t is her leisure (so $1 - x_t$ is the consumer's supply of labor producing the same amount of goods), and g_t is (exogenous) government consumption.

The consumer's preferences in a given period θ are given by the intertemporal utility function

$$\sum_{t=\theta}^{\infty} \beta^{t-\theta} U(c_t, x_t, m_t), \tag{2}$$

where $\beta \in (0, 1)$ is a discount factor and $U(c_t, x_t, m_t)$ is the period utility function. For simplicity, the period utility function is assumed additively separable, so the cross derivatives satisfy $U_{cx} = U_{cm} = U_{xm} = 0$, although we shall indicate that our results do not depend on this simplification. We let

$$m_t \equiv M_{t-1}/P_t \tag{3}$$

[3] The real part of the model in LS and PPS are identical, except that PPS abstract from uncertainty. LS introduce money via a cash/credit goods distinction, whereas PPS introduce it via money in the utility function.

denote beginning-of-period real balances, where M_{t-1} is money carried over from the previous period and held in the beginning of period t and P_t is the price level in period t. Thus, importantly, beginning-of-period real balances, M_{t-1}/P_t, rather than end-of-period real balances, M_t/P_t, provide liquidity services and facilitate transactions during period t.[4] The period utility function is concave, twice continuously differentiable, and strictly increasing in c_t and x_t, and increasing in m_t.

In period t, the consumer faces the budget constraint:

$$q_{\theta,t}[(1-\tau_t)(1-x_t)+M_{t-1}/P_t]+\sum_{s=t}^{\infty} q_{\theta,s}({}_{t-1}b_s+{}_{t-1}B_s/P_s) \geq q_{\theta,t}(c_t+M_t/P_t)+\sum_{s=t}^{\infty} q_{\theta,s}({}_tb_s+{}_tB_s/P_s). \tag{4}$$

Here, $q_{\theta,t}$ denotes the present value in period θ of goods in period t, and τ_t denotes proportional taxes on labor income levied by the government. Furthermore, ${}_{t-1}b_s \gtreqless 0$ denotes net claims by the consumer when entering period t on the amount of goods to be delivered by the government in period s, and ${}_{t-1}B_s \gtreqless 0$ denotes net claims on money to be delivered by the government in period s. From the point of view of the government in period t, ${}_{t-1}b_s$ and ${}_{t-1}B_s$ denote indexed and nominal debt service (the sum of maturing principal and interest payments) due in period s. Hence, ${}_{t-1}b \equiv \{{}_{t-1}b_s\}_{s=t}^{\infty}$ and ${}_{t-1}B \equiv \{{}_{t-1}B_s\}_{s=t}^{\infty}$ denote the maturity structure of the indexed and nominal government debt, respectively, that is outstanding at the beginning of period t.

The nominal interest rate between period t and $t+1$, i_{t+1}, is defined by[5]

$$\frac{1}{1+i_{t+1}} \equiv \frac{q_{\theta,t+1}/P_{t+1}}{q_{\theta,t}/P_t}. \tag{5}$$

Adding the period budget constraints (4) for $t \geq \theta$ and using (5), we can write the consumer's intertemporal budget constraint in period θ,[6]

$$\sum_{t=\theta}^{\infty} q_{\theta,t}(1-\tau_t)(1-x_t)+q_{\theta,\theta}M_{\theta-1}/P_\theta+\sum_{t=\theta}^{\infty} q_{\theta,t}({}_{\theta-1}b_t+{}_{\theta-1}B_t/P_t) \geq \sum_{t=\theta}^{\infty} q_{\theta,t}c_t+\sum_{t=\theta}^{\infty} q_{\theta,t+1}i_{t+1}m_{t+1}. \tag{6}$$

For given current and future present-value prices, interest rates, and taxes, and for given initial money stock and indexed and nominal claims on the government, optimal choices by the consumer

[4] As pointed out to us by a referee, the beginning-of-period convention is the only one that makes sense, if one views money in the utility function as shortcut for an explicit transaction technology: by the budget constraint, only nominal balances M_{t-1} are used to buy goods c_t at t. The assumption that beginning-of-period real balances give liquidity services is used, for instance, in Danthine and Donaldson [4].

[5] We surpress the dependence of i_{t+1} on θ. As is evident from equation (9) below, there is no such dependence in a consumer equlibrium.

[6] Throughout, we assume that the appropriate no-Ponzi-game and transversality conditions are fullfilled.

of $\{c_t, x_t, M_t\}_{t=\theta}^{\infty}$ result in the first-order conditions,

$$\begin{aligned} q_{\theta,t} &= \beta^{t-\theta} U_{ct}, & (7) \\ \tau_t &= 1 - \frac{U_{xt}}{U_{ct}}, & (8) \\ i_{t+1} &= \frac{U_{m,t+1}}{U_{c,t+1}} & (9) \end{aligned}$$

for $t \geq \theta$, where $U_{ct} \equiv \partial U(c_t, x_t, m_t)/\partial c_t$, and so forth, and we normalize present-value prices to units of utility in period θ.

The government in period t finances its exogenous consumption by taxing labor income, increasing the money supply, and net borrowing, given the initial money stock and the initial indexed and nominal debt. This implies a period-t budget constraint,

$$q_{t,t}\tau_t(1-x_t) + q_{t,t}(M_t - M_{t-1})/P_t + \sum_{s=t+1}^{\infty} q_{t,s}({}_t b_s + {}_t B_s/P_s) - \sum_{s=t}^{\infty} q_{t,s}({}_{t-1}b_s + {}_{t-1}B_s/P_s) - q_{t,t}g_t \geq 0, \tag{10}$$

where the third term is the value of the indexed and nominal debt held at the end of period t (beginning of period $t+1$). Multiplying by $\beta^{t-\theta}$, using (7), summing (10) for $t \geq \theta$, and using (5) result in the intertemporal budget constraint in period θ,

$$\sum_{t=\theta}^{\infty} q_{\theta,t}\tau_t(1-x_t) + \sum_{t=\theta}^{\infty} q_{\theta,t+1} i_{t+1} m_{t+1} - q_{\theta,\theta} M_{\theta-1}/P_\theta - \sum_{t=\theta}^{\infty} q_{\theta,t}({}_{\theta-1}b_t + {}_{\theta-1}B_t/P_t) - \sum_{t=\theta}^{\infty} q_{\theta,t} g_t \geq 0. \tag{11}$$

3 Optimal policy under commitment

What is the optimal policy for a government that, in period θ, can decide on current and future taxes and money supplies, $\{\tau_t, M_t\}_{t=\theta}^{\infty}$, and commit future governments to implement these decisions? The government chooses these policy instruments to maximize the consumer's intertemporal utility, subject to its budget constraint, (11), the initial money stock, $M_{\theta-1}$, the initial indexed and nominal debt, ${}_{\theta-1}b$ and ${}_{\theta-1}B$, the economy's resource constraint, (1), and consumer optimization, represented by (7)–(9).

It is convenient to reformulate this problem such that government in period θ directly chooses the price level, P_θ, and the allocation of current and future consumption and real balances, $X_\theta \equiv \{c_t, m_{t+1}\}_{t=\theta}^{\infty}$, instead of the policy instruments, $\{\tau_t, M_t\}_{t=\theta}^{\infty}$: First, we use the binding resource constraint to eliminate x_t in the consumer's intertemporal utility function, and define the government's objective function in period θ as

$$V_\theta(P_\theta, X_\theta) \equiv U(c_\theta, 1 - g_\theta - c_\theta, M_{\theta-1}/P_\theta) + \sum_{t=\theta+1}^{\infty} \beta^{t-\theta} U(c_t, 1 - g_t - c_t, m_t).$$

Second, we use the resource constraint to eliminate x_t and write the government's budget constraint in period θ as

$$\sum_{t=\theta}^{\infty} q_{\theta,t}[\tau_t(c_t+g_t) - g_t - {}_{\theta-1}b_t] + \sum_{t=\theta+1}^{\infty} q_{\theta,t} i_t m_t - q_{\theta,\theta}\left(M_{\theta-1} + \sum_{t=\theta}^{\infty} Q_{\theta,t}\, {}_{\theta-1}B_t\right)/P_\theta \geq 0. \tag{12}$$

The term inside the parenthesis in the third term on the left side is the *net nominal liabilities* of the government in period θ. Dividing this by P_θ and multiplying by $q_{\theta,\theta}$ give the real present value (in units of utility) of the government's net nominal liabilities. Here, $Q_{\theta,t}$ denotes the nominal present value in period θ of one unit of money in period t,

$$\begin{aligned} Q_{\theta,\theta} &\equiv 1, \\ Q_{\theta,t} &\equiv \frac{q_{\theta,t}/P_t}{q_{\theta,\theta}/P_\theta} \equiv \prod\nolimits_{s=\theta+1}^{t} \frac{1}{1+i_s} \qquad (t \geq \theta+1). \end{aligned} \tag{13}$$

Next, we use the resource constraint to eliminate x_t in the first-order-conditions (7)–(9), take the additive separability of the utility function into account, and define the functions $q_{\theta,t} = q_{\theta,t}(c_t)$ and $\tau_t = \tau(c_t)$ for $t \geq \theta$, and $i_t = i(c_t, m_t)$ for $t \geq \theta+1$, according to[7] [8]

$$q_{\theta,t}(c_t) \equiv \beta^{t-\theta} U_c(c_t), \tag{14}$$

$$\tau(c_t) \equiv 1 - \frac{U_x(1-g_t-c_t)}{U_c(c_t)}, \tag{15}$$

$$i(c_t, m_t) \equiv \frac{U_m(m_t)}{U_c(c_t)}. \tag{16}$$

Finally, under the convention that $q_{\theta,t}$, τ_t, and i_t in (12) are functions of (c_t, m_t) and that $Q_{\theta,t}(X_\theta)$ is the function defined by (13) and (16), we can restate the problem for the government in period θ as

$$\max_{(P_\theta, X_\theta)} V_\theta(P_\theta, X_\theta) \tag{17}$$

subject to the implementability constraint,

$$W_\theta(P_\theta, X_\theta) \geq 0, \tag{18}$$

where we can interpret

$$\begin{aligned} W_\theta(P_\theta, X_\theta) \equiv& \sum_{t=\theta}^{\infty} q_{\theta,t}(c_t)[\tau(c_t)(c_t+g_t) - g_t - {}_{\theta-1}b_t] + \sum_{t=\theta+1}^{\infty} q_{\theta,t}(c_t) i(c_t, m_t) m_t \\ &- q_{\theta,\theta}(c_\theta)\left(M_{\theta-1} + \sum_{t=\theta}^{\infty} Q_{\theta,t}(X_\theta)\, {}_{\theta-1}B_t\right)/P_\theta, \end{aligned} \tag{19}$$

[7] Without the assumption of separability, the arguments $(c_t, 1-g_t-c_t, m_t)$ would enter in all derivatives of the utility function.

[8] From our assumption about concavity, twice continuous differentiability of the period utility function, and additive separability, the derivatives of the functions defined by (14)–(16) fulfill $\partial q_{\theta,t}/\partial c_t < 0$, $\partial \tau_t/\partial c_t < 0$, $\partial i_t/\partial c_t > 0$, $\partial i_t/\partial m_t < 0$.

as the generalized *net wealth* of the government in period θ. In equilibrium, the net wealth of the government will always be zero. We shall refer to an increase (decrease) in W_θ as a *slackening* (*tightening*) of the government's intertemporal budget constraint.

Thus, according to this reformulation, the government directly chooses the allocation $X_\theta = \{c_t, m_{t+1}\}_{t=\theta}^{\infty}$ and the initial price level, P_θ. The Lagrangian for the problem is

$$L_\theta = V_\theta(P_\theta, X_\theta) + \lambda_\theta W_\theta(P_\theta, X_\theta), \tag{20}$$

where $\lambda_\theta \geq 0$ is the Lagrange multiplier of (18). The first-order conditions for an optimal policy in an equilibrium under commitment, the *Ramsey policy*, are

$$\frac{\partial V_\theta(P_\theta, X_\theta)}{\partial P_\theta} + \lambda_\theta \frac{\partial W(P_\theta, X_\theta)}{\partial P_\theta} = 0, \tag{21}$$

$$\frac{\partial V_\theta(P_\theta, X_\theta)}{\partial c_t} + \lambda_\theta \frac{\partial W(P_\theta, X_\theta)}{\partial c_t} = 0 \qquad (t \geq \theta), \tag{22}$$

$$\frac{\partial V_\theta(P_\theta, X_\theta)}{\partial m_t} + \lambda_\theta \frac{\partial W(P_\theta, X_\theta)}{\partial m_t} = 0 \qquad (t \geq \theta + 1), \tag{23}$$

with the complementary slackness condition

$$\lambda_\theta W_\theta(P_\theta, X_\theta) \geq 0.$$

We assume that the exogenous government consumption and the initial debt structure is such that $\lambda_\theta > 0$, so the government's intertemporal budget constraint is strictly binding. Then, the first-order conditions (21)–(23) together with (18) (with equality) determine P_θ, $\{c_t, m_{t+1}\}_{t=\theta}^{\infty}$, and λ_θ in the Ramsey equilibrium. The corresponding prices and interest rates $\{q_{\theta,t}, i_{t+1}\}_{t=\theta}^{\infty}$ are then determined by (14) and (16), and leisure $\{x_t\}_{t=\theta}^{\infty}$ by the binding resource constraint, (1). Given P_θ, the future price levels, $\{P_t\}_{t=\theta+1}^{\infty}$, then follow from (5). Finally, the policy instruments, $\{\tau_t, M_t\}_{t=\theta}^{\infty}$, are determined by (15) and (3).

Let $v_\theta(M_{\theta-1}, {}_{\theta-1}b, {}_{\theta-1}B, \{g_t\}_{t=\theta}^{\infty})$ denote the optimal value of this problem. By (19), (20), and the envelope theorem, we have

$$\frac{\partial v_\theta}{\partial\, {}_{\theta-1}b_t} = -\,\lambda_\theta q_{\theta,t}. \tag{24}$$

Evidently, we can interpret $\lambda_\theta \geq 0$ as the *marginal cost of public funds*, a measure of the distortion caused by taxation. If $\lambda_\theta = 0$, taxation is nondistortionary, as it would be if we allowed for lumpsum taxes.[9] We will only study equilibria where λ_θ is positive. Then, higher government indexed debt

[9] Note that, since the left side of (24) and $q_{\theta,t}$ on the right side both have the dimension of utility per good, λ_θ is defined such that it is a dimensionless number.

service to the private sector in period t requires an increase in taxation which reduces consumer utility, even though the consumer directly receives the debt payment.

The first-order conditions, (21)–(23), and the definition of W_θ, (19), illustrate that, in general, the Ramsey policy depends on the initial debt structure. This is because net government wealth depends on the market value of the outstanding debt and because the government's policy choices have an effect of the market value through its effect on nominal and real interest rates (present-value prices).[10] When the indexed and nominal debt service inherent in the initial maturity structure is not constant over time, the Ramsey policy does not generally prescribe constant taxes and interest rates over time, even if government spending is constant.

4 Time consistency under discretion

Consider now the situation when the government in office in any period t can reoptimize under discretion. As demonstrated by LS—and more recently by AKN—the Ramsey policy is, in general, time inconsistent under discretion, because the incentives to manipulate price levels and interest rates change over time. We now argue, as in PPS, that these incentives can be neutralized: by leaving a uniquely defined indexed and nominal debt structure, each government can induce the next one to implement the Ramsey policy, even if the next government reoptimizes under discretion.

Suppose the government in period θ (called government θ) has solved the Ramsey problem in the previous section for the optimal price level P_θ and allocation $\{c_t, m_{t+1}\}_{t=\theta}^{\infty}$, and the corresponding $\{q_{\theta,t}, i_{t+1}\}_{t=\theta}^{\infty}$, $\{P_t\}_{t=\theta+1}^{\infty}$, and $\{\tau_t, M_t\}_{t=\theta}^{\infty}$. It would like the government in the next period, government $\theta+1$, to choose the continuation of this Ramsey policy, when reoptimizing for given M_θ, ${}_\theta b$, and ${}_\theta B$. What debt structure, ${}_\theta b$ and ${}_\theta B$, should government θ leave to government $\theta+1$?

We can answer this question by fixing $P_{\theta+1}$ and $\{c_t, m_{t+1}\}_{t=\theta+1}^{\infty}$ at the values preferred by government θ and finding the debt structure that satisfies the first-order conditions (21)–(23) for government $\theta+1$. The first-order condition for $P_{\theta+1}$, (21), for government $\theta+1$ can be written

$$U_{m,\theta+1} M_\theta = \lambda_{\theta+1} q_{\theta+1,\theta+1} \left(M_\theta + \sum_{t=\theta+1}^{\infty} Q_{\theta+1,t\;\theta} B_t \right), \tag{25}$$

where $U_{m,\theta+1}$ denotes $U_m(M_\theta/P_{\theta+1})$ (without the assumption of additive separability, $c_{\theta+1}$ and $1-g_{\theta+1}-c_{\theta+1}$ would also enter as arguments). We assume that government θ knows $\lambda_{\theta+1} > 0$, the

[10] The real interest rate between period t and period $t+1$, r_{t+1}, will satisfy

$$\frac{1}{1+r_{t+1}} \equiv \frac{q_{\theta,t+1}}{q_{\theta,t}} = \frac{\beta U_c(c_{t+1})}{U_c(c_t)}.$$

cost of public funds for government $\theta+1$; we show below how it is determined. The left side of (25) corresponds to government $\theta+1$'s direct *marginal cost of unanticipated inflation* in period $\theta+1$, that is, an unanticipated rise in the price level, $P_{\theta+1}$. Unanticipated inflation lowers the real balances in the beginning of period $\theta+1$, $M_\theta/P_{\theta+1}$, in proportion to the given beginning-of-period money stock, M_θ. This imposes a marginal utility cost measured by the left side of (25). It is positive as long as the Ramsey policy chosen by government θ implies a positive value of $i_{\theta+1} = U_{m,\theta+1}/U_{c,\theta+1}$. The right side of (25) corresponds to government $\theta+1$'s *marginal benefit of unanticipated inflation.* Within parenthesis is the government's net nominal liabilities at the beginning of period $\theta+1$, the sum of the money stock and the nominal value of the nominal debt, the real value of which are eroded by an unanticipated rise in the price level. The resulting slackening of the government's intertemporal budget constraint allows the government to reduce the distortions due to labor taxes or anticipated inflation. Multiplication by the cost of public funds gives the corresponding increase in consumer utility. To satisfy condition (25) at the predetermined value of M_θ and thus eliminate the incentive for a surprise inflation, the value of the nominal debt, $\sum_{t=\theta+1}^{\infty} Q_{\theta+1,t\;\theta}B_t$, must be such that *net nominal liabilities are positive.*

Condition (25) can also be written as

$$\sum_{t=\theta+1}^{\infty} Q_{\theta+1,t\;\theta}B_t = -\,M_\theta\left(1-\frac{i_{\theta+1}}{\lambda_{\theta+1}}\right), \qquad (26)$$

where we have used (14) and (16). If $i_{\theta+1} < \lambda_{\theta+1}$, according to (26), government θ should leave government $\theta+1$ with *negative* nominal debt (positive nominal bond holdings), although less in absolute value than the money stock, so as to leave net nominal liabilities positive. If $i_{\theta+1} > \lambda_{\theta+1}$, government θ should leave government $\theta+1$ with *positive* nominal debt. The nominal debt is lower (the nominal bond holdings are larger) (i) the lower is the interest rate, $i_{\theta+1}$ (and thereby the cost of unanticipated inflation in (25), which is proportional to $U_{m,\theta+1}$ and $i_{\theta+1}$), and (ii) the higher is the cost of public funds, $\lambda_{\theta+1}$ (and thereby the benefit of unanticipated inflation in (25)).

The incentives to renege on $P_{\theta+1}$ and the way to neutralize them are quite easy enough to grasp. But the time consistency problem associated with the other policy instruments is more subtle. The first-order condition for m_t ($t \geq \theta+2$) for government $\theta+1$ is

$$\beta^{t-\theta-1}U_{mt} = \lambda_{\theta+1}\left(-\,q_{\theta+1,t}i_t - q_{\theta+1,t}m_t\frac{\partial i_t}{\partial m_t} + q_{\theta+1,\theta+1}\sum_{s=t}^{\infty} Q_{\theta+1,s\;\theta}B_s\frac{-\,\partial i_t/\partial m_t}{1+i_t}/P_{\theta+1}\right), \qquad (27)$$

where the derivative $\partial i_t/\partial m_t$ is the derivative of the function defined by (16) (without the assumption of additive separability, derivatives of $q_{\theta+1,t}$ and τ_t with respect to m_t would also enter), and where we use that

$$\begin{aligned}\frac{\partial Q_{\theta+1,s}}{\partial m_t} &= 0 \qquad (s<t, \quad t\geq\theta+2),\\ \frac{\partial Q_{\theta+1,s}}{\partial m_t} &= Q_{\theta+1,s}\frac{-\,\partial i_t/\partial m_t}{1+i_t} \qquad (s\geq t, \quad t\geq\theta+2).\end{aligned}$$

The left side of (27) is the direct *marginal benefit of increasing real balances* in period $t\geq\theta+2$. The bracketed term on the right side is the corresponding tightening of the government's budget constraint: the fall in the present value of the government's net wealth, due to a fall in seigniorage and a rise in the present value of the nominal debt because of a lower interest rate i_t (note that $\partial i_t/\partial m_t<0$ by footnote 8). Multiplication by $\lambda_{\theta+1}$, the cost of public funds, gives the *marginal cost of increasing real balances* in period t from the viewpoint of government $\theta+1$. As both the debt structure $\sum_{t=\theta+1}^{\infty} Q_{\theta+1,t}\,{}_{\theta}B_t$ and the cost of public funds, $\lambda_{\theta+1}$, generally take different values in period $\theta+1$ than in period θ, (27) generally implies a different value of m_t than the optimal value for government θ. To imply the same solution for $\{m_{t+1}\}_{t=\theta+1}^{\infty}$ (when we hold $\{c_t\}_{t=\theta+1}^{\infty}$ constant at the Ramsey values), it has to be that

$$\sum_{s=t}^{\infty} Q_{\theta+1,s}\,{}_{\theta}B_s = \frac{P_{\theta+1}}{q_{\theta+1,\theta+1}}\left(\frac{E_t}{\lambda_{\theta+1}}+F_t\right) \qquad (t\geq\theta+2), \tag{28}$$

where

$$E_t \equiv (1+i_t)\beta^{t-\theta-1}\frac{U_{mt}}{-\,\partial i_t/\partial m_t}, \tag{29}$$

$$F_t \equiv (1+i_t)\,q_{\theta+1,t}\left(\frac{i_t}{-\,\partial i_t/\partial m_t}-m_t\right). \tag{30}$$

Equation (28) determines the maturity structure ${}_{\theta}B_t$ for $t\geq\theta+2$ and equation (26) determines ${}_{\theta}B_{\theta+1}$. Therefore, we have determined the complete nominal debt structure for any value of $\lambda_{\theta+1}$. The equilibrium value of $\lambda_{\theta+1}$ is determined below.

In a similar vein, the first-order condition for c_t $(t\geq\theta+1)$ for government $\theta+1$ is

$$U_{c,\theta+1}-U_{x,\theta+1}=\lambda_{\theta+1}\left\{\begin{array}{l} -\,[\tau_{\theta+1}(c_{\theta+1}+g_{\theta+1})-g_{\theta+1}-\,{}_{\theta}b_{\theta+1}]\,\frac{\partial q_{\theta+1,\theta+1}}{\partial c_{\theta+1}}\\ -\,q_{\theta+1,\theta+1}[\tau_{\theta+1}+(c_{\theta+1}+g_{\theta+1})\frac{\partial\tau_{\theta+1}}{\partial c_{\theta+1}}]\\ +\,(M_{\theta}+\sum_{s=\theta+1}^{\infty}Q_{\theta+1,s}\,{}_{\theta}B_s)\frac{\partial q_{\theta+1,\theta+1}}{\partial c_{\theta+1}}/P_{\theta+1}\end{array}\right\}, \tag{31}$$

$$\beta^{t-\theta-1}(U_{ct}-U_{xt})=\lambda_{\theta+1}\left\{\begin{array}{l} -\,[\tau_t(c_t+g_t)+i_tm_t-g_t-\,{}_{\theta}b_t]\,\frac{\partial q_{\theta+1,t}}{\partial c_t}\\ -\,q_{\theta+1,t}[\tau_t+(c_t+g_t)\frac{\partial\tau_t}{\partial c_t}+m_t\frac{\partial i_t}{\partial c_t}]\\ +\,q_{\theta+1,\theta+1}\sum_{s=t}^{\infty}Q_{\theta+1,s}\,{}_{\theta}B_s\frac{-\,\partial i_t/\partial c_t}{1+i_t}/P_{\theta+1}\end{array}\right\} \qquad (t\geq\theta+2), \tag{32}$$

where the derivatives of $q_{\theta+1,t}$, τ_t, and i_t refer to the functions (14)–(16) (the same derivatives would enter also without the assumption of additive separability). The left side is the direct marginal utility gain of increasing c_t (and simultaneously reducing x_t). On the right side within the curly brackets is the marginal cost of tightening the government's intertemporal budget constraint, due to the changes in present-value prices, tax rates, and interest rates. How can we guarantee that these conditions imply time consistent choices for $\{c_t\}_{t=\theta+1}^{\infty}$? If we fix $c_{\theta+1}$ at its Ramsey value and the nominal debt structure at the value determined by (26) and (28), any (positive) $\lambda_{\theta+1}$ determines a unique ${}_\theta b_{\theta+1}$ that satisfies equation (31). Similarly, equation (32) determines ${}_\theta b_t$ for $t \geq \theta+2$. Using (25) and (28)–(30) to eliminate the nominal claims in (31) and (32), we can rewrite the equations for ${}_\theta b$ as

$$
\begin{aligned}
{}_\theta b_{\theta+1} &= \tau_{\theta+1}(c_{\theta+1}+g_{\theta+1}) - g_{\theta+1} - \frac{G_{\theta+1}}{\lambda_{\theta+1}} + H_{\theta+1}, && (33)\\
{}_\theta b_t &= \tau_t(c_t+g_t) - g_t + i_t m_t - \frac{G_t}{\lambda_{\theta+1}} + H_t \qquad (t \geq \theta+2), && (34)
\end{aligned}
$$

where

$$
\begin{aligned}
G_{\theta+1} &\equiv \frac{U_{c,\theta+1} - U_{x,\theta+1}}{-\,\partial q_{\theta+1,\theta+1}/\partial c_{\theta+1}} + \frac{U_{m,\theta+1} m_{\theta+1}}{q_{\theta+1,\theta+1}},\\
H_{\theta+1} &\equiv -\,q_{\theta+1,\theta+1} \frac{\tau_{\theta+1} - (c_{\theta+1}+g_{\theta+1})\left(-\,\partial \tau_{\theta+1}/\partial c_{\theta+1}\right)}{-\,\partial q_{\theta+1,\theta+1}/\partial c_{\theta+1}},\\
G_t &\equiv \beta^{t-\theta-1} \frac{U_{ct} - U_{xt} + U_{mt}\frac{\partial i_t/\partial c_t}{-\,\partial i_t/\partial m_t}}{-\,\partial q_{\theta+1,t}/\partial c_t} \qquad (t \geq \theta+2),\\
H_t &\equiv -\,q_{\theta+1,t} \frac{\tau_t - (c_t+g_t)\left(-\,\partial \tau_t/\partial c_t\right) + i_t \frac{\partial i_t/\partial c_t}{-\,\partial i_t/\partial m_t}}{-\,\partial q_{\theta+1,t}/\partial c_t} \qquad (t \geq \theta+2).
\end{aligned}
$$

Hence, equations (33) and (34) determine the indexed debt structure, ${}_\theta b$, that government θ should leave to government $\theta+1$.

Equations (26), (28), (33), and (34) pin down the incentive-compatible debt structure for government $\theta+1$, given its cost of public funds, $\lambda_{\theta+1}$. The last step of our solution is to ensure that, at the equilibrium value of $\lambda_{\theta+1}$, this debt structure is consistent with the budget constraints of governments θ and $\theta+1$. Thus, we find the value of $\lambda_{\theta+1}$ that makes the value of the total government debt (${}_\theta b$, ${}_\theta B$) consistent with the budget constraint of government $\theta+1$, which in turn makes it consistent with the budget constraint of government θ. To do that, we subtract ${}_\theta b_{\theta+1}$ and ${}_\theta b_t$ from both sides of (33) and (34), respectively, multiply by $q_{\theta+1,\theta+1}$ and $q_{\theta+1,t}$, sum for $t \geq \theta+1$, and write the result as

$$
0 = \left\{ \sum_{t=\theta+1}^{\infty} q_{\theta+1,t}[\tau_t(c_t+g_t) - g_t - {}_\theta b_t] + \sum_{t=\theta+2}^{\infty} q_{\theta+1,t} i_t m_t \right\} - \frac{\sum_{t=\theta+1}^{\infty} q_{\theta+1,t} G_t}{\lambda_{\theta+1}} + \sum_{t=\theta+1}^{\infty} q_{\theta+1,t} H_t.
$$

We then use the budget constraint (12) with equality to replace the term in curly brackets by

$$q_{\theta+1,\theta+1}\left(M_\theta+\sum_{t=\theta+1}^{\infty}Q_{\theta+1,t\ \theta}B_t\right)/P_{\theta+1}.$$

This ensures that the cost of public funds and the debt structure are consistent with the budget constraint of government θ. We finally use (25) to replace this term and get the expression

$$\frac{U_{m,\theta}m_{\theta+1}}{\lambda_{\theta+1}}-\frac{\sum_{t=\theta+1}^{\infty}q_{\theta+1,t}G_t}{\lambda_{\theta+1}}+\sum_{t=\theta+1}^{\infty}q_{\theta+1,t}H_t=0.$$

Solving for $\lambda_{\theta+1}$ gives

$$\lambda_{\theta+1}=\frac{\sum_{t=\theta+1}^{\infty}q_{\theta+1,t}G_t-U_{m,\theta+1}m_{\theta+1}}{\sum_{t=\theta+1}^{\infty}q_{\theta+1,t}H_t}. \tag{35}$$

This establishes our main result: Given the equilibrium cost of public funds determined by (35), equations (26), (28), (33), and (34) determine the unique nominal and indexed debt structure that induces government $\theta+1$ to implement the Ramsey policy under discretion.

A working-paper version of this paper (Persson, Persson, and Svensson, [12]) contains two numerical examples that illustrate the results in this section.[11]

5 Relation to earlier work

Persson, Persson, and Svensson (1987) PPS assumed that end-of-period real balances enter the period utility function. That is, the period utility function is $U(c_t,x_t,\tilde{m}_t)$, where

$$\tilde{m}_t\equiv M_t/P_t \tag{36}$$

denotes end-of-period real balances. The objective function for government θ becomes

$$\tilde{V}_\theta(\tilde{X}_\theta)\equiv\sum_{t=\theta}^{\infty}\beta^{t-\theta}U(c_t,1-g_t-c_t,\tilde{m}_t),$$

where $\tilde{X}_\theta\equiv\{c_t,\tilde{m}_t\}_{t=\theta}^{\infty}$. Importantly, the objective function no longer depends directly on the price level in period θ, P_θ. This means that unanticipated inflation has no direct effect on consumer utility, only an indirect effect via the government's intertemporal budget constraint and changes in the real value of the government's nominal liabilities and distortionary taxation.

The consumer's intertemporal budget constraint becomes

$$\sum_{t=\theta}^{\infty}q_{\theta,t}(1-\tau_t)(1-x_t)+q_{\theta,\theta}M_{\theta-1}/P_\theta+\sum_{t=\theta}^{\infty}q_{\theta,t}({}_{\theta-1}b_t+{}_{\theta-1}B_t/P_t)\geq\sum_{t=\theta}^{\infty}q_{\theta,t}c_t+\sum_{t=\theta}^{\infty}q_{\theta,t}\frac{i_{t+1}}{1+i_{t+1}}\tilde{m}_t,$$

[11] The working-paper version and Matlab programs for the examples are available at www.princeton.edu/~svensson/.

where we use (5) and (36). Optimal consumer choices lead to the first-order conditions (7) and (8) with $q_{\theta,t}$ and τ_t, so the functions $q_{\theta,t} = q_{,t}(c_t)$ and $\tau_t = \tau(c_t)$ are still given by (14) and (15). However, the first-order condition (9) with i_{t+1} is replaced by

$$\frac{i_{t+1}}{1+i_{t+1}} = \frac{U_m(\tilde{m}_t)}{U_c(c_t)}. \tag{37}$$

Thus, the function $i_t = i(c_t, m_t)$ for $t \geq \theta + 1$ defined by (16) is replaced by $i_{t+1} = \tilde{\imath}(c_t, \tilde{m}_t)$ for $t \geq \theta + 1$ defined by (37), and the function $Q_{\theta,t}(X_\theta)$ is replaced by $Q_{\theta,t}(\tilde{X}_\theta)$ defined as in (13) and (37). The net wealth of government θ satisfies

$$\begin{aligned} \tilde{W}_\theta(P_\theta, \tilde{X}_\theta) \;\; \equiv \;\; & \sum_{t=\theta}^{\infty} q_{\theta,t}(c_t)[\tau(c_t)(c_t + g_t) - g_t - {}_{\theta-1}b_t] + \sum_{t=\theta}^{\infty} q_{\theta,t}(c_t) \frac{\tilde{\imath}(c_t, \tilde{m}_t)}{1+\tilde{\imath}(c_t, \tilde{m}_t)} \tilde{m}_t \\ & - q_{\theta,t}(c_\theta) \left(M_{\theta-1} + \sum_{t=\theta}^{\infty} Q_{\theta,t}(\tilde{X}_\theta)\, {}_{\theta-1}B_t \right) / P_\theta. \end{aligned} \tag{38}$$

The optimization problem of government θ can be written as

$$\max_{P_\theta, \tilde{X}_\theta} \tilde{V}_\theta(\tilde{X}_\theta) \;\; \text{subject to} \tag{39}$$

$$\tilde{W}_\theta(P_\theta, \tilde{X}_\theta) \geq 0, \tag{40}$$

with the following first-order conditions for an optimum:

$$\lambda_\theta \frac{\partial \tilde{W}(P_\theta, \tilde{X}_\theta)}{\partial P_\theta} = 0, \tag{41}$$

$$\frac{\partial \tilde{V}_\theta(\tilde{X}_\theta)}{\partial c_t} + \lambda_\theta \frac{\partial \tilde{W}(P_\theta, \tilde{X}_\theta)}{\partial c_t} = 0 \qquad (t \geq \theta), \tag{42}$$

$$\frac{\partial \tilde{V}_\theta(\tilde{X}_\theta)}{\partial m_t} + \lambda_\theta \frac{\partial \tilde{W}(P_\theta, \tilde{X}_\theta)}{\partial m_t} = 0 \qquad (t \geq \theta). \tag{43}$$

In this case, the first-order condition for the initial price level of the subsequent government, $P_{\theta+1}$, (41), boils down to

$$M_\theta + \sum_{t=\theta}^{\infty} Q_{\theta+1,t\ \theta}B_t = 0. \tag{44}$$

Compared to (25), the direct utility effect of unanticipated inflation is missing. The first-order condition states what PPS proposed, namely that government θ should leave government $\theta + 1$ with positive nominal bond holdings (that is, $\sum_{t=\theta+1}^{\infty} Q_{\theta+1,t\ \theta}B_t$ negative) equal in value to the money stock such that the net nominal liabilities of government $\theta + 1$ are zero.

Calvo and Obstfeld (1990) Although the condition (44) appears simple and intuitive, CO showed, via an informal variation argument, that it actually does not correspond to an optimum. For given $P_{\theta+1}$, they considered a small deviation $\Delta\tilde{X}_{\theta+1}$ that leaves the objective function unchanged, $\frac{\partial V_{\theta+1}}{\partial \tilde{X}_{\theta+1}}\Delta\tilde{X}_{\theta+1} = 0$, but, via changes in the interest rates $i(c_s, \tilde{m}_s)$ for some $s \geq \theta + 1$, changes the term

$$\sum_{t=\theta+1}^{\infty} Q_{\theta+1,t}(\tilde{X}_{\theta+1})\, {}_{\theta}B_t = {}_{\theta}B_{\theta+1} + \sum_{t=\theta+2}^{\infty} \left({}_{\theta}B_t \prod_{s=\theta+1}^{t-1} \frac{1}{1+\tilde{\imath}(c_s, \tilde{m}_s)} \right), \tag{45}$$

so as to make the government's net nominal liabilities negative (positive). Given negative (positive) net nominal liabilities, the government can increase $\tilde{W}_{\theta+1}$ and slacken the government's intertemporal budget constraint by decreasing (increasing) $P_{\theta+1}$. This, in turn, allows the government to adjust $\tilde{X}_{\theta+1}$ to use up that slack and increase $\tilde{V}_{\theta+1}$. Consequently, the initial situation cannot be an optimum.

Note that this argument crucially hinges on unanticipated inflation having no direct effect on consumer utility. If $\tilde{V}_{\theta+1}$ would depend directly on $P_{\theta+1}$, as when beginning-of-period real balances enter into the utility function, the CO argument no longer goes through, as noted in Persson, Persson, and Svensson [11].

Alvarez, Kehoe, and Neumeyer (2004) AKN considered the same model with end-of-period real balances. In particular, they made assumptions on consumer preferences such that the Ramsey policy in period θ satisfies the Friedman rule, $i_{t+1} = 0$ $(t \geq \theta)$. They assumed that consumption and real balances are *weakly homogeneously separable* from leisure,

$$U(c_t, x_t, \tilde{m}_t) \equiv u(w(c_t, \tilde{m}_t), x_t), \tag{46}$$

where $w(c_t, \tilde{m}_t)$ is homothetic (and with no loss of generality can be assumed to be homogeneous). This implies that consumption and leisure are *quasi-separable* from leisure: the marginal rate of substitution between consumption and real balances along a given ray in the real balance–consumption plane is independent of leisure *along a given indifference surface*, that is, for a given utility level. Deaton [5] has shown that quasi-separability of a group of goods implies that uniform tax *rates* on the (constant) production costs of these goods are optimal.[12] The optimal *tax* on real balances is then the product of the optimal tax rate and the production cost of real balances. Since the production cost of real balances is assumed to be zero and the optimal tax rate is bounded, it

[12] The working-paper version of this paper (Persson, Persson, and Svensson [12]) gives an explicit example of a utility function for which the uniform-taxation result does not apply.

follows that the optimal tax on real balances is zero. Since we can interpret $\frac{i_t}{1+i_t}$ as the tax on real balances, the Friedman rule follows.[13]

Under the assumption of a satiation point for real balances (whatever the real allocation), we thus have

$$i_{t+1} = \tilde{\imath}(c_t, \tilde{m}_t) = U_{\tilde{m}}(c_t, 1 - g_t - c_t, \tilde{m}_t) = 0 \qquad (t \geq \theta) \tag{47}$$

for the optimal allocation $\tilde{X}_\theta = \{c_t, \tilde{m}_t\}_{t=\theta}^{\infty}$. Under the assumption that the period utility function is weakly increasing in $\tilde{m}_t$ and twice continuously differentiable, it also follows that $\tilde{U}_{\tilde{m}\tilde{m}} = 0$ and, by (37),

$$\frac{\partial i_{t+1}}{\partial c_t} = \frac{\partial i_{t+1}}{\partial \tilde{m}_t} = 0, \tag{48}$$

when (47) holds.

As in PPS, the first-order condition for government $\theta+1$ for $P_{\theta+1}$, (41), is only satisfied when net nominal liabilities (at zero interest rates) are zero,

$$M_\theta + \sum_{t=\theta+1}^{\infty} {}_\theta B_t = 0. \tag{49}$$

AKN proposed that government θ imposes the following maturity structure on its successor (see below)

$$\begin{aligned} {}_\theta B_{\theta+1} &= -M_\theta, & (50)\\ {}_\theta B_t &= 0 \qquad (t \geq \theta+2), & (51) \end{aligned}$$

that is, government θ leaves only nominal bonds that mature in period $\theta+1$ and no longer-maturity nominal assets or liabilities. The first-order condition for $\tilde{m}_t$ for $t \geq \theta+1$, (43), is

$$\beta^{t-\theta-1} U_{\tilde{m}t} = \lambda_{\theta+1} \left\{ \begin{array}{l} -\, q_{\theta+1,t} \frac{i_{t+1}}{1+i_{t+1}} - q_{\theta+1,t} \tilde{m}_t \frac{\partial}{\partial \tilde{m}_t} \frac{i_{t+1}}{1+i_{t+1}} \\ +\, q_{\theta+1,\theta+1} \sum_{s=t+1}^{\infty} Q_{\theta+1,s}\, {}_\theta B_s \frac{-\partial i_{t+1}/\partial \tilde{m}_t}{1+i_{t+1}} / P_{\theta+1} \end{array} \right\}. \tag{52}$$

Under (47) and (48), all terms in (52) are zero, even if (51) is not satisfied. Finally, the first-order condition for c_t for $t \geq \theta+1$, (42), is

$$\begin{aligned} \beta^{t-\theta-1}(U_{ct} - U_{xt}) &= \lambda_{\theta+1} \left\{ \begin{array}{l} -\, [\tau_t(c_t+g_t) - g_t - {}_\theta b_t] \frac{\partial q_{\theta+1,t}}{\partial c_t} \\ -\, q_{\theta+1,t}[\tau_t + (c_t+g_t)\frac{\partial \tau_t}{\partial c_t} + \tilde{m}_t \frac{\partial}{\partial c_t} \frac{i_{t+1}}{1+i_{t+1}}] \\ +\, q_{\theta+1,\theta+1} \sum_{s=t+1}^{\infty} Q_{\theta+1,s}\, {}_\theta B_s \frac{-\partial i_{t+1}/\partial c_t}{1+i_{t+1}} / P_{\theta+1} \end{array} \right\} \\ &= \lambda_{\theta+1} \left\{ \begin{array}{l} -\, [\tau_t(c_t+g_t) - g_t - {}_\theta b_t] \frac{\partial q_{\theta+1,t}}{\partial c_t} \\ -\, q_{\theta+1,t}[\tau_t + (c_t+g_t)\frac{\partial \tau_t}{\partial c_t}] \end{array} \right\} \qquad (t \geq \theta+1), \end{aligned} \tag{53}$$

[13] Teles [14] provides a survey of some results on the optimality of the Friedman rule and emphasizes the crucial role of the (near-)zero production costs of real balances for the separability and uniform-taxation assumptions to imply (approximately) the Friedman rule.

where, under the Friedman rule, the last line follows from (47) and (48). If (51) is satisfied, the term involving nominal debt on the right side is zero regardless of (48).

Condition (53) is equivalent to the first-order condition for c_t $(t \geq \theta+1)$ for government $\theta+1$ in a real economy without money, as in LS and Persson and Svensson [8]. It determines the indexed debt structure ${}_{\theta}b \equiv \{{}_{\theta}b_t\}_{t=\theta+1}^{\infty}$ that ensures time consistency under discretion of the optimal policy under commitment. Moreover, the conditions (50) and (51) make net nominal liabilities zero and eliminate any nominal bonds with maturity longer than one period. The condition of zero net nominal liabilities removes any incentive for surprise inflation or deflation. Furthermore, the condition of no long nominal debt implies that the informal variation argument CO used for PPS does not apply, because it requires nominal debt of longer maturity than one period.

AKN explicitly assumed that government θ must have inherited zero net nominal liabilities from government $\theta - 1$, and so forth. Indeed, the first government in history that computes the Ramsey policy must have initial net nominal liabilities at all maturities equal to zero. If the initial net nominal liabilities are not all zero, the initial government would find it optimal to manipulate the initial price level directly, or along the lines of the CO variational argument. If initial net nominal liabilities are negative, by lowering the initial price level the government can effectively impose a sufficient lumpsum tax instead of distortionary labor taxes. In this case, the Ramsey policy would be trivial, as the government would not need to impose any distortions when raising revenue. If initial net nominal liabilities are positive, the government would attempt to increase the price level beyond any finite level, so as to reduce the real value of those liabilities to zero. Obviously, the condition of *zero* net nominal liabilities at all maturities is very strong. In our case with beginning-of-period real balances and a direct utility cost of surprise inflation, by contrast, a nontrivial Ramsey policy requires only that the first government's initial net nominal liabilities be *positive*, which they usually are in the real world.

As AKN observed, under the Friedman rule, the economy essentially becomes a real economy at the Ramsey optimum. On the margin, money does not supply any transactions services and is just a store of value in the same way as indexed bonds. Since anticipated inflation does not raise any revenue, the only meaningful tradeoff in the government's optimal tax problem concerns labor tax distortions at different points in time. But the empirically relevant case for many countries and periods is a genuinely monetary economy where the inflation tax is a source of some revenue to be traded off against other distorting means of raising revenue. With beginning-of-period real balances and a direct utility cost of surprise inflation, we can find conditions for time consistent

Ramsey policies in such economies, as demonstrated by our analysis in sections 3–4.

6 Conclusion

Earlier work by Calvo [2], Lucas and Stokey [6], Calvo and Obstfeld [3]), and Alvarez, Kehoe, and Neumeyer [1] suggests that time inconsistency of Ramsey policies in monetary economies is either unavoidable, or avoidable only in environments where the Friedman rule is optimal so that the monetary economy is isomorphic to a real economy.

In contrast, and in line with Persson, Persson, and Svensson's [10] unpublished extension of Persson, Persson, and Svensson [9], we show that time consistency of Ramsey policies is possible also in economies where monetary policy plays a significant role and positive interest rates optimally raise some revenue. Time consistency of the Ramsey policy requires an active debt-management policy, where each government leaves to its successor a unique maturity structure of the nominal and indexed debt. The Ramsey policy may very well entail non-constant interest rates, inflation, and taxes, even if private preferences and endowments and government consumption are constant.

We show these results in a model where agents derive liquidity services from the money balances held at the beginning, rather than the end, of any time period. More generally, the critical assumption is that unanticipated inflation, realistically, imposes some direct cost on the private sector.

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